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SAFE BUILDING.

SAFE BUILDING

A TREATISE

GIVING IN THE SIMPLEST FORMS POSSIBLE

THE PRACTICAL AND THEORETICAL RULES AND FORMULÆ USED IN THE CONSTRUCTION OF BUILDINGS

BY

LOUIS DE COPPET BERG, F. A. I. A.

ASSOCIATE AMERICAN SOCIETY OF CIVIL ENGINEERS

VOLUME ONE

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To

MY FRIEND AND PARTNER,

J. CLEVELAND CADY, M.A., F.A.I.A.,

THIS BOOK IS AFFECTIONATELY DEDICATED

BY THE WRITER.

2016057

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SAFE BUILDING.

INTRODUCTORY REMARKS.

IN the articles on this subject the writer proposes to furnish to any earnest student the opportunity to acquire, so far as books will teach, the knowledge necessary to erect *safely* any building. While, of course, the work will be based strictly on the science of mechanics, all useless theory will be avoided. The object will be to make the articles simply practical. To follow any of the mathematical demonstrations, arithmetic and a rudimentary knowledge of algebra and plane geometry will be sufficient.

The following outline will probably give a better idea of the work proposed: —

First will come an introductory chapter on the "Strength of Materials." This chapter will give the values of, and explain briefly, the different terms used, such as strain, stress, factor-of-safety, centre of gravity, neutral axis, moment of inertia, centre and radius of gyration, moment of resistance, and moduli of elasticity and rupture.

Then will follow the several formulæ to be used, with explanations giving their applications, viz.: compression in long and short columns; wrinkling strains and lateral flexure in top chords of girders and beams; tension and shearing strains; transverse strains, including rupture, deflection and bending moments in cantilevers and beams; parallelogram of forces and graphical method of calculating trusses and arches; also manner of obtaining amounts of loads. Accompanying the above will be the necessary tables used in calculations.

After this introductory chapter will follow a series of chapters, each dealing with some part of a building, giving practical advice and numerous examples of calculations of strength; for instance, chapters on foundations, walls and piers, columns, beams, riveted and other girders, cast-iron lintels, roof and other trusses, spires; masonry, inverted and floor arches, corrugated iron, stairs, sidewalks, chimneys, etc., and possibly also chapters on drainage, plumbing, heating and ventilating.

CHAPTER I.

STRENGTH OF MATERIALS.

(German, *Festigkeit;* French, *Résistance des matériaux.*)

ALL solid bodies or materials are made up of an infinite number of atoms, fibres or molecules. These adhere to each other and resist separation with more or less tenacity, varying in different materials. This tenacity or tendency of the fibres to resume their former relation to each other after the strain is removed is called the elasticity of the material. It is when this elasticity is overcome that the fibres separate, and the material breaks and gives way.

There are to be considered in calculating strengths of materials two kinds of forces, viz., the external (or applied) forces and the internal (or resisting) forces. The external forces are any kind of forces applied to a material and tending to disrupt or force the fibres apart. Thus a load lying apparently perfectly tranquil on a beam is really a very active force; for the earth is constantly attracting the load, which tends to force its way downwards by gravitation and push aside the fibres of the beam under it. These latter, however, resist separation from each other, and the amount of the elasticity of all these fibres being greater than the attraction of the earth, the load is unable to force its way downwards and remains apparently at rest.

Strain. The amount of this tendency to disrupt the fibres (produced by the external forces) at any point is called the "strain" at that point.

Stress. The amount of the resistance against disruption of the fibres at such point is called the "stress" at that point.

External (or applied) forces, then, produce *strains*. Internal (or resisting) forces produce *stresses*.

This difference must be well understood and constantly borne in mind, as strains and stresses are the opposing forces in the battle of all materials against their destruction.

Ultimate Stress. When the strain at every point of the material just equals the stress, the material remains in equilibrium. The greatest stress, at any point of a material that it is capable of exerting is the ultimate stress (that is, the ultimate strength of resistance) at that point. Were the strain to exactly equal that ultimate stress, the material, though on the point of breaking, would still be

safe theoretically. But it is impossible for us to calculate so closely. Besides we can never determine accurately the actual ultimate stress, for different pieces of the same material vary in practice very greatly, as has been often proved by experiment. Therefore the actual ultimate stress might be very much less than that calculated. **Factor-of-Safety.**

Again, it is impossible to calculate the exact strain that will always take place at a certain point; the applied forces or some other conditions might vary. Therefore, to provide for all possible emergencies, we must make our material strong enough to be surely safe; that is, we must calculate (allow) for a considerably greater ultimate stress at every point than there is ever likely to be strain at that point.

The amount of extra allowance of stress varies greatly, according to circumstances and material. The number of times that we calculate the ultimate stress to be greater than the strain is called the factor-of-safety (that is, the ratio between stress and strain).

If the elasticity of different pieces of a given material is practically uniform, and if we can calculate the strain very closely in a given case, and further, if this strain is not apt to ever vary greatly, or the material to decay or deteriorate, we can of course take a low or small factor-of-safety; that is, the ultimate stress need not exceed many times the probable greatest strain.

On the other hand, if the elasticity of different pieces of a given material is very apt to vary greatly, or if we cannot calculate the strain very closely, or if the strain is apt to vary greatly at times, or the material is apt to decay or to deteriorate, we must take a very high or large factor-of-safety, that is, the ultimate stress must exceed many times the probable greatest strain.

Factors-of-safety are entirely a matter of practice, experience, and circumstances. In general, we might use for stationary loads:

A factor of safety of 3 to 4 for wrought-iron and steel,
" " " 6 for cast-iron,
" " " 4 to 10 for wood,
" " " 10 for brick and stone.

For moving-loads, such as people dancing, machinery vibrating, dumping of heavy loads, etc., the factor-of-safety should be one-half larger, or if the shocks are often repeated and severe, at least double of the above amounts. Where the constants to be used in formulæ are of doubtful authority (as is the case with most of them for woods and stones), the factor-of-safety chosen should be the highest one.

In building-materials we meet with four kind of strains, and, of course, with the four corresponding stresses resisting them, viz.: —

STRAINS.

Compression, or crushing strains,
Tension, or pulling strains,
Shearing, or sliding strains, and
Transverse, or cross-breaking strains.

STRESSES.

The resistance to *Compression*, or crushing-stress,
The resistance to *Tension*, or pulling-stress,
The resistance to *Shearing*, or sliding-stress, and
The resistance to *Transverse* strains, or cross-breaking stress.

Materials yield to *Compression* in three different ways: —

1. By direct crushing or crumbling of the material, or
2. By gradual bending of the piece sideways and ultimate rupture, or
3. By buckling or wrinkling (corrugating) of the material lengthwise.

Materials yield to *Tension*,

1. By gradually elongating (stretching), thereby reducing the size of the cross-section, and then,
2. By direct tearing apart.

Materials yield to *Shearing* by the fibres sliding past each other in two different ways, either

1. Across the grain, or
2. Lengthwise of the grain.

Materials yield to *Transverse* strains,

1. By deflecting or bending down under the load, and (when this passes beyond the limit of elasticity),
2. By breaking across transversely.

In calculating strains and stresses, there are certain rules, expressions, and formulæ which it is necessary for the student to understand or know, and which will be here given without attempting elaborate explanations or proofs. For the sake of clearness and simplicity, it is essential that in *all* formulæ the same letters should *always* represent the same value or meaning; this will enable the student to *read* every formula off-hand, without the necessity of an explanatory key to each one. The writer has further made it a habit to express, in *all* cases, his formulæ in pounds and inches (rarely using tons or feet); this will frequently make the calculation a little more elabo-

rate, but it will be found to greatly simplify the formulæ, and to make their *understanding* and *retention* more easy.

In the following articles, then, a capital letter, if it were used, would invariably express a quantity (respectively), either in tons or feet, while a small letter *invariably* expresses a quantity (respectively), either in pounds or inches.

The following letters, *in all cases*, will be found to express the same meaning, *unless distinctly otherwise stated*, viz.: —

a signifies *area*, in square inches.
b " *breadth*, in inches.
c " constant for *ultimate resistance to compression*, in pounds, per square inch. (See Tables IV and V.)
d signifies *depth*, in inches.
e " constant for *modulus of elasticity*, in pounds-inch, that is, pounds per square inch. (See Table IV.)
f " *factor of safety*.
g " constant for *ultimate resistance to shearing*, per square inch, across the grain. (See Tables IV and V.)
g_1 " constant for *ultimate resistance to shearing*, per square inch, lengthwise of the grain. (See Table IV.)
h " *height*, in inches.
i " *moment of inertia*, in inches. (See Table I.)
k " *ultimate modulus of rupture*, in pounds, per square inch. (See Tables IV and V.)
l " *length*, in inches.
m " *moment* or *bending moment*, in pounds-inch. (See Table IX.)
n " *constant* in Rankine's formula for compression of long pillars. (See Table II.)
o " the *centre*.
p " the *amount* of the *left-hand re-action* (or support) of beams, in pounds.
q " the *amount* of the *right-hand re-action* (or support) of beams, in pounds.
r " *moment of resistance*, in inches. (See Table I.)
s " *strain*, in pounds.
t " constant for *ultimate resistance to tension*, in pounds, per square inch. (See Tables IV and V.)
u " *uniform load*, in pounds.
v " *stress*, in pounds.
w " *load at centre*, in pounds.
x, *y*, and *z* signify *unknown quantities*, either in pounds or inches.

δ signifies *total deflection*, in inches.
ρ^2 " *square of the radius of gyration*, in inches.
$\mathfrak{D}$ " *diameter*, in inches.
$\mathfrak{r}$ " *radius*, in inches.
$\pi = 3.14159$, or, say, 3.1-7 signifies the *ratio of the circumference and diameter of a circle.*

If there are more than one of each kind, the second, third, etc., are indicated with Roman numerals, as for instance, a, a_{I}, a_{II}, a_{III}, etc., or b, b_{I}, b_{II}, b_{III}, etc.

In taking moments, or bending moments, strains, stresses, etc., to signify at what point they are taken, the letter signifying that point is added, as for instance: —

m signifies moment or bending moment at *centre.*
m_{A} " " " " *point A.*
m_{B} " " " " *point B.*
m_{X} " " " " *point X.*
s " strain at *centre.*
s_{B} " " *point B.*
s_{X} " " " *X.*
v " stress at *centre.*
v_{D} " " *point D.*
v_{X} " " *X.*
w signifies load at *centre.*
w_{A} " " " *point A.*

CENTRE OF GRAVITY.

(German, *Schwerpunkt;* French, *Centre de gravité.*)

Centre of Gravity. The centre of gravity of a figure, or body, is that point upon which the figure, or body, will balance itself in whatever position the figure or body may be placed, provided no other force than gravity acts upon the figure or body.

To find the centre of gravity of a plane figure, find two neutral axes, in different directions, and their point of intersection will be the centre of gravity required.

NEUTRAL AXIS.

(German, *Neutrale Achse;* French, *Axe neutre.*)

Neutral Axis. The neutral axis of a body, or figure, is an imaginary line upon which the body, or figure, will always balance, provided the body, or figure, is acted on by no other force than gravity. The neutral axis always passes through the centre of gravity, and may run in any direction. In calculating transverse strains, the neutral axis

designates an imaginary line of the body, or of the cross-section of the body, at which the forces of compression and tension meet. The strain on the fibres at the neutral axis is always naught. **Extreme Fibres.** On the upper side of the neutral axis the fibres are compressed, while those on the lower side are elongated. The amount of compression or elongation of the fibres increases directly as their distance from the neutral axis; the greatest strain, therefore, being in the fibres along the upper and lower edges, these being farthest from the neutral axis, and therefore called the *extreme fibres*. It is necessary to calculate only the ultimate resistance of these extreme fibres, as, if they will stand the strain, certainly all the other fibres will, they all being nearer the neutral axis, and consequently less strained. Where the ultimate resistances to compression and tension of a material vary greatly, it is necessary to so design the cross-section of the body, that the "extreme fibres" (farthest edge) on the side offering the weakest resistance, shall be nearer to the neutral axis than the "extreme fibres" (farthest edge), on the side offering the greatest resistance, the distance of the "extreme fibres" from the neutral axis being on each side in direct proportion to their respective capacities for resistance. Thus, in cast-iron the resistance of the fibres to compression is about six times greater than their resistance to tension; we must therefore so design the cross-section, that the distance of the neutral axis from the top-edge will be six-sevenths of the total depth, and its distance from the lower edge one-seventh of the total depth.

To find the neutral axis of any plane-figure, some writers recommend cutting, in stiff card-board, a duplicate of the figure (of which the neutral axis is sought), then to experiment until it balances on the edge of a knife, the line on which it balances being, of course, the neutral axis. This is an awkward and unscientific method of procedure, though there may be some cases where it will recommend itself as saving time and trouble. **How to find Neutral Axis.**

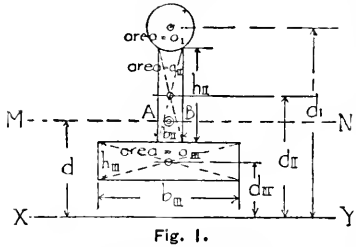


Fig. 1.

The following general formula, however, covers every case: To find the neutral axis *M-N* in any desired direction, draw a line *X-Y* at random, but parallel to the desired direction. Divide the figure into any number of *simple* figures, of which the areas and cen-

tres of gravity can be readily found, then the distance of the neutral axis M–N from the line X–Y will be equal to the *sum* of the *products*[1] of each of the small areas, multiplied by the distance of the centre of gravity of each area from X–Y, the whole to be divided by the entire area of the whole figure. An *Example* will make this more clear.

Find the horizontal neutral axis of the cross-section of a deck-beam, standing vertically on its bottom-flange.

Draw a line (X–Y) horizontally (Fig. 1), then let d_{I}, d_{II}, d_{III}, represent the respective distances from X–Y of the centres of gravity of the small subdivided *simple* areas a_{I}, a_{II}, a_{III}, then let a stand for the whole area of section, that is:—

$$a_{\text{I}} + a_{\text{II}} + a_{\text{III}} = a,$$

then the required distance (d) of the neutral axis M–N from X–Y, will be

$$d = \frac{a_{\text{I}}\, d_{\text{I}} + a_{\text{II}}\, d_{\text{II}} + a_{\text{III}}\, d_{\text{III}}}{a}.$$

To find the centre of gravity of the figure, we might find another neutral axis, but in a different direction, the point of intersection of the two being the required centre of gravity. But as the figure is uniform, we readily see that the centre of gravity of the whole figure must be half-way between points A and B.

Centres of Gravity. The centre of gravity of a circle is always its centre. The centre of gravity of a parallelogram is always the point of intersection of its two diagonals. The centre of gravity of a triangle is found by bi-secting two sides, and connecting these points each with its respective opposite apex of the triangle, the point of intersection of the two lines being the required centre of gravity, and which is always at a distance from each base equal to one-third of the respective height of the triangle. Any line drawn through either centre of gravity is a neutral axis.

MOMENT OF INERTIA.

(German, *Trägheitsmoment;* French, *Moment d'énertie,* or *Moment de giration.*)

Moment of Inertia. (See Table I.) The moment of inertia, sometimes called the moment of gyration, is the formula representing the inactivity (or state of rest) of any body rotating around any axis. The reason of the connection of this formula with the calculation of strains and the manner of obtaining it cannot be gone into here, as it would be quite beyond the scope of these articles. The moment of

[1] If line X–Y is inside of (bisects) figure, take sum of products on one side only and *deduct* sum of products on other side.

inertia of any body or figure is the *sum* of the products of each particle of the body or figure multiplied by the *square* of its distance from the axis around which the body or figure is rotating.

A table of moments of inertia, of various sections, will be given later on and will be all the student will need for practical purposes.

THE CENTRE OF GYRATION AND RADIUS OF GYRATION.

(German, *Trägheitsmittelpunkt;* French *Centre de giration.*)

The centre of gyration "is that point at which, if the whole mass of a body rotating around an axis or point of suspension were collected, a given force applied would produce the same angular velocity as it would if applied at the same point to the body itself." The distance of this centre of gyration from the axis of rotation is called the *radius of gyration* (German, *Trägheitshalbmesser;* French, *Rayon de giration*); this latter is used in the calculation of strains, and is found by dividing the moment of inertia of the body by the area or mass of the body, and extracting the square root of the quotient, or,

Square of Radius of Gyration. (See Table I.)

$$\varrho = \sqrt{\frac{i}{a}}, \text{ or}$$

$$\varrho^2 = \frac{i}{a}.$$

A table will be given, later on, of the "squares of the radius of gyration" (German, *Quadrat des Trägheitshalbmessers;* French, *Carré du rayon de giration*).

THE MOMENT OF RESISTANCE.

(German, *Widerstandsmoment;* French, *Moment de résistance*).

The moment of resistance of *any* fibre of a body, revolving around an axis, is equal to the moment of inertia of the whole body, divided by the distance of said fibre from the (neutral) axis, around which the body is revolving.

Moment of Resistance. (See Table I.)

A table of moments of resistance will be given later on.

MODULUS OF ELASTICITY.

(German, *Elasticitäts modulus*, French, *Module d'élasticité*).

The modulus of elasticity of a given material is the force required to elongate a piece of the material (whose area of cross-section is equal to one square inch) through space a distance equal to its primary length. Thus, if a bar of iron, twelve inches long, and of one square inch area of cross-section, could be made so elastic as to stretch to

Modulus of Elasticity. (See Table IV.)

twice its length, the force required to stretch it until it were twenty-four inches long would be its modulus of elasticity in weight per square inch.

MODULUS OF RUPTURE.

(German *Bruchcoefficient;* French, *Module de rupture.*)

Modulus of Rupture. (See Tables IV and V.)

It has been found by actual tests that though the different fibres of materials under transverse strains are either in compression or tension, the ultimate resistance of the "extreme fibres" neither entirely agrees with their ultimate resistance to compression nor tension. Attempts have been made to account for this in many different ways; but the fact remains. It is usual, therefore, where the cross-section of the material is uniform above and below the neutral axis, to use a constant derived from actual tests of each material, and this constant (which should always be applied to the "extreme fibres," *i. e.*, those along upper or lower edge) is called the modulus of rupture, and is usually expressed in pounds per square inch.

TO FIND THE MOMENT OF INERTIA OF ANY CROSS-SECTION.

How to find moment of inertia of any cross-section.

Divide the cross-section into simple parts, and find the moment of inertia of each simple part around its own neutral axis (parallel to main neutral axis); then, if we call the moment of inertia of the whole cross-section i, and that of each part i_{I}, i_{II}, i_{III}, i_{IIII}, etc., and, further, if we call the area of each part a_{I}, a_{II}, a_{III}, a_{IIII}, etc., and the distance of the centre of gravity of each part from the main neutral axis of the whole cross-section, d_{I}, d_{II}, d_{III}, d_{IIII}, etc., we have:—

$$i=(d_{\text{I}}{}^2a_{\text{I}}+i_{\text{I}})+(d_{\text{II}}{}^2a_{\text{II}}+i_{\text{II}})+(d_{\text{III}}{}^2a_{\text{III}}+i_{\text{III}})+(d_{\text{IIII}}{}^2a_{\text{IIII}}+i_{\text{IIII}})+, \text{ etc.}$$

Referring back to Figure 1, we should have for Part I:—

$$i_{\text{I}}=\frac{11}{14}r_{\text{I}}{}^4. \qquad \textbf{(See Table I., column 8.)}$$

For Part II we should have:—

$$i_{\text{II}}=\frac{b_{\text{II}}h_{\text{II}}{}^3}{12}$$

And for Part III:—

$$i_{\text{III}}=\frac{b_{\text{III}}h_{\text{III}}{}^3}{12}$$

For the distances of individual centres of gravity from main centre of gravity we should have for Part I: d_{I}–d.

For Part II: d_{II}–d.

And for Part III: d–d_{III}.

Therefore the moment of inertia, i, of the whole deck-beam would be:—

$$i=\left\{(d_{\mathrm{I}}-d)^2.a_{\mathrm{I}}+\frac{11}{14}r_{\mathrm{I}}^4\right\}+\left\{(d_{\mathrm{II}}-d)^2.a_{\mathrm{II}}+\frac{b_{\mathrm{II}}h_{\mathrm{II}}^3}{12}\right\}+$$

$$\left\{(d-d_{\mathrm{III}})^2.a_{\mathrm{III}}+\frac{b_{\mathrm{III}}h_{\mathrm{III}}^3}{12}\right\}$$

But $a_{\mathrm{I}}=\frac{22}{7}r_{\mathrm{I}}^2$

Further, $a_{\mathrm{II}}=b_{\mathrm{II}}\,h_{\mathrm{II}}$,

And $a_{\mathrm{III}}=b_{\mathrm{III}}\,h_{\mathrm{III}}$, which, inserted above, gives for

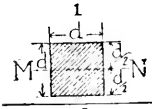
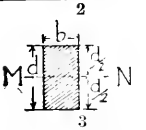
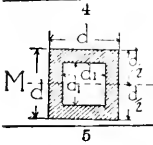
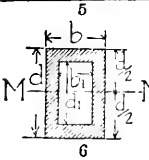
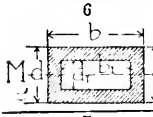
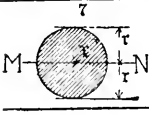
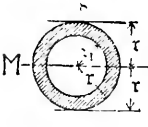
$$i=\frac{r_{\mathrm{I}}^2}{14}\left\{11r_{\mathrm{I}}^2+44(d_{\mathrm{I}}-d)^2\right\}+\frac{b_{\mathrm{II}}h_{\mathrm{II}}}{12}\left\{h_{\mathrm{II}}^2+12\,(d_{\mathrm{II}}-d)^2\right\}+$$

$$\frac{b_{\mathrm{III}}h_{\mathrm{III}}}{12}\left\{h_{\mathrm{III}}^2+12(d-d_{\mathrm{III}})^2\right\}$$

The following table (I) gives the values for the moment of inertia (i), moment of resistance (r), area (a), square of radius of gyration (ϱ^2), etc., for nearly every cross-section likely to be used in building. Those not given can be found from Table I by dividing the cross-section into several simpler parts, for which examples can be found in the table. Note, that it makes a great difference whether the neutral axis is located through the centre of gravity (of the part), or elsewhere. When making calculations we must, of course, insert in the different formulæ in place of i, r, a, ϱ^2, their values (for the respective cross-section), as given in Table I.

TABLE I.

DISTANCE OF EXTREME FIBRES, MOMENTS OF INERTIA AND RESISTANCE, SQUARE OF RADIUS OF GYRATION, AND AREAS OF DIFFERENT SHAPES OF CROSS-SECTIONS.

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r	Area. a.	Square of Radius of Gyration. ρ^2
1 	$\frac{d}{2}$	$\frac{d^4}{12}$	$\frac{d^3}{6}$	d^2	$\frac{d^2}{12}$
2  3	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	bd	$\frac{d^2}{12}$
4 	$\frac{d}{2}$	$\frac{d^4-d_1^4}{12}$	$\frac{d^4-d_1^4}{6d}$	$d^2-d_1^2$	$\frac{d^2+d_1^2}{12}$
5  6 	$\frac{d}{2}$	$\frac{bd^3-b_1d_1^3}{12}$	$\frac{bd^3-b_1d_1^3}{6d}$	$bd-b_1d_1$	$\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}$
7 	r	$\frac{11}{14}r^4$	$\frac{11}{14}r^3$	$\frac{22}{7}r^2$	$\frac{r^2}{4}$
	r	$\frac{11}{14}(r^4-r_1^4)$	$\frac{11}{14}\frac{(r^4-r_1^4)}{r}$	$\frac{22}{7}(r^2-r_1^2)$	$\frac{r^2+r_1^2}{4}$

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. i^2
9	$\frac{d}{2}$	$\frac{b_1 d^3 - b_1 d_1^3}{12}$	$\frac{b_1 d^3 - b_1 d_1^3}{6d}$	$b_1 d - b_1 d_1$	$\frac{b_1 d^3 - b_1 d_1^3}{12(b_1 d - b_1 d_1)}$
10					
11					
12					
13	$\frac{d}{2}$	$\frac{1}{12}(b_1 d^3 - b_1 d_1^3 - b_{11} d_{11}^3 - b_{111} d_{111}^3)$	$\frac{b_1 d^3 - b_1 d_1^3 - b_{11} d_{11}^3 - b_{111} d_{111}^3}{6d}$	$b_1 d - b_1 d_1 - b_{11} d_{11} - b_{111} d_{111}$	$\frac{1}{12} \cdot \frac{b_1 d^3 - b_1 d_1^3 - b_{11} d_{11}^3 - b_{111} d_{111}^3}{b_1 d - b_1 d_1 - b_{11} d_{11} - b_{111} d_{111}}$

Number and Form of Section.		
Distance of Neutral axis M.....N from extreme fibres.	$\frac{d}{2}$	$\frac{d}{2}$
Moment of Inertia i.	$\frac{1}{12}\left\{(b-z)d^3-b_1d_1^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{III})(d_{III}^3-(d_{III}-z)^3)\right\}$	$\frac{d^3(b-b_1)+b_1(d-d_1)^3}{12}$
Moment of Resistance. r.	$\frac{(b-z)d^3-b_1d_1^3-(b_{II}-z)d_{II}^3-b_{III}d_{III}^3-(b_{III}+b_{III})(d_{III}^3-(d_{III}-z)^3)}{6d}$	$\frac{d^3(b-b_1)+b_1(d-d_1)^3}{6d}$
Area. a.	$bd-b_1d_1-b_{II}d_{II}-b_{III}d_{III}-z(d-d_{II})-z(b_{III}+b_{III})$	$d(b-b_1)+b_1(d-d_1)$
Square of Radius of Gyration. ρ^2	$\frac{1}{12}\left\{bd^3-b_1d_1^3-b_{II}d_{II}^3-b_{III}d_{III}^3-z(d-d_{II})-z(b_{III}+b_{III})\right\}$	$\frac{d^3(b-b_1)+b_1(d-d_1)^3}{12}\left\{d^3(b-b_1)+b_1(d-d_1)^3\right\}$

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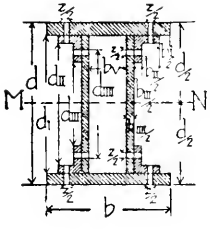
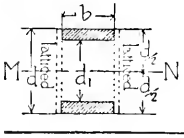
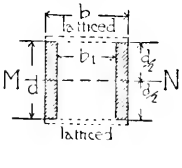
15

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. ρ^2
16 	$\frac{d}{2}$	$\frac{1}{12}\left\{(b-z)d^3-(b_1+b_v)d_1^3-(b_{11}-z)d_{11}^3-b_{111}d_{111}^3-(b_{111}+b_{1111})(\overline{d_{1111}}^3-(d_{1111}-z)^3)\right\}$	$\frac{(b-z)d^3-(b_1+b_v)d_1^3-(b_{11}-z)d_{11}^3-b_{111}d_{111}^3-(b_{111}+b_{1111})(\overline{d_{1111}}^3-(d_{1111}-z)^3)}{6d}$	$bd-(b_1+b_v)d_1-b_{11}d_{11}-b_{111}d_{111}-z(d-d_{11})-z(b_{111}+b_{1111})$	$\frac{(b-z)d^3-(b_1+b_v)d_1^3-(b_{11}-z)d_{11}^3-b_{111}d_{111}^3-(b_{111}+b_{1111})\cdot(\overline{d_{1111}}^3-(d_{1111}-z)^3)}{12\left\{bd-(b_1+b_v)d_1-b_{11}d_{11}-b_{111}d_{111}-z(d-d_{11})-z(b_{111}+b_{1111})\right\}}$
17 	$\frac{d}{2}$	$\frac{b}{12}(d^3-d_1^3)$	$\frac{b}{6d}(d^3-d_1^3)$	$b(d-d_1)$	$\frac{d^3-d_1^3}{12(d-d_1)}$
18 	$\frac{d}{2}$	$\frac{d^3}{12}(b-b_1)$	$\frac{d^2}{6}(b-b_1)$	$d(b-b_1)$	$\frac{d^2}{12}$

Number and Form of Section.				
Distance of Neutral axis M.....N from extreme fibres.	Lower Fibres $\frac{bd^2+b_1d_1\left(d+\frac{d_1}{2}\right)}{bd+b_1d_1}$	Upper Fibres. $\frac{b_1d_1^2+bd(d_1+\frac{d}{2})}{bd+b_1d_1}$	$x+y$ should be $=d+d_1$ and $x:y=c:t$, where c = ultimate resistance to compression, t = ultimate resistance to tension, per square inch.	
Moment of Inertia i.	$\frac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3}$			
Moment of Resistance. r.	Lower Fibres. $\frac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3y}$	Upper Fibres. $\frac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3x}$		
Area. a.	$bd+b_1d_1$			
Square of Radius of Gyration. ρ^2	$\frac{b_1x^3+by^3-(y-d)^3.(b-b_1)}{3(bd+b_1d_1)}$			

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21

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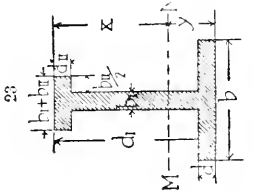
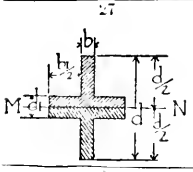
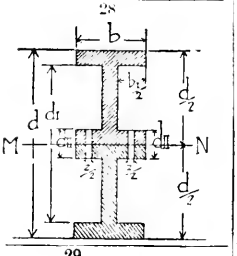
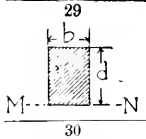
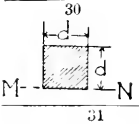
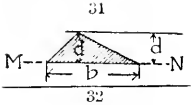
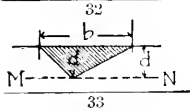
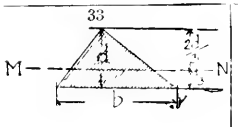
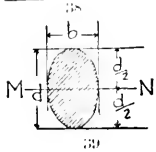
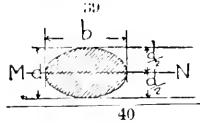
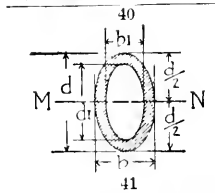
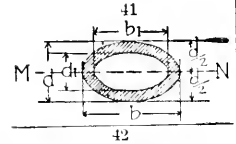
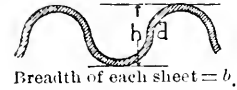
TABLE I, CONTINUED.

Square of Radius of Gyration. ρ^2	$\dfrac{b_1x^3+b_{11}\{x^3-(x-d_{11})^3\}+by^3-(b-b_1).(y-d)^3}{3(bd+b_1d_1+b_{11}d_{11})}$	
Area. a.	$bd+b_1d_1+b_{11}d_{11}$	
Moment of Resistance. r.	Lower Fibres. $\dfrac{b_1x^3+b_{11}\{x^3-(x-d_{11})^3\}+by^3-(b-b_1).(y-d)^3}{3y}$	Upper Fibres. $\dfrac{b_1x^3+b_{11}\{x^3-(x-d_{11})^3\}+by^3-(b-b_1).(y-d)^3}{3x}$
Moment of Inertia i.	$\dfrac{b_1x^3+b_{11}\{x^3-(x-d_{11})^3\}+by^3-(b-b_1).(y-d)^3}{3}$	
Distance of Neutral axis M.....N from extreme fibres.	Lower Fibres. $y=\dfrac{\frac{bd^2}{2}+b_1d_1(d+\frac{d_1}{2})+b_{11}d_{11}(d+d_1-\frac{d_{11}}{2})}{bd+b_1d_1+b_{11}d_{11}}$	Upper Fibres. $x=\dfrac{\frac{b_{11}d_{11}^2}{2}+b_1\frac{d_1^2}{2}+bd(d_1+\frac{d}{2})}{bd+b_1d_1+b_{11}d_{11}}$ where $x+y=d+d_1$ and $x:y=c:t$.
Number and Form of Section.		

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia. $i.$	Moment of Resistance. $r.$	Area. $a.$	Square of Radius of Gyration. ρ^2
24	$\frac{d}{2}$	$\frac{(l-z)r^3-(l_1-z)r_1^3-l_{11}r_{11}^3}{12}$	$\frac{(l-z)r^3-(l_1-z)r_1^3-l_{11}r_{11}^3}{6r}$	$(l-z)r-(l_1-z)r_1-l_{11}r_{11}$	$\frac{(l-z)r^3-(l_1-z)r_1^3-l_{11}r_{11}^3}{12\{(l-z)r-(l_1-z)r_1-l_{11}r_{11}\}}$
25	$\frac{d}{2}$	$\frac{(l-l_1)r^3-l_{11}r_{11}^3+(l_{11}-z)^3\{r_{11}+l_1r_1^3-l_{11}r_{11}^3\}}{12}$	$\frac{(l-l_1)r^3-l_{11}r_{11}^3+(l_{11}-z)^3\{r_{11}+l_1r_1^3-l_{11}r_{11}^3\}}{6l}$	$(l-l_1)r+l_1r_1-l_{11}r_{11}$	$\frac{(l-l_1)r^3-l_{11}r_{11}^3+(l_{11}-z)^3\{r_{11}+l_1r_1^3-l_{11}r_{11}^3\}}{12(l-l_1)r+l_1r_1-l_{11}r_{11}\}}$
26	$\frac{d}{2}$	$\frac{(l-l_1)(r^3-r_1^3)+l_1(r_{11}^3-r_1^3)}{12}$	$\frac{(l-l_1)(r^3-r_1^3)+l_1(r_{11}^3-r_1^3)}{6l}$	$(l-l_1)r+l_1(r_{11}-r_1)$	$\frac{(l-l_1)(r^3-r_1^3)+l_1(r_{11}^3-r_1^3)}{12\{(l-l_1)r+l_1(r_{11}-r_1)\}}$

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia i.	Moment of Resistance. r.	Area. a.	Square of Radius of Gyration. f^2
27 	$\frac{d}{2}$	$\frac{bd^3+b_1d_1^3}{12}$	$\frac{bd^3+b_1d_1^3}{6d}$	$bd+b_1d_1$	$\frac{bd^3+b_1d_1^3}{12(bd+b_1d_1)}$
28 	$\frac{d}{2}$	$\frac{bd^3-b_1(d_1^3-d_{11}^3)-zd_{11}^3}{12}$	$\frac{bd^3-b_1(d_1^3-d_{11}^3)-zd_{11}^3}{6d}$	$bd-b_1(d_1-d_{11})-zd_{11}$	$\frac{bd^3-b_1(d_1^3-d_{11}^3)-zd_{11}^3}{12(bd-b_1(d_1-d_{11})-zd_{11})}$
29 	d	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	bd	$\frac{d^2}{3}$
30 	d	$\frac{d^4}{3}$	$\frac{d^3}{3}$	d^2	$\frac{d^2}{3}$
31 	d	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{bd}{2}$	$\frac{d^2}{6}$
32 	d	$\frac{bd^3}{4}$	$\frac{bd^2}{4}$	$\frac{bd}{2}$	$\frac{d^2}{2}$
33 	Lower Fibres. $\frac{d}{3}$ Upper Fibres. $\frac{2}{3d}$	$\frac{bd^3}{36}$	Lower Fibres. $\frac{bd^2}{12}$ Upper Fibres. $\frac{bd^2}{24}$	$\frac{bd}{2}$	$\frac{d^2}{18}$

Number and Form of Section.	Distance of Neutral axis M.....N from extreme fibres.	Moment of Inertia k	Moment of Resistance. r_1	Area. a.	Square of Radius of Gyration. ρ^2
34	$\frac{b}{2} = \frac{b}{1.4142}$	$\frac{b^4}{12}$	$0.1179\ b^3$	b^2	$\frac{b^2}{12}$
35	$\frac{d}{2}$	$\frac{d^4}{12}\ \frac{11}{11}\ r^4$	$\frac{d^3}{6}\ \frac{11}{7}\ r^4$	$d^2 - \frac{22}{7} r^2$	$\frac{7 d^4 - 66 r^4}{12(7 d^2 - 22 r^2)}$
36	r	$\frac{11}{14} r^4 - \frac{d^4}{12}$	$\frac{11}{14} r^3 - \frac{d^4}{12 r}$	$\frac{22}{7} r^2 - d^2$	$\frac{66 r^4 - 7 d^4}{12(22 r^2 - 7 d^2)}$
37	$\frac{d}{2}$	$\frac{d^4}{20} + \frac{d_1^4}{12} + \frac{h(bb^2 + d^2 - d_1^2)}{12} - \frac{h}{12}\left\{d_1^3 - (d_1 - z)^3 - zb^2\right\}$	$\frac{d^4 - d_1^4}{10d} + \frac{h(bb^2 + d^2 - d_1^2)}{6d} - \frac{h}{6d}\left\{d_1^3 - (d_1 - z)^3 - zb^2\right\}$	$\frac{11}{14}(d^2 - d_1^2) + 2h(b - z)$	$\frac{7}{30}\left\{3(d^4 - d_1^4) + 5h(bb^2 + d^2 - d_1^2) - 5h\left\{d_1^3 - (d_1 - z)^3 - zb^2\right\}\right\}$ $\frac{11(d^2 - d_1^2) + 28h(b - z)}{11(d^2 - d_1^2) + 28h(b - z)}$

Number and Form of Section.	Distance of Neutral axis M - - - - N from extreme fibres.	Moment of Inertia i.	Moment of Resistance, r.	Area, a.	Square of Radius of Gyration. k^2
38  39 	$\frac{d}{2}$	$\frac{11}{224} bd^3$	$\frac{11}{112} bd^2$	$\frac{11}{14} bd$	$\frac{d^2}{16}$
40  41 	$\frac{d}{2}$	$\frac{11}{224}(bd^3-b_1d_1^3)$	$\frac{11}{112}\frac{(bd^3-b_1d_1^3)}{d}$	$\frac{11}{14}(bd-b_1d_1)$	$\frac{bd^3-b_1d_1^3}{16(bd-b_1d_1)}$
42 Corrugated Iron.  Breadth of each sheet $= b$.			$\frac{4}{15} dbh$		

CALCULATION OF STRAINS AND STRESSES.

As we have already noticed, the stress should exceed the strain as many times as the adopted factor-of-safety, or: —

$$\frac{\text{Stress}}{\text{Strain}} = \text{factor-of-safety.}$$

Or, stress = strain × factor-of-safety.

This holds good for all calculations, and can be expressed by the following simple fundamental formula: —

$$v = s.f \qquad (1)$$

Fundamental Formula.

Where v = the ultimate stress in pounds.

" s = " strain in pounds.

And where $f =$ the factor-of-safety.

COMPRESSION.

Compression, short columns. In pieces under compression the load is *directly* applied to the material. In short pieces, therefore, which cannot give sideways, the strain will just equal the load, or we have: —

$$s = w.$$

Where $s =$ the strain in pounds.

And where $w =$ the load in pounds.

The stress will be equal to the area of cross-section of the piece being compressed, multiplied by the amount of resistance to compression its fibres are capable of.[1] This amount of resistance to compression which its fibres are capable of is found by tests, and is given for each square inch cross-section of a material. A table of constants for the resistance to crushing of different materials will be given later on.

In all the formulæ these constants are represented by the letter c.

We have, then, for the stress of short pieces under compression: —

$$v = a.c$$

Where v is the ultimate stress in pounds.

Where a is the area of cross-section of the piece in inches.

And where c is the ultimate resistance to compression in pounds per square inch.

Inserting this value for v, and w for s in the fundamental formula (1), we have for short pieces under compression, which cannot yield sideways: —

$$a.c = w.f, \text{ or: —}$$

$$w = a.\left(\frac{c}{f}\right). \qquad (2)$$

Short Columns. Where $w =$ the *safe* total load in pounds.

Where $a =$ the area of cross-section in inches.

And where $\left(\frac{c}{f}\right) =$ the *safe* resistance to crushing per square inch.

Example.

What is the safe load which the granite cap of a 12″ x 12″ *pier will carry, the cap being twelve inches thick?*

The cap being only twelve inches high, and as wide and broad as

[1] This is not theoretically correct, as there is in every case a tendency for the material under compression to spread; but it is near enough for all practical purposes.

high, is evidently a short piece under compression, therefore the above formula (2) applies.

The area is, of course: $a = 12.12 = 144$ square inches.

The ultimate resistance of granite to crushing per square inch is, say, fifteen thousand pounds, and using a factor-of-safety of ten, we have for the safe resistance: —

$$\frac{c}{f} = \frac{15000}{10} = 1500 \text{ lbs.}$$

Therefore the safe load w on the block would be: —

$$w = 144.\ 1500 = 216000 \text{ pounds.}$$

Compression, Long Columns.

Where long pieces (pillars) are under compression, and are not secured against yielding sideways, it is evident they would be liable to bend before breaking. To ascertain the *exact* strain in such pieces is probably one of the most difficult calculations in strains, and in consequence many authors have advanced different theories and formulæ. The writer has always preferred to use Rankine's formula, as in his opinion it is the most reliable. According to this, the greatest strain would be at the centre of the length of the pillar, and would be equal to the load, plus an amount equal to the load multiplied by the square of the length in inches, and again multiplied by a certain constant, n, the whole divided by the "square of the radius of gyration" of the cross-section of the pillar. We have therefore for the total strain at the centre of long pillars: —

$$s = w + \frac{w.l^2 n}{\rho^2}$$

Where s = the strain in pounds.

" w = the total load in pounds.

" l = the length in inches.

" ρ^2 = the square of the radius of gyration of the cross-section.

" n = a constant, as follows: —

TABLE II.

VALUE OF n IN FORMULA FOR COMPRESSION.

Material of pillar.	Both ends of pillar smooth (turned or planed.)	One end smooth (turned or planed) other end a pin end.	Both ends pin ends.
Cast-iron	0.0003	0.0004	0.00057
Wrought-iron	0.000025	0.000033	0.00005
Steel	0.00002	0.000025	0.000033
Wood	0.00033	0.00044	0.00067
Stone	0.002		
Brick	0.0033		

The stress of course will be as before: —

$$v = a.\ c.$$

Where v = the ultimate stress in pounds.

" a = the area of cross-section in inches.

" c = the ultimate resistance to crushing per square inch.

Inserting the values for strain, s, and stress, v, in the fundamental formula (1) we have: —

$$c.a = \left(w + \frac{w.l^2.n}{\rho^2}\right).$$

or: —

$$a.\left(\frac{c}{f}\right) = w\left(1 + \frac{l^2 n}{\rho^2}\right)$$

or: —

$$w = \frac{a.\left(\frac{c}{f}\right)}{1 + \frac{l^2.n}{\rho^2}} \qquad (3)$$

Long Columns.

Where w = the *safe* total load on the pillar.

" a = the area of cross-section in inches.

" ρ^2 = the square of the radius of gyration of the cross-section.

" l = the length in inches.

" = the *safe* resistance to crushing per square inch.

Example.

What safe load will a 12″ x 12″ *brick pier carry, if the pier is ten feet long, and of good masonry?*

The area of cross-section will be: —

$$a = 12.12 = 144 \text{ square inches.}$$

The square of the radius of gyration according to Section No. 1 in Table I would be: —

$\frac{d^2}{12}$, and as $d = 12$, we have $\rho^2 = \frac{12.12}{12} = 12$

For the safe resistance to crushing per square inch, we have, using a factor-of-safety of ten, and considering the ultimate resistance to be 2,000 pounds per square inch,

$$\left(\frac{c}{f}\right) = \frac{2000}{10} = 200 \text{ lbs.}$$

The length will be ten feet, or one hundred and twenty inches; therefore: —

$$l^2 = 14400$$

For n we must use (according to Table II), for brickwork: —

$$n = 0.0033;$$

Therefore the safe total load on the pier would be: —

$$w = \frac{144.\ 200}{1 + \frac{14400.\ 0{,}0033}{12}} = \frac{28800}{1+3{,}96} = 5806 \text{ lbs.}$$

In all formulæ where constants and factors of safety are used, it will be found simpler and avoiding confusion to immediately reduce the constant by dividing it by the factor-of-safety, and then using only the reduced or *safe* constant.

Thus if $c = 48{,}000$ pounds, and if $f = 4$, do not write into your formula for $\left(\frac{c}{f}\right) = \frac{48000}{4}$, but use at once for $\left(\frac{c}{f}\right) = 12000$.

Materials in compression that have an even bearing on all parts of the bed will stand very much more compression to the square inch than materials with rough, uneven or rounded beds, or where the bearing is on part of the cross-section only, as in the case of pins (in trusses) bearing on eye-bars. It is usual in calculating to make allowance for this. Columns with perfectly even bearing on all parts of the bed (planed or turned iron or dressed stone) will stand the largest amount of compression. Columns with rough, rounded or uneven ends are calculated the same as for pin-ends of eye-bars. In the table (II) giving the values for n of Rankine's formula for compression, the different values for smooth and also for pin ends are given.

WRINKLING STRAINS.

Thin pieces of wrought-iron under compression endwise may neither crush nor deflect (bend), but give way by *wrinkling*, that is, buckling or corrugating, provided there are no stiffening-ribs lengthwise.

Fig. 2.

Thus a square, tubular column, if the sides are very thin might give way, as shown in Figure 2, which is called wrinkling. Or, in a similar way, the top plate of a boxed girder, if very thin, might wrinkle, as shown in Figure 3, under heavy compressive strains. To calculate this strain use the following formula:

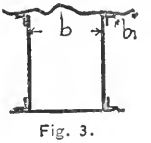


Fig. 3.

$$b = d.\left(\frac{w_r}{w}\right)^2 \qquad (4)$$

Where w = the amount of ultimate compression in pounds per square inch, which will wrinkle the material.

w_r = a constant,

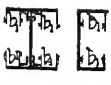
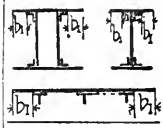
d = the thickness of plate in inches,

b = the unstiffened breadth of plate in inches.

If a plate has stiffening ribs along both edges, use for b the actual breadth between the stiffening ribs; if the plate is stiffened along one edge only, use $4b_1$ in place of b. Thus, in the case of the boxed girder, Figure 3, if we were considering the part of top plate between the webs, we should use for b in the formula, the actual breadth of b in inches; while, if we were considering the overhanging part b_1 of top plate, we should use $4b_1$ in place of b in formula. For rectangular columns use 160,000 pounds for w_r; for tubular beams, top plates of girders, and single plates use 200,000 pounds for w_r. With a factor-of-safety of 3, we should have $\frac{160000}{3} = 53000$ pounds for rectangular columns, and $\frac{200000}{3} = 66000$ pounds for tubular beams, top plates of riveted girders and single plates.

For w we shall use, of course, $\frac{36000}{3} = 12000$ pounds, which is the safe allowable compressive strain. This would give the following table for safe unstiffened breadth of wrought-iron plates, to prevent wrinkling of plates.

TABLE III.

Thickness of Plate in inches.	Safe breadth in inches of Plate stiffened along both edges. (use b.)		Safe breadth in inches of Plate stiffened along one edge only. (use $4b_1$.)	
	Rectangular Columns.	Tubular Beams, riveted Girders, and single Plates.	Rectangular Columns.	Riveted Girders and single Plates.
				
$\frac{1}{8}$	$2\frac{7}{16}$	$3\frac{3}{4}$	$\frac{5}{8}$	$1\frac{15}{16}$
$\frac{1}{4}$	$4\frac{1}{8}$	$7\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{7}{8}$
$\frac{3}{8}$	$7\frac{5}{16}$	$11\frac{3}{8}$	$1\frac{7}{8}$	$2\frac{13}{16}$
$\frac{1}{2}$	$9\frac{3}{4}$	$15\frac{1}{8}$	$2\frac{7}{16}$	$3\frac{3}{4}$
$\frac{5}{8}$	$12\frac{3}{16}$	$18\frac{7}{8}$	3	$4\frac{11}{16}$
$\frac{3}{4}$	$14\frac{5}{8}$	$22\frac{11}{16}$	$3\frac{11}{16}$	$5\frac{5}{8}$
$\frac{7}{8}$	$17\frac{1}{16}$	$26\frac{1}{2}$	$4\frac{1}{4}$	$6\frac{9}{16}$
1	$19\frac{1}{2}$	$30\frac{1}{4}$	$4\frac{7}{8}$	$7\frac{1}{16}$
$1\frac{1}{4}$	$24\frac{3}{8}$	$37\frac{13}{16}$	$6\frac{1}{8}$	$9\frac{7}{16}$
$1\frac{1}{2}$	$29\frac{1}{4}$	$45\frac{3}{8}$	$7\frac{5}{16}$	$11\frac{5}{16}$
$1\frac{3}{4}$	$34\frac{1}{8}$	$52\frac{15}{16}$	$8\frac{9}{16}$	$13\frac{3}{16}$
2	39	$60\frac{1}{2}$	$9\frac{3}{4}$	$15\frac{1}{8}$

The above table will cover every case likely to arise in buildings.

Two facts should be noticed in connection with wrinkling:

1. That the length of plate does not in any way affect the resistance to wrinkling, which is dependent only on the breadth and thickness of the part of plate unstiffened, and

2. That the resistance of plates to wrinkling being dependent on their breadth and thickness only, to obtain equal resistance to wrinkling at all points (in rectangular columns with uneven sides), the thickness of each side should be in proportion to its breadth.

Thus, if we have a rectangular column $30'' \times 15''$ in cross section and the $30''$ side is $1''$ thick, we should make the $15''$ side but $\frac{1}{2}''$ thick, for as $30'' : 1'' :: 15'' : \frac{1}{2}''$.

Of course, we must also calculate the column for direct crushing and flexure, and in the case of beams for rupture and deflection, as well as for wrinkling.

Example of Wrinkling.

It is desired to make the top plate of a boxed girder as wide as possible, the top flange is to be $1\frac{1}{4}''$ *thick, and is to be subjected to the full amount of the safe compressive strain, viz:* 12,000 *pounds per square inch; how wide apart should the webs be placed, and how much can the plate overhang the angles without danger of wrinkling? Each web to be* $\frac{1}{2}''$ *thick, and the angles* $4'' \times 4''$ *each?*

For the distance between webs we use b in Formula (4).

$$b = 1\tfrac{1}{4} \cdot \left(\frac{66000}{12000}\right)^2 = 1\tfrac{1}{4} \cdot 5\tfrac{1}{2}^2 = 37\tfrac{13}{16}'',$$

which is the safe width *between* webs to avoid wrinkling

For the overhanging part of top plate we must use $4b_1$ in place of b in Formula (4).

$$4b_1 = 1\tfrac{1}{4}\left(\frac{66000}{12000}\right)^2 = 37\tfrac{13}{16}, \text{ therefore,}$$

$$b_1 = \frac{37\frac{13}{16}}{4} = 9{,}453, \text{ or say, } b_1 = 9\tfrac{7}{16}''.$$

The total width of top plate will be, therefore, including $1''$ for two webs and $8''$ for the two angles, or $9''$, and remembering that there is an overhanging part, b_1, each side,

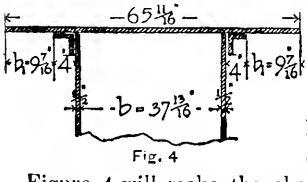


Fig. 4

$$9'' + b + b_1 + b_1$$
$$= 9 + 37\tfrac{13}{16} + 9\tfrac{7}{16} + 9\tfrac{7}{16}$$
$$= 65\tfrac{11}{16}''.$$

By referring to Table III, we should have obtained the same result, without the necessity of any calculation.

Figure 4 will make the above still more clear.

LATERAL FLEXURE IN TOP FLANGES OF BEAMS, GIRDERS, OR TRUSSES, DUE TO COMPRESSION.

The usual formulæ for rupture and deflection assume the beam, girder or truss to be supported against possible lateral flexure (bending sideways). Now, if the top chord of a truss or beam is comparatively narrow and not supported sideways, the heavy, compressive strains caused in same may bend it sideways. To calculate this lateral flexure, use the formula given for long columns in compression, but in place of l use only two-thirds of the span of the beam, girder or truss, that is $\frac{2}{3}l$, and for w use one-third of the greatest compressive strain in top chord, which is usually at the centre.

Inserting this in Formula (3) we have:

$$\frac{w}{3}=\frac{a\left(\frac{c}{f}\right)}{1+\frac{4l^2n}{9\varrho^2}} \text{ transposing, we have, } w=\frac{3a\left(\frac{c}{f}\right)}{1+\frac{4l^2n}{9\varrho^2}} \quad (5)$$

where a the area of the cross-section of the top chord in inches,

ϱ^2 is the square of the radius of gyration of the top chord around its *vertical axis;* we must therefore reverse the usual positions of b and d, that is the breadth of top chord, becomes the depth or d, and the depth of top chord becomes the thickness, or b (both in formulæ given in last column of Table I.)

$\frac{w}{3}$ is the greatest allowable compressive strain in pounds at any point to resist lateral flexure safely at that point.

$\left(\frac{c}{f}\right)$ is the safe resistance of the material to compression per square inch in pounds.

l is the total length of span in inches.

n is given in Table II.

Example.

A trussed girder is 60′ long between bearings, and not supported sideways; the top chord consists of two plates each 22″ deep and 1″ thick; the plates are 2″ apart, as per Figure 5. The greatest compressive strain on top chord has previously been ascertained to be on the central panel, and to be 525000 pounds. Is there danger of the girder bending sideways?

The girder is safe against lateral flexure so long as the strain at centre does not exceed $\frac{w}{3}$ in Formula (5).

Now, the area $a=2.1.22=44$.

Using 48000 pounds per square inch for ultimate resistance to compression of wrought-iron, and a factor-of-safety of 4, we have

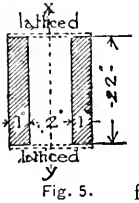


Fig. 5.

$$\left(\frac{c}{f}\right) = \frac{48000}{4} = 12000$$

The length is 60′, or 720″, therefore

$$l^2 = 518400.$$

From Table II we have

$$n = 0{,}000025.$$

And from Table I, section Number 16, we have for the above cross-section,

$$\varrho^2 = \frac{d^3 - d_1^3}{12(d - d_1)}$$

As we are considering the section for bending sideways, we must, of course, take the neutral axis $x \cdots y$ vertically, therefore d becomes 4″ and d_1 becomes 2″. This supposes the plates to be stiffly latticed or bolted together, with separators between. We have then

$$\varrho^2 = \frac{4^3 - 2^3}{12(4-2)} = 2\tfrac{1}{3}$$

Then for w we have,

$$w = \frac{3.44.12000}{1 + \frac{4.518400}{9.\ 2\frac{1}{3}}\ 0.000025.}$$

$$= \frac{1584000}{1+2{,}47} = \frac{1584000}{3{,}47} = 456\,484 \text{ lbs.}$$

Or, we find that there is danger of the girder bending sideways long before the actual compressive strain of 525000 pounds has been reached. It will, therefore, be necessary to re-design the top chord, so that it will be stiffer sideways. This subject will be more fully treated when considering trusses.

TENSION.

In tension the load is applied *directly* to the material, and it is, therefore, evident that no matter of what shape the material may be, the strain will always be the same. This strain, of course, will be just equal to the load, and we have, therefore:—

$$s = w.$$

Where $s =$ the amount of strain.

Where $w =$ the amount of load.

The weakest point of the piece under tension will, of course, be where it has the smallest area of cross-section; and the stress at

such point will be equal to the area of cross-section, multiplied by the amount of resistance its fibres are capable of.[1]

The amount of resistance to tension the fibres of a material are capable of is found by experiments and tests, and is given for each material per square inch of cross-section. A table of constants for the ultimate and safe resistances to tension of different materials will be given later; in all the formulæ these constants are represented by the letter t.

We have, then, for the stress:—

$$v = a.\ t$$

Where v = the amount of ultimate stress.

Where a = the area of cross-section.

Where t = the ultimate resistance to tension, per square inch of the material.

Therefore, the fundamental formula (1), viz.: $v = s.\ f$, becomes for pieces under tension:—

$$a.\ t = w.\ f, \text{ or :—}$$

$$w = a.\left(\frac{t}{f}\right) \qquad (6)$$

Where w = the safe load or amount of tension the piece will stand.

Where a = the area of cross-section at the weakest point (in square inches).

Where $\left(\frac{t}{f}\right)$ = the safe resistance to tension per square inch of the material.

Example.

A weight is hung at the lower end of a vertical wrought-iron rod, which is firmly secured at the other end. The rod is 3″ at one end and tapers to 2″ at the other end. How much weight will the rod safely carry?

The smallest cross-section of the rod, where it would be likely to break, would be somewhere very close to the 2″ end, or, say, 2″ in diameter. Its area of cross-section at this point will be:—

$$a = \frac{22}{7}\ .\ \frac{2^2}{4} = 3\tfrac{1}{7} \text{ square inches.}$$

The ultimate resistance to tension of wrought-iron per square inch is, from forty-eight thousand pounds to sixty thousand pounds. We do not know the exact quality, and, therefore, take the lower figure;

[1] This, again, is not theoretically correct, as a piece under tension is apt to stretch and so reduce the area of its cross-section; but the above is sufficiently correct for all practical purposes.

using a factor-of-safety of four, we have for the safe resistance to tension per square inch: —

$$\left(\frac{t}{f}\right) = \frac{48\,000}{4} = 12\,000 \text{ pounds.}$$

Therefore, the safe load will be: —

$$w = 3\tfrac{1}{4}.\quad 12\,000 = 37\,714 \text{ pounds.}$$

SHEARING.

In compression the fibres are shortened by squeezing; in tension they are elongated by pulling. In shearing, however, the fibres are not disturbed in their individualities, but slide past each other.

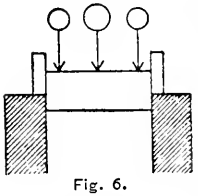


Fig. 6.

When this sliding takes place across the grain of the fibres, the action of shearing is more like cutting across. When this sliding takes place along the grain, the action of shearing is more like splitting. Thus, if a very deep, but thin, beam is of short span and heavily loaded, it might not break transversely, nor deflect excessively, but shear off at the supports, as shown in Figure 6, the action of the loads and supports being like a large cutting-machine, the weights cutting off the central part of beam and forcing it downwards past the support. This would be shearing across the grain.

If the foot of a main rafter is toed-in to the end of a tie-beam, and the foot forces its way outwardly, pushing away the block or part of tie-beam resisting it (splitting it out as it were), this would be shearing along the grain.

In most cases (*except in transverse strains*) the load is directly applied to the point being sheared off; the strain will, therefore, just equal the load, and we have: —

$$s = w.$$

Where s = the amount of the shearing strain.

" w = " " load.

The stress will be equal to the area of cross-section (affected by the shearing strain) multiplied by the amount of resistance to separation from each other that its fibres are capable of.

This amount of resistance is found by tests and experiments, and is given for each material per square inch of cross-section. A table of constants for resistance to shearing of different materials will be given later; in the formulæ these constants are represented by the letter g for shearing across the grain, and g_1 for shearing along the grain.

We have, then, for the stress: —

$$v = a.\ g.$$

Where v = the amount of ultimate stress.

Where a = the area of cross-section in square inches.

Where g = the ultimate resistance to shearing across the grain per square inch.

Therefore, the fundamental formula (1) $v = s.\ f$, becomes for pieces under shearing strains across the grain: —

$$a.\ g = w.\ f, \text{ or: —}$$

$$w = a.\left(\frac{g}{f}\right) \qquad (7)$$

And similarly, of course, we shall find: —

$$w = a.\left(\frac{g_1}{f}\right) \qquad (8)$$

Where w = the safe-load.

Where a = the area of cross-section in square inches, at the point where there is danger of shearing.

Where $\left(\frac{g}{f}\right)$ = the safe-resistance to shearing *across* the fibres per square inch.

Where $\left(\frac{g_1}{f}\right)$ = the safe-resistance to shearing *along* the fibres per square inch.

Example.

At the lower end of a vertical wrought-iron flat bar is suspended a load of eight thousand pounds. The bar is in two lengths, riveted together with one rivet. What diameter should the rivet be?

The strain on the rivet will, of course, be a shearing strain across the grain, and will be equal to the amount of tension on the bar, which we know is equal to the load. We use Formula (7), and have: —

$$w = 8000 \text{ pounds.}$$

The safe shearing for wrought-iron is about ten thousand pounds per square inch; inserting this in formula, we have: —

$$8000 = a.\ 10000, \text{ or } a. = \frac{8000}{10000} = \frac{4}{5}.$$

The area of rivet must, therefore, be four-fifths of a square inch. To obtain diameter, we know that: —

$$d = \sqrt{\tfrac{14}{11}.a} = \sqrt{\tfrac{14}{11}.\tfrac{4}{5}} = \sqrt{\tfrac{56}{55}} = \sqrt{1{,}01818}$$

This is, practically, equal to one; therefore, the diameter of rivet should be 1″.

In transverse strains the (vertical) cross-shearing is generally not equal to the load, but varies at different points of the beam or cantilever. The manner of calculating transverse strains, however, allows for straining only the edges (extreme fibres) up to the maximum; so that the intermediate fibres, not being so severely tested, generally have a sufficient margin of unstrained strength left to more than offset the shearing strain. In solid beams it can, therefore, as a rule, be overlooked, except at the points of support. (In plate-girders it must be calculated at the different points where weights are applied.) The amount of the shearing at each support is equal to the amount of load coming on or carried by the support.

We must, therefore, substitute for w in Formula (7) either p or q, as the case may be, and have at the left-hand end of beam for the safe resistance to shearing: —

$$p = a.\left(\frac{g}{f}\right) \tag{9}$$

And at the right-hand end of beam: —

$$q = a.\left(\frac{g}{f}\right) \tag{10}$$

Where p = the amount of load, in pounds, carried on the left-hand support.

Where q = the amount of load, in pounds, carried on the right-hand support.

Where a = the area of cross-section, in inches, at the respective support.

Where $\left(\frac{g}{f}\right)$ = the safe resistance, per square inch, to cross-shearing.

Example.

A spruce beam of 5′ *clear span is* 24″ *deep and* 3″ *wide; how much uniform load will it carry safely to avoid the danger of shearing off at either point of support?*

The beam being uniformly loaded, the supports will each carry one-half of the load; if, therefore, we find the safe resistance to shearing at either support, we need only double it to get the safe load (instead of calculating for the other support, too, and adding the results).

Let us take the left-hand support. From Formula (9) we have: —

$$p = a.\left(\frac{g}{f}\right)$$

Now, we know that $a = 24.3 = 72$ square inches.

The ultimate resistance of spruce to cross-shearing is about thirty-six hundred pounds per square inch; using a factor-of-safety of ten, we have for the safe resistance per square inch: —

$$\left(\frac{q}{f}\right) = \frac{3600}{10} = 360 \text{ pounds.}$$

We have, now: —

$$p = 72.360 = 25920 \text{ pounds.}$$

Similarly, we should have found for the right-hand support: —

$$q = 25920 \text{ pounds.}$$

And as: —

$$u = p + q = 51840 \text{ pounds,}$$

that will, of course, be the safe uniform load, so far as danger of shearing is concerned.

The beam must also be calculated for transverse strength, deflection and lateral flexure, before we can consider it entirely safe. These will be taken up later on.

Should it be desired to find the amount of vertical shearing strain x at any point of a beam, other than at the points of support, use: —

$$x = \left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\} - \Sigma w \qquad (11)$$

Where x = the amount of vertical shearing strain, in pounds, at any point of a beam.

Where $\left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\}$ = the reaction, in pounds, (that is, the share of the total loads carried) at the *nearer* support to the point.

Where Σw = the sum of all loads, in pounds, between said *nearer* support and the point.

When x is found, insert it in place of w, in Formula (7), in order to calculate the strength of beam necessary at that point to resist the shearing.

Example.

A spruce beam, 20′ long, and 8″ deep, carries a uniform load of one hundred pounds per running foot. What should be the thickness of beam 5′ from either support, to resist safely vertical shearing?

Each support will carry one-half the total load; that is, one thousand pounds; so that we have for Formula (11): —

$$\left\{\begin{matrix} p \\ \text{or} \\ q \end{matrix}\right\} = 1000 \text{ pounds.}$$

The sum of all loads between the nearer support and a point 5′ from support will be: —

$$\Sigma w = 5. \ 100 = 500 \text{ pounds.}$$

Therefore, the amount of shearing at the point 5′ from support will be: —

$$x = 1000 - 500 = 500 \text{ pounds.}$$

Inserting this in Formula (7) we have: —

$$500 = a.\left(\frac{g}{f}\right), \text{ or, } a = \frac{500}{\left(\frac{g}{f}\right)}$$

We have just found that for sprnce,

$$\left(\frac{g}{f}\right) = 360 \text{ pounds.}$$

Therefore, $a = \frac{500}{360} = 1{,}39$ square inches.

And, as $b.\ d = a$, or $b = \frac{a}{d}$, we have, $b = \frac{1{,}39}{8} = \frac{1''}{6}$

This is such a small amount that it can be entirely neglected in an 8″ wooden beam.

To find the amount of vertical shearing at any point of a canti-lever, other than at the point where it is built in, use: —

$$x = \Sigma\ w \qquad (12)$$

Where x the amount of vertical shearing strain, in pounds, at any point of canti-lever.

Where $\Sigma\ w$ the sum of all loads between the *free* end and said point.

To find the strength of beam at said point necessary to resist the shearing, insert x for w in Formula (7).

In transverse strains there is also a *horizontal* shearing along the entire neutral axis of the piece. This stands to reason, as the fibres above the neutral axis are in compression, while those below are in tension, and, of course, the result along the neutral line is a tendency of the fibres just above and just below it, to slide past each other or to *shear* off along the grain.

We can calculate the *intensity* (not amount) of this horizontal shearing at any point of the piece under transverse strain.

If x represents the amount of vertical shearing at the point, then the intensity of horizontal shearing at the point is $= \frac{3}{2}\frac{x}{a}$.

If this intensity of shearing does not exceed the safe-constant $\left(\frac{g_1}{f}\right)$ for shearing along the fibres, the piece is safe, or: —

$$\frac{3}{2}.\ \frac{x}{a} = \left(\frac{g_1}{f}\right) \qquad (13)$$

Where x is found by formulæ (11) or (12) for any point of beam or,

Where $x = \left\{ \begin{matrix} p \\ \text{or} \\ q \end{matrix} \right\} =$ the amount of supporting force, in pounds, for either point of support.

Where $a =$ the area of cross-section in square inches.

Where $\left(\frac{q_1}{f}\right) =$ the amount of safe resistance, per square inch, to shearing along fibres.

Example.

Take the same beam as before. The amount of vertical shearing 5′ from support we found to be five hundred pounds, or: —

$$x = 500.$$

The area was 8″ multiplied by thickness of beam, or: —

$$a = 8\ b.$$

The ultimate shearing along the fibres of spruce is about four hundred pounds per square inch, and with a factor-of-safety of ten, we should have: —

$$\left(\frac{q_1}{f}\right) = \frac{400}{10} = 40.$$

Inserting this in Formula (13) $\frac{3}{2} \cdot \frac{500}{8b} = 40$

or $b = \frac{1500}{16.40} = 2,''34.$

The beam should, therefore, be at least $2\frac{1}{3}''$ thick, to avoid danger of longitudinal shearing at this point. At either point of support the vertical shearing will be equal to the amount supported there; that is, one-half the load, or one thousand pounds. Substituting this for x in Formula (13), we have: —

$$\frac{3}{2} \cdot \frac{1000}{8b} = 40, \text{ or } b = \frac{3000}{16.40} = 4,''68.$$

The beam would, therefore, have to be $4\frac{2}{3}''$ thick at the points of support, to avoid danger of longitudinal shearing. The beam, as it is, is much too shallow for one of such span, a fact we would soon discover, if calculating the transverse strength or deflection of beam, which will be taken up later on. It will also be found that the greater the depth of the beam, the smaller will be the danger from longitudinal shearing, and, consequently, to use thinner beams, it would be necessary to make them deeper.

TABLE IV.

STRENGTH OF MATERIALS PER SQUARE INCH.

WOODS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Acacia	—	—	11000	1100	—	—	7000	1400	1150000	48
Alder	6900	700	9500	1000	—	—	4000	800	800000	38
Apple	—	—	13000	1300	—	—	4500	900	—	50
Ash, English	8000	800	10500	1000	—	—	7800	1300	1200000	48
" American	8000	800	11000	1100	$g_1 = 650$	65	8000	1350	1250000	40
" " across.	2060	200	—	—	$g = 6000$	600	—	—	—	—
" Canadian	5000	500	6500	650	$g_1 = 1400$	140	5000	850	1000000	38
Bamboo	—	—	6000	600	—	—	—	—	—	25
Baywood	7200	720	10500	1000	—	—	—	—	—	—
Beech	8000	800	8000	800	$g = 5000$	500	6000	1000	1350000	48
Birch, English	5000	500	10000	1000	—	—	6000	1000	1250000	47
" American	8500	850	10000	1000	$g_1 = 650$	65	5800	950	1300000	43
" " across.	1800	180	—	—	$g = 5600$	560	—	—	—	—
Box	10000	1000	15000	1500	—	—	7500	1250	2000000	62
Cedar	6000	600	8000	800	—	—	7000	800	500000	41
" across	1950	200	—	—	$g = 1400$	140	—	—	—	—
Cherry	8000	800	—	—	—	—	4500	900	1200000	40
" across	2400	240	—	—	$g = 2900$	290	—	—	—	—
Chestnut, American	5300	530	8500	850	$g_1 = 700$	70	5400	900	750000	41
" " across	840	84	—	—	$g = 1500$	150	—	—	—	—
" English	—	—	10000	1000	—	—	8000	800	1000000	38
Crab	6800	680	—	—	—	—	4500	750	—	—
Deal	6000	600	8500	850	$g_1 = 625$	60	4800	800	—	30
Ebony	18000	1800	18000	1800	$g = 7500$	750	18000	2000	—	76
Elder	8700	870	10000	1000	—	—	4500	900	—	43
Elm, English	8000	800	8000	800	$g_1 = 1200$	120	5000	800	700000	35
" " across	—	—	—	—	$g = 3000$	300	—	—	—	—
" Canadian	8500	850	9000	900	$g_1 = 1400$	140	9000	900	2000000	47
" " across	—	—	—	—	$g = 5000$	500	—	—	—	—
Fir, Riga	6000	600	7500	750	—	—	7000	700	870000	40
" New England	6700	670	7500	750	$g_1 = 630$	70	4000	800	1000000	40
" " across	1200	120	690	70	—	—	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

WOODS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Greenheart	13000	1300	8000	800	—	—	9500	950	1500000	65
Gum	7000	700	10500	1000	5300	600	6000	900	1400000	62
Hazel	—	—	12000	1200	—	—	—	—	—	—
Hawthorn	—	—	7500	750	—	—	—	—	—	—
Hemlock	4500	450	7000	700	$g_1 = 500$	50	4500	750	800000	26
" **across**	500	50	550	55	$g = 2750$	275	—	—	—	—
Hickory, American	9500	1000	10000	1000	—	—	8000	1000	1100000	50
" " across	3000	300	—	—	$g = 6500$	650	—	—	—	—
Holly	5250	525	10500	1000	—	—	4500	900	—	—
Hornbeam	6600	660	14000	1400	—	—	6000	1000	750000	47
Jugeb	—	—	12500	1250	—	—	—	—	—	—
Lancewood	—	—	18000	1800	—	—	—	—	—	52
Larch	4500	450	7000	700	$g_1 = 1300$	130	5500	700	1000000	35
" across	1300	130	1300	130	$g = 3000$	300	—	—	—	—
Lignum-vitæ	9900	1000	10000	1000	—	—	8000	1000	1080000	62
Locust, American	10000	1000	12000	1200	$g_1 = 1150$	120	10000	1100	1500000	46
" " across	3000	300	—	—	$g = 7200$	720	—	—	—	—
Mahogany	8200	820	11000	1100	—	—	6000	1000	1200000	43
" across	2400	240	—	—	—	—	—	—	—	—
Maple	6500	650	8000	800	$g_1 = 450$	45	7000	700	877000	45
" across	1875	200	—	—	$g = 6000$	600	—	—	—	—
Oak, American, Red	6000	600	8000	800	$g_1 = 750$	75	6500	800	1200000	52
" " **White**	7200	720	11000	1100	$g_1 = 800$	80	7200	1100	900000	52
" " " **across**	2400	240	2300	230	$g = 4400$	440	—	—	—	—
" Black bog	—	—	7700	770	—	—	—	—	—	—
" Canadian	6000	600	7500	750	—	—	7000	750	—	54
" Dantzic	7500	750	6000	600	$g = 2300$	230	6000	800	1165000	47
" English	8300	830	12000	1200	$g_1 = 2316$	230	6800	1100	873000	54
" " across	1800	200	2400	240	$g = 4000$	400	—	—	—	—
" Live	6850	700	11000	1100	$g_1 = 700$	70	7300	1100	—	69
" " across	4500	450	—	—	$g = 8500$	850	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

WOODS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Pear	7500	750	8000	800	—	—	3400	700	800000	42
Pine, American yellow	5400	540	9000	900	—	—	6000	800	1100000	34
" " " across	—	—	—	—	g = 4800	500	—	—	—	—
" " red	6500	650	9000	900	g_1 = 650	65	6100	800	900000	36
" " white	5500	550	9000	900	g_1 = 450	45	4000	900	850000	28
" " " across	700	70	550	60	g = 2500	250	—	—	—	—
" Canadian yellow	5300	530	6000	600	g_1 = 350	40	4800	600	1000000	33
" " " across	1425	150	—	—	g = 4300	430	—	—	—	—
Pine, Georgia yellow	7400	750	12000	1200	g_1 = 500	50	7200	1200	1200000	48
" Georgia Yel'w across	1850	200	—	—	g = 5700	570	—	—	—	—
" Pitch	8900	900	10000	1000	g_1 = 510	50	6600	1000	1225000	65
" " across	—	—	—	—	g = 5000	500	—	—	—	—
Plum	9000	900	8000	800	—	—	7000	900	—	45
Poon	—	—	9000	900	—	—	8000	900	1600000	—
Poplar (white wood)	5000	500	5500	550	g_1 = 400	40	4500	600	762000	27
" across	1000	100	1800	180	g = 4400	440	—	—	—	—
Redwood, California	6000	600	7300	750	—	—	4500	750	—	23
Spruce	6500	650	10000	1000	g_1 = 400	40	4400	1000	850000	30
" across	750	75	550	60	g = 3600	360	—	—	—	—
Sycamore	8100	810	10000	1000	—	—	6000	1000	1000000	40
Teak	12000	1200	14000	1400	—	—	8000	1300	1400000	46
Walnut	7000	700	8000	800	—	—	5500	800	900000	39
" across	2100	210	—	—	g = 4000	400	—	—	—	39
Willow	6000	600	8500	850	—	—	4500	800	566000	25
Yew	—	—	6000	600	—	—	7000	700	—	50

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

METALS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Aluminium bronze	130000	25000	73000	12000	—	—	—	—	—	—
Antimony, cast	—	—	1100	200	—	—	—	—	—	418
Bismuth	—	—	3200	500	—	—	—	—	—	614
Brass, yellow, cast	80000	8000	28000	4500	37000	6000	20000	5000	9000000	506
" wire, unannealed	—	—	80000	8000	—	—	—	—	14000000	530
" " annealed	—	—	50000	5000	—	—	—	—	—	—
Bronze, gun-metal	120000	25000	32000	8000	—	—	53000	12500	10000000	529
" wire, unannealed	—	—	150000	30000	—	—	—	—	10000000	520
" " annealed	—	—	63000	13000	—	—	—	—	—	—
Copper, pure cast	117000	25000	30000	10000	33000	11000	22000	7000	10000000	543
" cast	—	—	25000	8000	—	—	—	—	—	—
" sheet wrought	103000	21000	31000	11000	—	—	—	—	15000000	556
" wire, unannealed	—	—	63000	20000	—	—	—	—	18000000	—
" " annealed	—	—	36000	12000	—	—	—	—	—	—
" bolts	—	—	32000	11000	—	—	—	—	—	556
Gold, cast	—	—	20000	5000	—	—	—	—	—	1204
" wire, unannealed	—	—	30000	7500	—	—	—	—	—	1217
" " annealed	—	—	5000	1250	—	—	—	—	—	—
Iron, cast, Am., 1″ thick	85000	15000	16500	2500	18000	2500	40000	5000	15000000	454
" " English	70000	13000	13000	2000	14000	2000	36000	4000	12000000	450
" **wrought, American**	47000	12000	50000	12000	40000	8000	43000	12000	27000000	483
" " English	35000	9000	45000	11000	36000	7000	34000	10000	26000000	480
" corrugated	—	—	—	—	—	—	40000	10000	—	—
" wire ropes, unannealed	—	—	80000	20000	—	—	—	—	25300000	480
" " " annealed	—	—	50000	12000	—	—	—	—	15000000	—
" chains	—	—	35000	9000	—	—	—	—	—	—
" welded with steel, 5 ply	—	—	55500	14000	—	—	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

METALS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Lead, sheet	—	—	2500	500	—	—	—	—	—	—
" cast	7700	1200	2000	400	—	—	—	—	720000	712
" drawn pipe	—	—	1700	350	—	—	—	—	—	709
" wire	—	—	1400	300	—	—	—	—	—	—
Mercury at 32°	—	—	—	—	—	—	—	—	—	851
Nickel	—	—	—	—	—	—	—	—	—	488
Palladium, wire	—	—	50000	10000	—	—	—	—	—	—
" drawn	—	—	26000	5200	—	—	—	—	—	—
" annealed	—	—	7100	1500	—	—	—	—	—	—
Pewter	—	—	—	—	—	—	—	—	—	453
Platinum, wire, unannealed	—	—	55000	11000	—	—	—	—	—	1350
" " annealed	—	—	25000	5000	—	—	—	—	—	1379
Plumbago	—	—	—	—	—	—	—	—	—	142
Silver, cast	—	—	41000	8000	—	—	—	—	—	655
" annealed	—	—	4300	900	—	—	—	—	—	658
" wire, unannealed	—	—	38000	7600	—	—	—	—	—	—
Solder, soft	—	—	7500	1200	—	—	—	—	—	—
Steel, American, wrought	90000	15000	108000	18000	75000	12000	100000	15000	29000000	488
" English	—	—	95000	15000	65000	10000	80000	13000	28000000	490
" wire, unannealed	—	—	160000	25000	—	—	—	—	—	490
" " annealed	—	—	80000	13000	—	—	—	—	—	—
" rope, galvanized	—	—	6700	1200	—	—	—	—	—	—
" chains	—	—	42000	8500	—	—	—	—	—	—
" cast	150000	25000	85000	15000	57000	10000	125000	15000	30000000	—
" welded hard & soft, 5 ply	—	—	63300	15000	—	—	—	—	—	—
Tin, cast	15500	2500	5500	1000	—	—	4000	1000	4000000	462
" wire	—	—	7000	1200	—	—	—	—	—	—

TABLE IV. (*Continued.*)

STRENGTH OF MATERIALS PER SQUARE INCH.

METALS.	Compression.		Tension.		Shearing.		Modulus of Rupture.		Modulus of Elasticity.	Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate g or g_1	Safe $\frac{g}{f}$ or $\frac{g_1}{f}$	Ultimate k	Safe $\frac{k}{f}$	e	
Zinc, sheet	—	—	12000	2000	—	—	—	—	13500000	449
" wire	—	—	20000	3300	—	—	—	—	—	—
" cast	40000	7000	3300	600	—	—	7500	1200	—	438
MISCELLANEOUS.										
Bone	—	—	5000	500	—	—	—	—	—	—
Cement, Portland	3000	300	500	50	—	—	900	90	800000	81
Glass, common green	28000	1500	2850	300	—	—	4000	400	8000000	173
" crystal	17000	1000	2413	240	—	—	—	—	5800000	157
" flint	25000	1400	2300	230	—	—	—	—	—	—
" white crown	30000	1600	2500	250	—	—	—	—	—	—
" plate	—	—	5000	500	—	—	—	—	—	—
Ice, 32°	230	20	—	—	—	—	—	—	6000000	57
Ivory	—	—	16000	1600	—	—	—	—	—	114
Limestone	7000	700	1000	100	—	—	1500	150	3300000	146
Marble, Italian	12000	1200	700	70	—	—	2080	200	2500000	168
Mortar, lime	400	100	50	5	—	—	125	12	1285000	98
" Portland cement	2000	200	300	30	—	—	700	70	600000	109
Ox leather	—	—	4200	400	—	—	—	—	240000	—
Portland stone	3500	350	900	90	—	—	1530	150	1600000	146
Silk fibre	—	—	5200	520	—	—	—	—	1300000	—
Skin, ox undressed	—	—	6300	600	—	—	—	—	—	—
Slate	11000	1100	10000	800	—	—	5000	500	12000000	160
Ropes, hempen	—	—	8000	800	—	—	—	—	—	—
Whalebone	—	—	7700	770	—	—	—	—	820000	81
Yarn, flaxen	—	—	25000	1500	—	—	—	—	—	—

TABLE V.

STRENGTH OF STONES, BRICKS, AND CEMENTS PER SQUARE INCH.

STONES.	Compression on bed.		Compression on edge.		Tension.		Modulus of Rupture.		Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate c_1	Safe $\frac{c_1}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate k	Safe $\frac{k}{f}$	
Bluestone	13500	1350	—	—	1400	140	2300	230	160
Granite, average	15000	1500	—	—	580	50	1860	180	170
" Aberdeen (Scotch)	10700	1000	—	—	—	—	—	—	165
" Blue, Staten Island, N. Y.	22250	2000	—	—	—	—	1300	130	179
" Canadian	12000	1200	—	—	—	—	—	—	162
" Connecticut	12000	1200	13000	1300	—	—	—	—	166
" Cornish (English)	6300	630	—	—	—	—	—	—	183
" Keene, N. H.	14500	1450	—	—	—	—	1500	150	—
" Kingston (Dublin), Ireland.	10450	1050	—	—	—	—	—	—	171
" Maine	15000	1500	15000	1500	—	—	—	—	168
" Massachusetts	16000	1600	9250	925	—	—	1870	180	165
" Millstone Point	15700	1570	—	—	—	—	1900	190	—
" Mt. Sorrel	12900	1300	—	—	—	—	—	—	—
" New Jersey	22000	2200	—	—	—	—	—	—	189
" New York State	15000	1500	15000	1500	—	—	—	—	168
" Peterhead (Scotch)	8300	830	—	—	—	—	—	—	166
" Virginia	14000	1400	—	—	—	—	—	—	164
" Greenstone, Giants' Causeway, Ireland	18000	1800	—	—	—	—	—	—	—
Limestone, average	7000	700	6000	600	1000	100	1500	150	146
" Anglesea, English	7600	760	—	—	—	—	—	—	—
" Bath, English	1450	150	—	—	—	—	370	37	—
" Caen, French	3550	350	—	—	150	15	600	60	119
" Chalk, average	1000	100	—	—	118	10	670	50	145
" Chilmark, English	3240	325	—	—	500	50	—	—	153
" Canton, Mo., drab	7200	720	—	—	—	—	—	—	146
" Marquette, Mich., drab	7900	790	7800	780	—	—	1500	150	146
" Dublin, Irish	17000	1700	—	—	—	—	—	—	—
" English, compact	8300	830	—	—	—	—	800	80	160
" Glen Falls, N. Y.	11500	1100	1 750	1000	—	—	—	—	169
" Lake Champlain, N. Y.	25000	2000	21500	1800	—	—	—	—	172
" North River, N. Y.	17000	1500	14150	1400	—	—	—	—	166
" Purbeck (hard bed), English	8000	800	—	—	—	—	—	—	153

TABLE V. (*Continued.*)

STRENGTH OF STONES, BRICKS, AND CEMENTS PER SQUARE INCH.

STONES.	Compression on bed.		Compression on edge.		Tension.		Modulus of Rupture.		Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate c_1	Safe $\frac{c_1}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate k	Safe $\frac{k}{f}$	
Limestone, Portland, English	3500	350	—	—	900	90	1530	150	146
" Joliet, Ill., white	13000	1300	7050	700	—	—	—	—	159
" Marblehead, Ohio, white	11500	1150	—	—	—	—	1500	150	152
Marble, average	8000	800	8000	800	700	70	1200	120	160
" Devonshire red (English)	7300	730	—	—	—	—	—	—	163
" Eastchester,	13000	1300	—	—	1600	160	1900	190	179
" Illinois	9700	970	9800	980	1000	100	1200	120	159
" Italian	12000	1200	—	—	700	70	2080	200	168
" Vermont	7600	760	8670	870	675	67	1200	120	166
Rock, average	12000	1000	11000	1000	—	—	—	—	165
" quartz (American)	22000	1500	13000	1000	—	—	—	—	165
" Holyhead (English)	25000	1500	13700	1000	—	—	—	—	165
" hard, New York City	11250	1000	12500	1000	—	—	—	—	182
Sandstone, average	5000	500	—	—	150	15	1000	100	144
" Berlin, Ohio	14250	1000	12000	1000	—	—	—	—	—
" Belville, N. J. (brown)	11700	1000	10250	800	—	—	650	50	141
" Corsehill (red Scotch)	7760	700	—	—	—	—	—	—	154
" Gatelawbridge (red Scotch)	8500	750	—	—	—	—	—	—	152
" Little Falls, N. Y. (brown)	9850	980	9150	900	—	—	1530	170	140
" Craigleith (Scotch)	5500	550	—	—	450	45	770	77	144
" Longmeadow, (Worcester) brown	10300	1000	—	—	—	—	—	—	142
" Longmeadow, (Ribbe) red	12700	1200	—	—	—	—	—	—	149
" Middletown, Conn. (brown)	6950	600	5550	500	590	50	1000	100	148
" Amherst, Ohio (brown-gray)	6000	600	5450	500	105	10	600	60	135
" Bramley Falls, English	5900	550	—	—	—	—	—	—	142
" Dorchester, Nova Scotia (freestone)	9200	900	—	—	100	10	800	80	150
" " (olive)	4250	400	6050	600	—	—	—	—	150
" Drab Berea (Ohio)	8750	800	—	—	105	10	700	70	134
" Runcorn red, (English)	2100	200	—	—	—	—	—	—	129
" Yorkshire paving (English)	5600	500	—	—	—	—	750	75	166
Slate	11000	1100	—	—	10000	800	5000	500	160
Whinstone (Scotch)	8100	800	—	—	1450	140	—	—	—
Stonework	with	level,	dressed	beds	allow	4-10 of	strength	of	stone.
Rubblework	500	100	—	—	—	—	—	—	150

TABLE V. (*Continued.*)

STRENGTH OF STONES, BRICKS, AND CEMENTS PER SQUARE INCH.

BRICKS.—CEMENTS, ETC.—SOILS.	Compression on bed.		Compression on edge.		Tension.		Modulus of Rupture.		Weight per Cubic ft.
	Ultimate c	Safe $\frac{c}{f}$	Ultimate c_1	Safe $\frac{c_1}{f}$	Ultimate t	Safe $\frac{t}{f}$	Ultimate k	Safe $\frac{k}{f}$	
BRICKS.									
Bricks, light red	640	65	—	—	40	4	—	—	—
" good common	10000	800	—	—	200	20	600	60	100
" best hard	12000	1000	—	—	400	40	750	75	107
" Philadelphia, pressed	6000	600	—	—	—	—	650	65	105
" brown glazed (English)	1300	130	—	—	—	—	—	—	—
" fire (English)	1700	170	—	—	—	—	—	—	138
Brickwork, common	1000	100	—	—	50	5	—	—	110
" good	1500	150	—	—	100	10	—	—	111
" best, in cement mortar	2000	200	—	—	300	30	—	—	112
Terra-cotta	5000	400	—	—	—	—	—	—	110
Terra-cotta work	2000	200	—	—	—	—	—	—	112
CEMENTS, ETC.									
Cement, Rosendale	1500	150	—	—	290	29	400	40	56
" Portland	3000	300	—	—	500	50	900	90	81
" lime	600	60	—	—	—	—	200	20	59
" plaster-of-Paris	600	60	—	—	70	7	—	—	78
Mortar, lime	400	100	—	—	50	5	125	12	98
" lime and Rosendale	550	125	—	—	75	8	200	20	100
" Rosendale cement	700	150	—	—	145	15	300	30	102
" Portland cement	2000	200	—	—	300	30	700	70	109
" plastering	400	50	—	—	—	—	—	—	86
Concrete, Portland cement	1500	200	—	—	427	40	162	16	150
" Rosendale cement	400	100	—	—	100	10	66	8	140
SOILS.									
Clay, horizontal layers	—	40	—	—	—	—	—	—	120
Firm earth	—	20	—	—	—	—	—	—	115
Gravel, firm with earth	—	100	—	—	—	—	—	—	125
Loam, soft	—	10	—	—	—	—	—	—	110
Sand, solid dry	—	7	—	—	—	—	—	—	112

The amounts given in Table IV for compression, tension and shearing are *along* fibres, except where marked *across*.

It will also be noticed that the factors-of-safety chosen are very different; the reason being that where figures seemed reliable the factor chosen was low, and became higher in proportion to the unreliability of the figures. The tables, as they are, are extremely unsatisfactory and unreliable, though the writer has spent much time in their construction. Any one, who will devote to the subject even the slightest research, will find that there are hardly any two original experimenters who agree, and in most cases, the experiments are so carelessly made or recorded that they are of but little value.

TABLE VI.

WEIGHT PER CUBIC FOOT OF MATERIALS.

(Not included in Tables IV and V.)

Material.	Weight.	Material.	Weight.
Ashes	59	Peat	85
Asphalt	150	Petrified wood	145
Butter	60	Pitch	75
Camphor	63	Plumbago	131
Charcoal	23	Pumice-stone	56
Coal, solid	93	Resin	68
" loose	54	Rock crystal	172
Coke	50	Rubber	62
Cork	15	Salt	134
Cotton in bales	20	Saltpetre	130
Fat	58	Snow, fresh fallen	6
Gunpowder	56	" solid	20
Hay in bales	17	Sugar	82
Isinglass	70	Sulphur	125
Lead, red	560	Tiles	115
Paper	55	Water	63

TRANSVERSE STRENGTH. — RUPTURE.

If a beam is supported at two ends, and loads are applied to the beam, it is evident: —

1st, that the beam will bend under the load, or *deflect*.

2d, that if the loading continues, the beam will eventually break, or be *ruptured*.

Deflection when non-important. The methods of calculating deflection and rupture differ very greatly. In some cases, where deflection in a beam would do no damage—such as cracking plaster, lowering a column, making a floor too uneven for machinery, etc.,—or where it would not look unsightly, we can leave deflection out of the question, and calculate for rupture only. Where, however, it is important to guard against deflection, we must calculate for both.

REACTION OF SUPPORTS.

If we imagine the loaded beam supported at both ends by two giants, it is evident that each giant would have to exert a certain amount of force upwards to keep his end of the beam from tipping.

We can therefore imagine in all cases the supports to be resisting or *reacting* with force sufficient to uphold their respective ends. The amount of this reaction for either support is equal to the load multiplied by its distance from the further support, the whole divided by the length, or

Amount of Reaction.

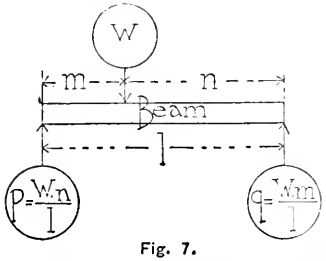


Fig. 7.

$$p = \frac{w.\ n}{l} \qquad (14)$$

Where p = the amount of the *left* hand reaction or supporting force.

$$\text{and } q = \frac{w.\ m}{l} \qquad (15)$$

Where q = the amount of the *right* hand reaction or supporting force.

If there are several loads the same law holds good for each, the reaction being the sum of the products, or

$$p = \frac{w_{\text{I}}\ n}{l} + \frac{w_{\text{II}}\ s}{l} \qquad (16)$$

$$\text{and } q = \frac{w_{\text{I}}\ m}{l} + \frac{w_{\text{II}}\ r}{l} \qquad (17)$$

As a check add the two reactions together and their sum must equal the whole load, that is, $p + q = w_{\text{I}} + w_{\text{II}}$

Example.

A beam 9′ 2″ long between bearings carries two loads, one of 200 lbs. 4′ 2″ from the left-hand support, and the other of 300 lbs. 3′ 4″ from the right-hand support. What are the right-hand and left-hand reactions?

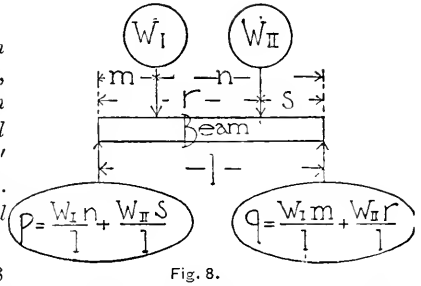


Fig. 8.

Referring to Figure 8 we should have $w_{\text{I}} = 200$ lbs., and $w_{\text{II}} = 300$ lbs., further $l = 110''$; $m = 50''$; $n = 60''$; $s = 40''$, and $r = 70''$, therefore the left-hand reaction would be: —

$$p = \frac{200.\ 60}{110} + \frac{300.\ 40}{110} = 218\tfrac{2}{11} \text{ pounds.}$$

and the right-hand reaction would be: —

$$q = \frac{200.\ 50}{110} + \frac{300.\ 70}{110} = 281\tfrac{9}{11} \text{ pounds.}$$

As a check add p and q together, and they should equal the whole load of 500 lbs., and we have in effect: —

$$p + q = 218\tfrac{2}{11} + 281\tfrac{9}{11} = 500 \text{ pounds.}$$

If the load on a beam is uniformly distributed, or is concentrated at the centre of the beam, or is concentrated at several points along the beam, each half of beam being loaded similarly, then each support will react just one half of the total load.

THE PRINCIPLE OF MOMENTS.

Law of Lever. The law of the lever is well known. The distance of a force from its fulcrum or point where it takes effect is called its *leverage*. The *effect* of the force at such point is equal to the amount of the force multiplied by its leverage.

Moment of a force. The *effect* of a force (or load) at any point of a beam is called the *moment* of the force (or load) at said point, and is equal to the amount of the force (or load) multiplied by the distance of the force (or load) from said point, *the distance measured at right angles to the line of the force.* If therefore we find the moments — for all of the forces acting on a beam — at any single point of the beam we know the total moment at said point, and this is called the *bending-moment* at said point. Of course, forces

Bending moment. acting in opposite directions will give opposite moments, and will counteract each other; to find the bending-moment, therefore, for any single point of a beam take the difference between the sums of the opposing moments of all forces acting at that point of the beam.

Now on any loaded beam we have two kinds of forces, the loads which are pressing downwards, and the supports which are resisting upwards (theoretically *forcing* upwards). Again, if we imagine that the beam will break at any certain point, and imagine one side of the beam to be *rigid*, while the other side is tending to break away from the rigid side, it is evident that the effect at the point of rupture will be from one side only; therefore we must take the forces on *one side* of the point *only*. It will be found in practice that no matter for what point of a beam the bending moment is sought, the bending moment will be found to be the same, whether we take the forces to the right side or left side of the point. This gives an excellent check on all calculations, as we can calculate the bending moment from the forces on each side, and the results of course should be the same.

Now to find the actual strain on the fibres of any cross-section of the beam, we must find the bending moment at the point where the cross-section is taken, and divide it by the moment of resistance of the fibre, or,

$$\frac{m}{r} = s$$

Where m = the bending moment in lbs. inch.

Where r = the moment of resistance of the fibre in inches.

Where s = the strain.

The stress, of course, will be equal to the resistance to cross-breaking the fibres are capable of. In the case of beams which are of uniform cross-section above and below the neutral axis, this resistance is called the Modulus of Rupture (k). It is found by experiments and tests for each material, and will be found in Tables IV and V. We have, then, for *uniform cross-sections*: —

$$v = k$$

Where v = the ultimate stress per square inch.

Where k = the modulus of rupture per square inch. Inserting this and the above in the fundamental formula (1), viz.: $v = s.f.$, we have: —

$$k = \frac{m}{r}.f, \text{ or}$$

Transverse strength uniform cross section.

$$\frac{m}{\left(\frac{k}{f}\right)} = r \qquad (18)$$

Where m = the bending moment in lbs. inch at a given point of beam.

Where r = the moment of resistance in inches of the fibres at said point.

Where $\left(\frac{k}{f}\right)$ = the safe modulus of rupture of the material, per square inch.

Transverse strength section not uniform.

If the *cross-section is not uniform* above and below the neutral axis, we must make two distinct calculations, one for the fibres above the neutral axis, the other for the fibres below; in the former case the fibres would be under compression, in the latter under tension. Therefore, for the fibres *above the neutral axis*, the ultimate stress would be equal to the ultimate resistance of the fibres to compression, or $v = c$.

Inserting this in the fundamental formula (1), we have:—

$$c = \frac{m}{r} \cdot f, \text{ or}$$

Upper fibres.

$$\frac{m}{\left(\frac{c}{f}\right)} = r \qquad (19)$$

Where m = the bending moment in lbs. inch, at a given point of beam.

Where r = the moment of resistance in inches of the fibres at said point.

Where $\left(\frac{c}{f}\right)$ = the safe resistance to crushing of the material, per square inch.

For the *fibres below the neutral axis*, the ultimate stress would be equal to the ultimate resistance of the fibres to tension, or, $v = t$.

Inserting this in the fundamental formula (1) we have:—

$$t = \frac{m}{r} \cdot f, \text{ or}$$

Lower fibres.

$$\frac{m}{\left(\frac{t}{f}\right)} = r \qquad (20)$$

Where m = the bending moment in lbs. inch at a given point of beam.

Where r = the moment of resistance in inches of the fibres at said point.

Where $\left(\frac{t}{f}\right)$ = the safe resistance to tension of the material, per square inch.

The same formulæ apply to cantilevers as well as beams.

The moment of resistance r of any fibre is equal to the moment of inertia of the whole cross-section, divided by the distance of the fibre from the neutral axis of the cross-section.

Greatest strains on extreme fibres.

The greatest strains are along the upper and lower edges of the beam (the extreme fibres); we, therefore, only need to calculate their resistances, as all the intermediate fibres are nearer to the neutral axis, and, consequently, *less* strained. The distance of fibres chosen in calculating the moment of resistance is, therefore, the distance from the neutral axis of either the upper or lower edges, as the case may be. The moments of resistance given in the fourth column, of Table I, are for the upper and lower edges (the extreme fibres), and should be inserted in place of r, in all the above formulæ.

Point of greatest bending moment.

To find at what point of a beam the greatest bending moment takes place (and, consequently, the greatest fibre strains, also), begin at either support and move along the beam towards the other support, passing by load after load, until the amount of loads that have been passed is equal to the amount of the reaction of the support (point of start); the point of the beam where this amount is reached is the point of greatest bending moment.

In cantilevers (beams built in solidly at one end and free at the other end), the point of greatest bending moment is *always* at the point of the support (where the beam is built in).

In light beams and short spans the weight of the beam itself can be neglected, but in heavy or long beams the weight of the beam should be considered as an independent uniform load.

RULES FOR CALCULATING TRANSVERSE STRAINS.

1. *Find Reaction of each Support.*

Summary of Rules.

If the loads on a girder are uniformly or symmetrically distributed, each support carries or reacts with a force equal to one-half of the total load. If the weights are unevenly distributed, each support carries, or the reaction of each support is equal to, the *sum* of the products of *each* load into its

distance from the *other* support, divided by the whole length of span. See Formulæ (14), (15), (16), and (17).

2. *Find Point of Greatest Bending Moment.*

The greatest bending moment of a uniformly or symmetrically distributed load is always at the centre. To find the *point* of greatest bending moment, when the loads are unevenly distributed, begin at either support and pass over load after load until an amount of loads has been passed *equal* to the amount of reaction at the support from which the start was made, and this is the desired point. In a cantilever the point of greatest bending moment is always at the wall.

3. *Find the Amount of the Greatest Bending Moment.*

In a beam (supported at both ends) the greatest bending moment is at the centre of the beam, provided the load is uniform, and this moment is equal to the product of the whole load into one-eighth of the length of span, or

$$m = \frac{u.l}{8} \qquad (21)$$

Where m = the greatest bending moment (at centre), in lbs. inch, of a uniformly-loaded beam supported at both ends.

Where u = the total amount of uniform load in pounds.

Where l = the length of span in inches.

If the above beam carried a central load, in place of a uniform load, the greatest bending moment would still be at the centre, but would be equal to the product of the load into one-quarter of the length of span, or

$$m = \frac{w.l}{4} \qquad (22)$$

Where m = the greatest bending moment (at centre), in lbs. inch, of a beam with concentrated load at centre, and supported at both ends.

Where w = the amount of load in pounds.

Where l = the length of span in inches.

To find the greatest bending moment of a beam, supported at both ends, with loads unevenly distributed, imagine the girder cut at the point (previously found) where the greatest bending moment is known to exist; then the *amount* of the bending moment at that point will be equal to the product of the reaction (of either support) into its distance from said point, less the *sum* of the products of *all* the loads on the *same* side into their respective distances from said point, *i. e.*, the point where the beam is supposed to be cut. To check the whole calculation, try the reaction and loads of the discarded side of the beam, and the same result should be obtained.

To put the above in a formula, we should have:—

Amount of greatest bending moment.

$$m_A = p.x - \Sigma\,(w_{\rm I}\,x_{\rm I} + w_{\rm II}\,x_{\rm II} + w_{\rm III}\,x_{\rm III}, \text{ etc.}) \quad (23)$$

And as a *check* to above:

$$m_A = q.(l-x) - \Sigma\,(w_{\rm III}\,x_{\rm III} + w_{\rm V}\,x_{\rm V} + w_{\rm VI}\,x_{\rm VI}) \quad (24)$$

etc.

Where A = is the point of greatest bending moment.

Where m_A = is the amount of bending moment, in lbs. inches, at A.

Where p = is the left-hand reaction, in pounds.

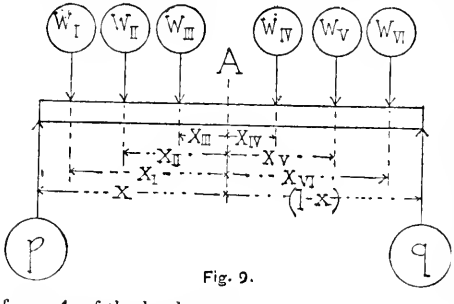


Fig. 9.

Where q = is the right-hand reaction, in pounds.

Where x and $(l-x)$ = the respective distances in inches, of the left and right reactions from A.

Where $x_{\rm I}$, $x_{\rm II}$, $x_{\rm III}$, etc., = the respective distances, in inches, from A, of the loads $w_{\rm I}$, $w_{\rm II}$, $w_{\rm III}$, etc.

Where $w_{\rm I}$, $w_{\rm II}$, $w_{\rm III}$, etc., = the loads, in pounds.

Where Σ = the sign of summation.

The same formulæ, of course, would hold good for any point of beam.

In a cantilever (supported and built in at one end only), the greatest bending moment is *always* at the point of support.

For a uniform load, it is equal to the product of the whole load into one-half of the length of the free end of cantilever, or

$$m = \frac{u.l}{2} \quad (25)$$

Where m = the amount, in lbs. inch, of the greatest bending moment (at point of support).

Where u = the amount of the whole uniform load, in pounds.

Where l = the length, in inches, of the free end of cantilever.

For a load concentrated at the free end of a cantilever, the greatest bending moment is at the point of support, and is equal to the product of the load into the length of the free end of cantilever, or

$$m = w.\,l \quad (26)$$

Where m = the amount, in lbs. inch, of the greatest bending moment (at point of support).

Where w = the load, in pounds, concentrated at free end.

Where l = the length, in inches, of free end of cantilever.

For a load concentrated at any point of a cantilever, the greatest bending moment is at the point of support, and is equal to the product of the load into its distance from the point of support, or

$$m = w.\ x \qquad (27)$$

Where m = the amount, in lbs. inch, of the greatest bending moment (at point of support).

Where w = the load, in pounds, at any point.

Where x = the distance, in inches, from load to point of support of cantilever.

Note, that in all cases, when measuring the distance of a load, we must take the shortest distance (at right angles) of the vertical neutral axis of the load, (that is, of a vertical line through the centre of gravity of the load.)

4. *Find the Required Cross-section.*

To do this it is necessary first to find what will be the required moment of resistance.

If the cross-section of the beam is *uniform* above and below the neutral axis, we use Formula (18), viz.:—

$$r = \frac{m}{\left(\frac{k}{f}\right)}$$

If the cross-section is *unsymmetrical*, that is, not uniform above and below the neutral axis, we use for the *fibres above the neutral axis*, formula (19), viz.:—

$$r = \frac{m}{\left(\frac{c}{f}\right)}$$

and for the *fibres below the neutral axis*, Formula (20), viz.:—

$$r = \frac{m}{\left(\frac{t}{f}\right)}$$

In the latter two cases, for economy, the cross-section should be so designed that the respective distances of the upper and lower edges (extreme fibres), from the neutral axis, should be proportioned to their respective stresses or capacities to resist compression and tension. This will be more fully explained under cast-iron lintels.

A simple example will more fully explain all of the above rules.

Example.

Three weights of respectively 500 *lbs.*, 1000 *lbs., and* 1500 *lbs., are placed on a beam of* 17′ 6″ (*or* 210″) *clear span,* 2′ 6″ (*or* 30″), 7′ 6″ (*or* 90″), *and* 10′ 0″ (*or* 120″) *from the left-hand support. The modulus of rupture of the material is* 2800 *lbs. per square inch. The factor-of-safety to be used is* 4. *The beam to be of uniform cross-section. What size of beam should be used?*

1. *Find Reactions* (*see formulæ* 16 *and* 17).

Reaction p will be in pounds, $= \frac{500.180}{210} + \frac{1000.120}{210} + \frac{1500.90}{210} =$ $1642\frac{6}{7}$ pounds.

Reaction q will be in pounds, $= \frac{500.30}{210} + \frac{1000.90}{210} + \frac{1500.120}{210} =$ $1357\frac{1}{7}$ pounds.

Check, $p + q$ must equal whole load, and we have in effect: —

$$p + q = 1642\tfrac{6}{7} + 1357\tfrac{1}{7} = 3000,$$ which

being equal to the sum of the loads is correct, for: —

$$500 + 1000 + 1500 = 3000.$$

2. *Find Point of Greatest Bending Moment.*

Begin at p, pass over load 500, plus load 1000, and we still need to pass $142\frac{6}{7}$ pounds of load to make up amount of reaction p ($1642\frac{6}{7}$ lbs.); therefore, the greatest bending moment must be at load 1500; *check*, begin at q and we arrive only at the first load (1500) before passing amount of reaction q ($1357\frac{1}{7}$ lbs.), therefore, at load 1500 is the point sought.

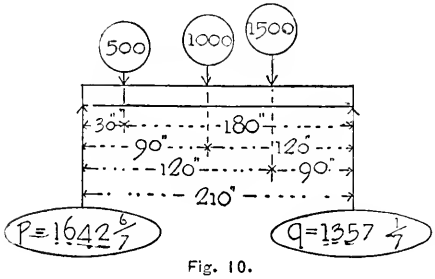


Fig. 10.

3. *Find Amount of Greatest Bending Moment.*

Suppose the beam cut at load 1500, then take the left-hand side of beam, and we have for the bending moment at the point where the beam is cut.

$$\begin{aligned} m &= 1642\tfrac{6}{7}.120 - (500.90 + 1000.30 + 1500.0) \\ &= 197143 - (45000 + 30000 + 0) \\ &= 197143 - 75000 \\ &= 122143 \text{ lbs. inch.} \end{aligned}$$

As a check on the calculation, take the right-hand side of beam and we should have: —

$$m = 1357\tfrac{1}{2}.90 - 1500.0$$
$$= 122143 - 0$$
$$= 122143 \text{ lbs. inch},$$

which, of course, proves the correctness of former calculation.

4. *Find the Required Cross-section of Beam.*

We must first find the required moment of resistance, and as the cross-section is to be uniform, we use formula (18), viz.: —

$$r = \frac{m}{\left(\frac{k}{f}\right)}$$

Now, $m = 122143$, and $\frac{k}{f} = \frac{2800}{4} = 700$, therefore,

$$r = \frac{122143}{700} = 174{,}49 \text{ or say } = 174{,}5$$

Consulting Table I, fourth column, for section No. 2, we find

$$r = \frac{bd^2}{6}, \text{ we have, therefore,}$$

$$\frac{bd^2}{6} = 174{,}5 \text{ or } bd^2 = 1047.$$

If the size of either b or d is fixed by local conditions, we can, of course, find the other size (d or b) very simply; for instance, if for certain reasons of design we did not want the beam to be more than 4″ wide, we should have

$$b = 4, \text{ therefore, } 4.d^2 = 1047, \text{ and}$$
$$d^2 = \frac{1047}{4} = 262, \text{ therefore, } d = (\text{about})\ 16'',$$

or, if we did not want the beam to be over 12″ deep, we should have

$$d = 12, \text{ and } d^2 = 12.12 = 144, \text{ therefore,}$$
$$b.144 = 1047, \text{ and } b = \frac{1047}{144} = 7{,}2'' \text{ or say } 7\tfrac{1}{4}''.$$

The deepest beam the most economical. One thing is very important and, *must be remembered*, that the *deeper* the beam is, the more economical, and the stiffer will it be. If the beam is too shallow, it might deflect so as to be utterly unserviceable, besides using very much more material. As a rule, it will therefore be necessary to calculate the beam for deflection as well as for its transverse strength.

Safe deflection. The deflection should not exceed 0,″03 that is, three one-hundredths of an inch for each foot of span, or else the plastering would be apt to crack, we have then the formula: —

$$\delta = L.\ 0{,}03 \qquad (28)$$

Where $\delta =$ the greatest allowable total deflection, in inches, at centre of beam, to prevent plaster cracking.

Where $L =$ the length of span, in feet.

In case the beam is so unevenly loaded that the greatest deflection will not be at the centre, but at some other point, use: —

$$\delta = X.\ 0{,}06 \qquad (29)$$

Where $\delta =$ the greatest allowable total deflection, in inches, at point of greatest deflection.

Where $X =$ the distance, in feet, to *nearer* support from point of greatest deflection.

If the beam is not stiffened sideways, it should also be calculated for lateral flexure. These matters will be more fully explained when treating of beams and girders.

COMPARATIVE STRENGTH AND STIFFNESS OF BEAMS AND COLUMNS.

(1) If a beam supported at both ends and loaded uniformly will safely carry an amount of load $= u$; then will the same beam:

(2) if both ends are built in solidly and load uniformly distributed, carry $1\frac{1}{2}.\ u$,

(3) if one end is supported and other built in solidly and load uniformly distributed, carry $1.\ u$,

(4) if both ends are built in solidly and load applied in centre, carry $1.\ u$,

(5) if one end is supported and other built in solidly and load applied in centre, carry $\frac{2}{3}.\ u$,

(6) if both ends are supported and load applied in centre, carry $\frac{1}{2}.\ u$,

(7) if one end is built in solidly and other end free (cantilever) and load uniformly distributed, carry $\frac{1}{4}.\ u$,

(8) if one end is built in solidly and other end free (cantilever) and load applied at free end, carry $\frac{1}{8}.\ u$.

That is, in cases (1), (3) and (4) the effect would be the same with the same amount of load; in case (2) the beam could safely carry $1\frac{1}{2}$ times as much load as in case (1); in case (5) the beam could safely carry only $\frac{2}{3}$ as much as in case (1), etc., *provided that the length of span is the same in each case.*[2]

If the amount of deflection in case (1) were δ_1 then would the amount of deflection in the other cases be as follows:

Case (2) $\delta_{II} = \frac{1}{2}.\ \delta_1$	Case (4) $\delta_{IV} = \frac{2}{5}.\ \delta_1$	Case (7) $\delta_{VII} = 9\frac{3}{5}.\ \delta_1$
Case (3) $\delta_{III} = \frac{2}{5}.\ \delta_1$	Case (5) $\delta_V = \frac{3}{4}.\ \delta_1$	Case (8) $\delta_{VIII} = 25\frac{3}{5}.\ \delta_1$
	Case (6) $\delta_{VI} = 1\frac{3}{5}.\ \delta_1$	

[2] To count on the end of a beam being built in solidly would be very bad practice in most cases of building construction; as, for instance, a wooden beam with end built in solidly could not fall out in case of fire, and would be apt to throw the wall. Even where practicable, it would require very careful supervision to get the beam built in properly; then, too, it causes upward strains which must be overcome, complicating the calculations unnecessarily. In most cases where it is necessary to "build in" beam ends, the additional strength and diminished deflection thereby secured had better be credited as an additional margin of safety. The above rules for deflection do not hold good if the beam is not of uniform cross-section throughout; the deflection being greater as the variation in cross-section is greater.

TABLE

BENDING-MOMENT (m) AND AMOUNT OF SHEARING-STRAIN (s)

Uniform load on cantilever.	Load at free end of cantilever.	Load at any point of Cantilever.
$m = \frac{u.l}{8}$	$m = \frac{w.l}{2}$	If y smaller than $\frac{l}{2}$: $m = o$
$s = \frac{u}{2}$	$s = w$	$s = o$
For x use: $m = \frac{u.(l-x)}{2}.x$	For x use: $m = w.(l-x)$	If x smaller than y use: $m = w.(y-x)$
$s = \frac{u.(l-x)}{l}$	$s = w$	$s = w$
At free end use: $m = o$	At free end use: $m = o$	If x greater than y use: $m = o$
$s = o$	$s = o$	$s = o$
If y greater than $\frac{l}{2}$: $m = w.y.\left(y - \frac{l}{2}\right)$		If $x = y$ use: $m = o$
$s = w$		$s = o$
at support p $m = \frac{u.l}{2}$	at support p $m = w.l$	at support p $m = w.y$
at support p $s = u$	at support p $s = w$	at support p $s = w$
at free end $\delta = \frac{1}{8} \cdot \frac{u.l^3}{e.i}$	at free end $\delta = \frac{1}{3} \cdot \frac{w.l^3}{e.i}$	at load $\delta = \frac{1}{3} \cdot \frac{w.y^3}{e.i}$
		at free end $\delta = \frac{1}{3} \cdot \frac{w.y.l^2}{e.i}$

VII. (See foot-note p. 60.)

OF BEAMS AND CANTILEVERS FOR VARIOUS LOADS.

Manner of Loading.	Description.	m and s at centre.	m and s at any point distant x from support p.	Location and amount of greatest bending-moment m.	Location and amount of greatest shearing-strain s.	Location and amount of greatest deflection δ.
	Uniform load on beam supported at both ends.	$m = \frac{u.l}{8}$ $s = 0$	$m = \frac{u}{2}.x.\left(1-\frac{x}{l}\right)$ $s = u.\left(\frac{1}{2}-\frac{x}{l}\right)$	at centre $m = \frac{u.l}{8}$	at support p or q. $s = \frac{u}{2}$	at centre $\delta = \frac{5}{384}.\frac{u.l^3}{e.i}$
	Load at centre of beam supported at both ends.	$m = \frac{w.l}{4}$ $s = 0$	When x greater than $\frac{l}{2}$ use $(l-x)$ in place of x. $m = \frac{w}{2}.x$ $s = \frac{w}{2}$	at centre $m = \frac{w.l}{4}$	at support p or q $s = \frac{w}{2}$	at centre $\delta = \frac{1}{48}.\frac{w.l^3}{e.i}$
	Load at any point on beam supported at both ends.	If y greater than $\frac{l}{2}$: $m = \frac{w.z}{2}$ $s = \frac{w.z}{l}$ If y smaller than $\frac{l}{2}$: $m = \frac{w.y}{2}$ $s = \frac{w.y}{l}$	At load use: $m = \frac{w.y.z}{l}$ $s = 0$ For x use: $m = \frac{w.z.x}{l}$ $s = \frac{w.z}{l}$ For x_1 use: $m = \frac{w.y.x_1}{l}$ $s = \frac{w.y}{l}$	at load $m = \frac{w.y.z}{l}$	at nearer support p If y smaller than $\frac{l}{2}$ $s = \frac{w.z}{l}$ at support q, If y greater than $\frac{l}{2}$ $s = \frac{w.y}{l}$	near centre $\delta = \frac{w.y.z}{9.l.e.i}.\sqrt{\frac{y.(l+z)^3}{3}}$

Strength of beams of different cross-sections. The comparative transverse strength of two or more rectangular beams or cantilevers is directly as the product of their breadth into the square of their depth, provided the span, material and manner of supporting and loading are the same, or

$$x = b\ d^{\,2} \qquad (30)$$

Where x = a figure for comparing strength of beams of equal spans.

Where b = the breadth of beam, in inches.

Where d = the depth of beam, in inches.

Example.

What is the comparative strength between a 3″ x 12″ *beam, and a* 6″ x 12″ *beam? Also, between a* 4″ x 12″ *beam, and a* 3″ x 16″ *beam? All beams of same material and span, and similarly supported and loaded.*

The strength of the 3″ x 12″ beam would be

$$x_{\mathrm{i}} = 3.\ 12.\ 12 = 432.$$

The strength of the 6″ x 12″ beam would be

$$x_{\mathrm{ii}} = 6.\ 12.\ 12 = 864, \text{ therefore,}$$

the latter beam would be just twice as strong as the former.

Again, the strength of the 4″ x 12″ beam would be

$$x_{\mathrm{ii'}} = 4.\ 12.\ 12 = 576$$

and the strength of the 3″ x 16″ beam would be

$$x_{\mathrm{iii}} = 3.\ 16.\ 16 = 768.$$

The latter beam would therefore be $\frac{768}{576}$ or just $1\frac{1}{3}$ times as strong as the former, while the amount of material in each beam is the same, as

$$4.\ 12 = 3.\ 16 = 48 \text{ square inches in each.}$$

The reason the last beam is so much stronger is on account of its greater depth.

Strength of beams of different lengths. The comparative transverse strength of two or more beams or cantilevers of same cross-section and material, but of unequal spans, is inversely as their lengths, provided manner of supporting and loading are the same. That is, a beam of twenty-foot span is only half as strong as a beam of ten-foot span, a quarter as strong as one of five-foot span, etc.

All measurements in Table VII are in inches; all weights in pounds; e = modulus of elasticity in pounds inch; i = moment of inertia of cross-section of beam or cantilever around its neutral axis in inches; m = bending-moment in pounds inch; s = amount of shearing strain in pounds; δ = total amount of deflection in inches.

Stiffness of beams of different lengths. The stiffness of beams or cantilevers of same cross-section and material (and similarly loaded and supported), however, diminishes very rapidly, as the length of span increases, or what is the same thing, the deflection increases much more rapidly in proportion than the length; the comparative stiffness or deflection being directly as the cube of their respective lengths or L^3.

That is if a beam 10 feet long deflects under a certain load one-third of an inch, the same beam with same load, but 20 feet long will deflect an amount x as follows:

$$x : \tfrac{1}{3} = 20^3 : 10^3, \text{ or } x = \frac{20^3 . \frac{1}{3}}{10^3} = \frac{8000}{3000} = 2\tfrac{2}{3}''$$

Stiffness of beams of different cross-sections. The comparative stiffness, that is amount of deflection of two or more beams or cantilevers, similarly supported and loaded, and of same material and span, but of different cross-sections, is inversely as the product of their respective breadths into the cubes of their respective depths or

$$x = \frac{1}{b.d^3} \qquad (31)$$

Where $x =$ a figure for comparing the deflection of beams of same material, span and load.

Where $b =$ the breadth of beam, in inches.

Where $d =$ the depth of beam, in inches.

Example.

If a beam 3″ x 8″ *deflects* $\frac{1}{2}$″ *under a certain load, what will a beam* 4″ x 12″ *deflect, if of same material and span, similarly supported and with same load?*

For the first beam we should have

$$x_1 = \frac{1}{3.8^3} = \frac{1}{1536} = 0{,}00065$$

For the second beam we should have

$$x_{11} = \frac{1}{4.12^3} = \frac{1}{6912} = 0{,}00014$$

The deflection of the latter beam will be as

$$\delta : 0'',5 = 0{,}00014 : 0{,}00065, \text{ or } \delta = 0'',108$$

Strength of beams of different lengths & cross-sections. The comparative strength of rectangular beams or cantilevers of different cross-sections and spans, but of same materials and similarly loaded and supported, is, of course, directly as the product of their breadth into the

squares of their depths, divided by their length of span, or

$$x = \frac{b.d^2}{L} \qquad (32)$$

Where x = a figure for comparing the strength of different beams of same material, but of different cross-sections and spans.

Where b = the breadth, in inches.

Where d = the depth, in inches.

Where L = the length of span, in feet.

Stiffness of beams of different lengths & cross-sections. The comparative stiffness or amount of deflection of different rectangular beams or cantilevers of same material, and similarly loaded and supported, but of different cross-sections and spans, would be directly as the cubes of their respective lengths, divided by the product of their respective breadths into the cubes of their depths or

$$x = \frac{L^3}{b.d^3} \qquad (33)$$

Where x = a figure for comparing the amount of deflections of beams of same material and load, but of different spans and cross-sections.

Where L = the length of span, in feet.

Where b = the breadth, in inches.

Where d = the depth in inches.

Strength and deflection of wooden beams, one inch thick. If it is desired to calculate a wooden girder supported at both ends and to carry its full safe uniform load, and yet not to deflect enough to crack plaster, the following will simplify the calculation:

TABLE VIII.

	Spruce.	Georgia pine.	White pine.	White oak.	Hemlock.
Calculate (x) for transverse strength only if d is greater than	$1\frac{1}{6}L$	L	$1\frac{1}{19}L$	$1\frac{1}{5}L$	$1\frac{4}{15}L$
Calculate (x_1) for deflection only if d is less than	$1\frac{1}{6}L$	L	$1\frac{1}{19}L$	$1\frac{1}{5}L$	$1\frac{4}{15}L$

Where L = the length of span, in feet.

Where d = the depth of beam, in inches.

Where x or x_1 = is found according to Table IX.

To find the safe load (x) or (x_1) *per running foot of span*, which a beam supported at both ends, and 1″ *thick* will carry, use the following table. (Beams two inches thick will safely carry twice as much

per running foot, as found per table, beams three inches thick three times as much, four inches thick four times as much, etc.)

TABLE IX.

	Spruce.	Georgia pine.	White pine.	White oak.	Hemlock.
If calculating for transverse strength only use	$x = 111\left(\frac{d}{L}\right)^2$	$x = 133\left(\frac{d}{L}\right)^2$	$x = 100\left(\frac{d}{L}\right)^2$	$x = 122\left(\frac{d}{L}\right)^2$	$x = 83\left(\frac{d}{L}\right)^2$
If calculating for a deflection (not to crack plaster) use	$x_1 = 95\left(\frac{d}{L}\right)^3$	$x_1 = 133\left(\frac{d}{L}\right)^3$	$x_1 = 95\left(\frac{d}{L}\right)^3$	$x_1 = 100\left(\frac{d}{L}\right)^3$	$x_1 = 89\left(\frac{d}{L}\right)^3$

Where $x =$ the safe load in lbs., per running foot of span, which a beam one-inch thick will carry regardless of deflection, if supported at both ends, and

$x_1 =$ the same, but without deflecting the beam enough to crack plaster; for thicker beams multiply x or x_1 by breadth, in inches.

Calculate for either x or x_1 as indicated in Table VIII.

Where $d =$ the depth of beam, in inches.

Where $L =$ the length of span in feet.

If a beam is differently supported, or not uniformly loaded, also for cantilevers, add or deduct from above result, as directed in cases 1 to 8, page 57.

Example.

A floor of 19′ *clear span is to be built with spruce beams, to carry* 100 *lbs. per square foot; what size beams would be the most economical?*

According to Table VIII, if $d = 1\frac{1}{6}$. $L = 1\frac{1}{6} \cdot 19 = 22\frac{1}{6}''$ we can calculate for either deflection or rupture and the result would be the same. If we make the beam deeper it will be so stiff that it will break before deflecting enough to crack plastering underneath; while if we make the beam more shallow it will deflect enough to crack plaster before it carries its total safe load. The former would be more economical of material, but, of course, in practice we should certainly not make a wooden beam as deep as 22″. Whatever depth we select, therefore, less than 22″, we need calculate for deflection only. We have, then, according to Table IX, second column,

$$x_1 = 95.\left(\frac{d}{L}\right)^3$$

If we use a beam 12″ deep, we should have

$$x_1 = 95.\frac{12^3}{19^3} = 24$$

or a beam 1″ x 12″ would carry 24 lbs. per foot; as the load is 100 lbs. per foot we should need a beam $\frac{100}{24} = 4\frac{1}{6}$ wide, or say a beam 4″ x 12″, and of course 12″ from centres.

If we use a beam 14″ deep we should have

$$x_1 = 95.\frac{14.^3}{19^3} = 38$$

or a beam 1″ x 14″ would carry 38 lbs. per foot, we need, therefore, a beam of width

$$b = \frac{100}{38} = 2\frac{12''}{19}$$

or we must use a beam say 3″ x 14″ and 12″ from centre, or a beam 4″ x 14″ and 16″ from centre. For if the beams are 16″ from centres each beam will carry per running foot $1\frac{1}{3}$.100 lbs. = 133 lbs. and a 4″ x 14″ will carry per foot

$$4.x_1 = 4.38 = 152.$$

We could even spread the beams farther apart, except for the difficulty of keeping the cross-furring strips sufficiently stiff for lathing.

Of course the 14″ beam is the most economical, for in the 12″ beam we use 4″ x 12″ = 48 square inches (cross-section) of material, and our beam is a trifle weak. While with the 14″ beam we use only 3″ x 14″ = 42 square inches of material, and our beam has strength to spare. The 4″ x 14″ beam 16″ from centres would be just as strong and use just as much material as the 3″ x 14″ beam 12″ from centres. If we wished to be still more economical of material, we might use a still deeper beam, but in that case it would be less than 3″ thick and might twist and warp. If the beam is not cross-bridged or supported sideways it might be necessary to calculate its strength for lateral flexure. That it will not shear off transversely we can see readily, as the load is so light, nor is there much danger of longitudinal shearing, still for absolute safety it would be better to calculate each strain.

Strength of columns different lengths. The comparative strength of columns of same cross-section is approximately inversely as the square of their lengths. Thus, if x be the strength of a column, whose length

is L, and x_1 be the strength of a column whose length is L_1, then we have *approximately*

$$x : x_1 = L_1^2 : L^2, \text{ or } x_1 = \frac{x.\ L^2}{L_1^2} \qquad (34)$$

Where $x_1 =$ approximately the strength of a column, L_1 feet long.

Where $x =$ the strength (previously ascertained or known), of a column of same cross-section, and L feet long.

Where L and $L_1 =$ the respective lengths of columns in feet.

The nearer L and L_1 are to each other the closer will be the result.

Strength of columns different cross-sections. The comparative strength *per square inch* of cross-section of columns of same length, but of different cross-sections, is, approximately, as their least outside diameter, or side, or

$$x : x_1 = b : b_1, \text{ or } x_1 = \frac{x.\ b_1}{b} \qquad (35)$$

Where $x_1 =$ approximately the strength of a column, per square inch, whose least side or diameter (outside) is $= b_1$.

Where $x =$ the strength per square inch (previously ascertained or known) of a column of same length, but whose least side or diameter (outside) is $= b$.

The more similar and the nearer in size the respective cross-sections are, the closer will be the result. That is, the comparison between two circular columns, each 1″ thick, will be very much nearer correct than between two circular columns, one $\frac{3}{4}$″ thick and the other 2″ thick, or between a square and a circular column. The thicker the shell of a column the less it will carry per square inch. The formulæ (34 and 35) are hardly exact enough for safe practice, but will do for ascertaining approximately the necessary size of column, before making the detailed calculation required by formula (3).

The approximate thickness required for the flanges of plate girders is as follows:

Approximate thickness of flange of plate girders.

$$x = \frac{\frac{r}{d} - a_1}{b} \qquad (36)$$

Where $x =$ the approximate thickness, in inches, of either flange of a riveted girder.

Where $b =$ the breadth of flange, *less rivet holes*, all in inches.

Where $d =$ the total depth of girder in inches.

Where $r =$ the moment of resistance in inches.

Where $a_1 =$ the area (less rivet holes) of cross-section of both angles at flange, in square inches.

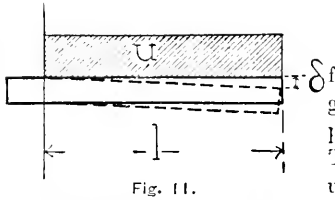


Fig. 11.

DEFLECTION.

The derivation of the following formulæ would be too lengthy to go into here, it will suffice for all practical purposes to give them. They are all based on the moduli of elasticity of the different materials, and are as follows:

FOR A CANTILEVER, UNIFORMLY LOADED.

$$\delta = \frac{1}{8} \cdot \frac{u.l^3}{e.i} \qquad (37)$$

FOR A CANTILEVER, LOADED AT FREE END.

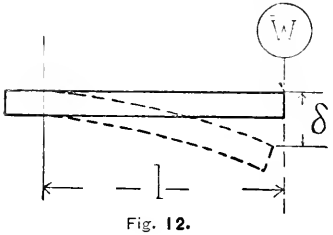


Fig. 12.

$$\delta = \frac{1}{3} \cdot \frac{w.l^3}{e.i} \qquad (38)$$

FOR A BEAM, UNIFORMLY LOADED.

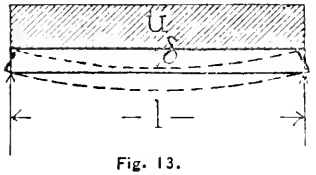


Fig. 13.

$$\delta = \frac{5}{384} \cdot \frac{u.l^3}{e.i} \qquad (39)$$

FOR A BEAM, LOADED AT CENTRE.

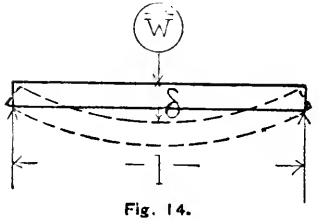


Fig. 14.

$$\delta = \frac{1}{48} \cdot \frac{w.l^3}{e.i} \qquad (40)$$

FOR A BEAM, LOADED AT ANY POINT.

Greatest deflection is near the centre, not at the point where load is applied.[1]

$$\delta = \frac{w.m.n.(l+n)}{9.l.e.i.} \cdot \sqrt{\frac{m.(l+n)}{3}} \qquad (41)$$

Where u = uniform load, in pounds.

Where w = concentrated load, in pounds.

Where l = length of span, in inches.

Where e = the modulus of elasticity, in pounds-inch, of the material, see Tables IV and V.

Where i = the moment of inertia, of cross-section, in inches.

Where m and n = the respective distances to supports, in inches.

Where δ = the greatest deflection, in inches (see Formulæ 28 and 29).

FOR A CANTILEVER, LOADED AT ANY POINT.

Greatest deflection is at free end; if y = distance from support to load, in inches, then:[2]

$$\delta = \frac{1}{3} \cdot \frac{w.y.l^2}{e.i} \qquad (42)$$

EXPANSION AND CONTRACTION OF MATERIALS.

Expansion of Iron trusses. All long iron trusses, *say about eighty feet long, or over*, should not be built-in solidly at both ends; otherwise the expansion and contraction due to variations of the temperature will either burst one of the supports, or else cause the truss to deflect so much, as to crack, and possibly endanger the work overhead. One end should be left free to move (lengthwise of truss) on rollers, but otherwise braced and anchored, the anchor sliding through slits in truss, as necessary. The expansion of iron for each additional single degree of temperature, Fahrenheit, is about equal to $\frac{1}{145000}$ of its length, that is, a truss 145

[1] The point of greatest deflection can never be further from the centre of beam than 2-25 of the entire length of span. It can as a rule, therefore, be safely assumed to be at the centre. If it is desired to find its exact location, use

$$x = \sqrt{\frac{l^2 - n^2}{3}} \qquad (43)$$

where n = the distance from weight to *nearer* support; x = the distance of point of greatest deflection from *farther* support; and l = the length of span; x, l and n should all be expressed either in feet or inches.

[2] Formula (42) is approximate only, but sufficiently exact for practical use.

feet long at 10° Fahrenheit, would gain in length (if the temperature advanced to 100° Fahrenheit), $\frac{90.145}{145000} = \frac{9}{100}$ of a foot, or, say, $1\frac{1}{2}$ inches, so that at 100° Fahrenheit the truss would be 145 feet and $1\frac{1}{2}$ inches long; this amount of expansion would necessitate rollers under one end. Of course the contraction would be in the same proportion. The approximate expansion of other materials for each additional degree Fahrenheit would be (in parts of their lengths), as follows:

Expansion and contraction of materials.		Pewter	$\frac{1}{78000}$
Wrought-iron	$\frac{1}{145000}$	Platina	$\frac{1}{208.00}$
Cast-iron	$\frac{1}{162000}$	Zinc	$\frac{1}{62000}$
Steel	$\frac{1}{151000}$	Glass	$\frac{1}{210000}$
Antimony	$\frac{1}{166000}$	Granite	$\frac{1}{208000}$
Gold, annealed	$\frac{1}{123000}$	Fire Brick	$\frac{1}{365000}$
Bismuth	$\frac{1}{130000}$	Hard Brick	$\frac{1}{600000}$
Copper	$\frac{1}{104000}$	White Marble	$\frac{1}{173000}$
Brass	$\frac{1}{95000}$	Slate	$\frac{1}{173000}$
Silver	$\frac{1}{95000}$	Sandstone	$\frac{1}{103000}$
Gun metal	$\frac{1}{90000}$	White pine	$\frac{1}{440000}$
Tin	$\frac{1}{87000}$	Cement	$\frac{1}{120000}$
Lead	$\frac{1}{63000}$		
Solder	$\frac{1}{70000}$		

The tension due to each additional degree of Fahrenheit would be equal to the modulus of elasticity of any material multiplied by the above fraction; or about 186 pounds per square inch of cross-section, for wrought-iron. Above figures are for linear dimensions, the superficial extension would be equal to twice the linear, while the cubical extension would be equal to three times the linear.

Water is at its maximum density at about 39° Fahrenheit; above that it expands by additional heat, and below that point it expands by less heat. At 32° Fahrenheit water freezes, and in so doing expands nearly $\frac{1}{12}$ part of its bulk, this strain equal to about 30,000 lbs. per square inch will burst iron or other pipes not sufficiently strong to resist such a pressure. The above table of expansions might be useful in many calculations of expansions in buildings; for instance, were

we to make the sandstone copings of a building in 10-foot lengths, and assume the variation of temperature from summer sun to winter cold would be about 150° Fahrenheit, each stone would expand $\frac{150.10}{103000} = \frac{1}{68}$ of a foot, or, say, about $\frac{1}{6}$ inches, quite sufficient to open the mortar joint and let the water in. The stones should, therefore, be much shorter.

GRAPHICAL METHOD OF CALCULATING STRAINS. — NOTATION.

Notation. The calculation of strains in trusses and arches is based on the law known as the "Parallelogram of Forces." Before going into same it will be necessary to explain the notation used. If Fig. 15 represents a truss, and the arrows the loads, and the two reactions (or supporting forces), we should call the left reaction O A and the right reaction F O. The loads would be, taking them in their order, A B, B C, C D, D E and E F. The foot, or lower half, of left rafter would be called B K, the upper half C I, while the respective parts of right rafter would be G E and H D. The King-post (tie) is I H, and the struts K I and H G, while the lower ties are K O and O G.

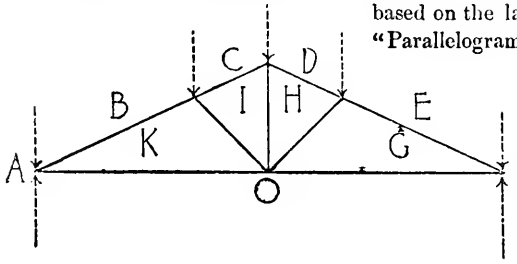


Fig. 15.

In the strain diagram, Fig. 16 (which will be explained presently), the notation is as usual; that is, loads A B, B C, C D, etc., are represented in the strain diagram by the lines *ab*, *bc*, *cd*, etc. Rafter pieces B K, C I, D H and E G are in the strain diagram *b k*, *c i*, *d h* and *e g* (*g* and *k* falling on the same point). I H in Fig 15 becomes *i h* in strain diagram. K I becomes *k i*, H G becomes *h g*, O K becomes *o k*, G O becomes *g o*, O A becomes *o a* and F O becomes *f o*.

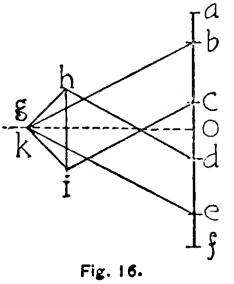


Fig. 16.

Or, in the drawing of the truss itself the lines are called, not by letters placed at the ends of the lines, but by letters placed *each side* of the lines, the lines being between; it is also usual to put these

letters in capitals to distinguish them from the letters representing the strain diagram, which are, as usual, at *each end* of the line they represent.

One thing is *very important*, however, and that is, always to read the pieces off in the *correct direction* and in their *proper order*. For instance, if we were examining the joint at middle of left rafter we must read off the pieces in their proper order, as B C, C I, I K, K B, and not jump, as B C, I K, C I, etc., as this would lead to error. Still more important is it to read around the joint in one direction, as from left to right (Fig. 17), that is, in the direction of the arrow. If we were to reverse the reading of the pieces, we should find the direction of the strain or stress reversed in the strain diagram. For instance, if we read K I and then find its corresponding line *k i* in the strain diagram, we find its direction downward, that is, pulling away from the joint, which would make K I a tie-rod, which, of course, is wrong, as we know it is a strut. If, however, we had read correctly *i k* it would be pushing upwards, which, of course, is correct and is the action of a strut.

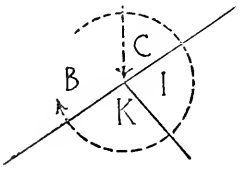


Fig. 17.

When we come to examine the joint at O, however, we reverse the above and here have to read *k i*, which is in the same relative direction for the point O, as was *i k* for the point at centre of left rafter.

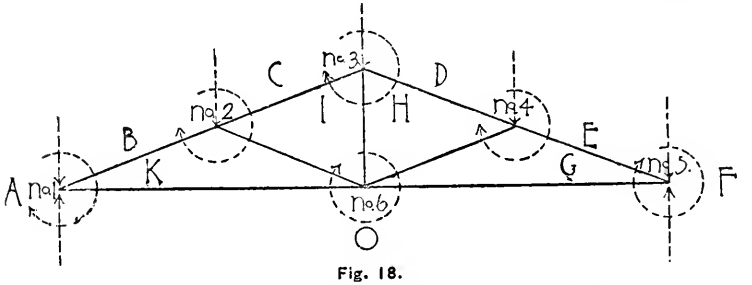


Fig. 18.

The arrows in the accompanying figure (18) show how each joint must be read, and *remember* always to read the pieces in their proper succession.

It makes no difference with which joint or with which piece of the joint we begin, so long as we read in correct succession and direction, thus: for joint No 1 we can read

A B, B K, K O and O A

or K O, O A, A B and B K,

or B K, K O, O A and A B, etc.

In the strain sheet of course we read in the same succession, and it will be found that the lines, as read, point *always* in the correct direction of the strain or stress.

PARALLELOGRAM OF FORCES.

Parallelogram of Forces. If a ball lying at the point A, Fig. 19, is propelled by a power sufficient to drive it in the direction of B, and as far as B in one minute, and at B is again propelled by a power sufficient to drive it in the direction of and as far as the point C in another minute, it will, of course, arrive at C at the end of two minutes, and by the route A B C.

If, on the other hand, both powers had been applied to the ball simultaneously, while lying at A, Fig. 20, it stands to reason that the ball would have reached C, but in one minute and by the route A C. A C (or E D), is, therefore, called the resultant of the forces A E and D A. If, now, we were to apply to the ball, while at A, simultaneously with the forces D A and A E, a third force (E D) sufficient to force the ball in the opposite direction to A C (that is, in the direction of C A), a distance equal to C A in one minute it stands to reason that the ball would remain perfectly motionless at A, as C A being the resultant (that is, the result) of the other two forces, if we oppose them with a power just equal to their own result, it stands to reason that they are completely neutralized. Now, applying this to a more practical case, if we had two sticks lying on A E and D A, Fig. 21, and holding the ball in place, and we apply to the ball a force E D = C A and in the direction C A, we can easily find how *much* each stick must resist or push against the ball. Draw a line *e d*, Fig. 22, parallel to E D, and of a length at any convenient scale equal in amount to force E D; through *e*, Fig. 22, draw *a e* parallel to A E, and through *d* draw *d a* parallel to D A, then the triangle *eda* (not *ead*) is the strain diagram for the Fig. 21, and *d a*,

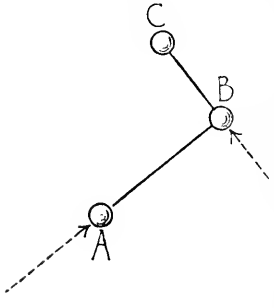


Fig. 19.

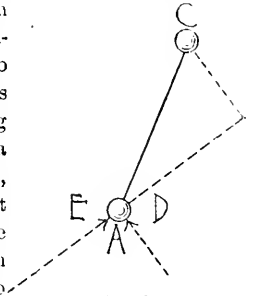
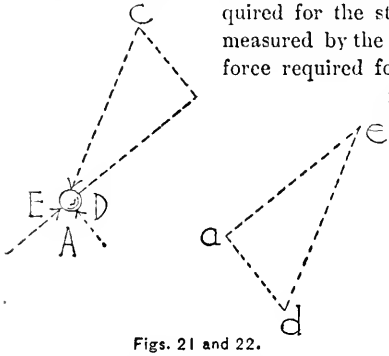


Fig. 20.

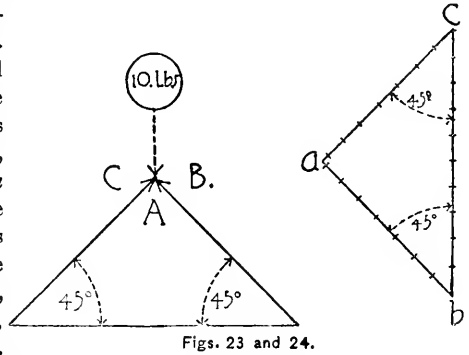
measured by the same scale as *e d*, is the amount of force required for the stick D A to exert, while *a e*, measured by the same scale, is the amount of force required for the stick A E to exert. If in place of the force E D we had had a load, the same truths would hold good, but we should represent the load by a force acting downward in a vertical and plumb line.



Figs. 21 and 22.

Thus, if two sticks, B A and A C, Fig. 23, are supporting a load of ten pounds at their summit, and the inclination of each stick from a horizontal line is 45°, we proceed in the same manner. Draw *c b*, Fig. 24, at any scale equal to ten units, through *b* and *c* draw *b a* and *a c* at angles of 45° each, with *c b*, then measure the number of (scale measure) units in *b a* and *a c*, which, of course, we find to be a little over seven. Therefore, each stick must resist with a force equal to a little over seven pounds.

Now, to find the direction of the forces. In Fig. 23 we read C B, B A and A C, the corresponding parts in the strain diagram, Fig. 24, are *c b*, *b a* and *a c*. Now the direction of *c b* is downwards, therefore C B acts downwards, which is, of course, the effect of a weight.



Figs. 23 and 24.

The direction, however, of *b a* and *a c* is upwards, therefore B A and A C must be pushing upwards, or *towards* the weight, and therefore they are in compression. The same truths hold good no matter how many forces we have acting at any point; that is, if the point remains in equilibrium (all the forces neutralizing each other), we can construct a strain diagram which will always be a *closed* polygon with as many sides as there are forces, and each side equal and parallel to one of the forces, and the sides being in the same succession

to each other as the forces are. We can now proceed to dissect a simple truss. Take a roof truss with two rafters and a single tie-beam. The rafters are supposed to be loaded uniformly, and to be strong enough not to give way transversely, but to transfer safely one-half of the load on each rafter to be supported on each joint at the ends of the rafter. We consider each joint separately.

Roof Trusses.

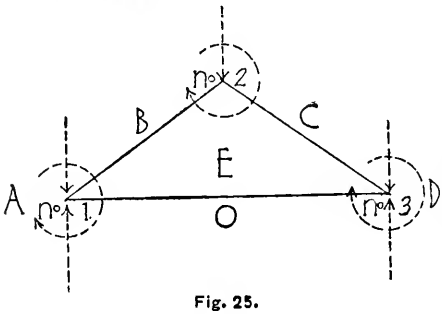


Fig. 25.

Take joint No. 1, Fig. 25.

We have four forces, one O A (the left-hand reaction), being equal to half the load on the whole truss; next, A B, equal to half the load on the rafter B E. Then we have the force acting along B E, of which we do not as yet know amount or direction (up or down), but only know that it is parallel to B E; the same is all we know, as yet, of the force E O. Now draw, at any scale, Fig. 26, No. 1, *o a* = and parallel to O A, then from *a* draw *a b* = and parallel to A B (*a b* will, of course, lap over part of *o a*, but this does not affect anything). Then from *b* draw *b e* parallel to B E, and through *o* draw *e o* parallel to E O. Now, in reading off strains, begin at O A, then pass in succession to A B, B E and E O. Follow on the strain diagram Fig. 26, No. 1, the direction as read off, with the finger (that is, *o a*, *a b*, *b e* and *e o*), and we have the actual directions of the strains. Thus *o a* is up, therefore pushing up; *a b* is down, therefore pushing down; *b e* is downwards, therefore pushing against joint No. 1 (and we know it is compression); lastly, *e o* is

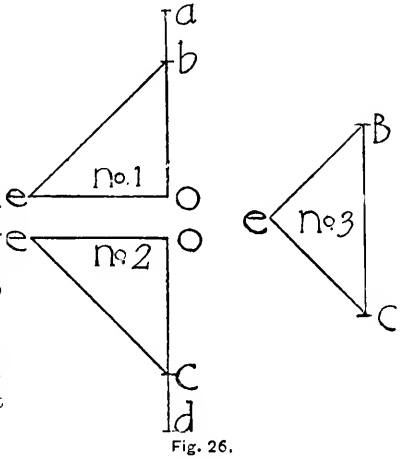


Fig. 26.

pushing to the right, therefore pulling away from the joint No. 1, and we know it is a tie-rod. In a similar manner we examine the joints 2 and 3, getting the strain diagrams No. 2 and No. 3 of Fig. 26. In Fig. 27, we get the same results exactly as in the above three diagrams of Fig. 26, only for simplicity they are combined into one diagram. If the single (combination) diagram, Fig. 27, should prove confusing to the student, let him make a separate diagram for each joint, if he will, as in Fig. 26. The above gives the principle of calculating the strength of trusses, graphically, and will be more fully used later on in practical examples.

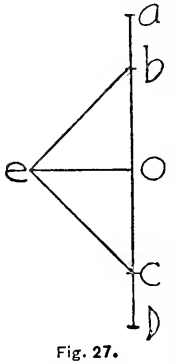


Fig. 27.

Should the student desire a fuller knowledge of the subject, let him refer to "*Greene's Analysis of Roof Trusses*," which is simple, short, and by far the best manual on the subject.

Line of Pressure Central. In roof and other trusses the line of pressure or tension will always be co-incident with the central line or longitudinal axis of each piece. Each joint should, therefore, be so designed that the central lines or axes of all the pieces will go through one point. Thus, for instance, the foot of a king post should be designed as per Fig. 28.

In roof-trusses where the rafters support purlins, the rafters must not only be made strong enough to resist the compressive strain on them, but *in addition to this enough material must be added* to stand the transverse strain. Each part of the rafter is treated as a separate beam, supported at each joint, and the amount of reaction at each joint must be taken as the load at the joint. The same holds good of the tie-beam, when it has a ceiling or other weights suspended from it; of course these weights must all be shown by arrows on the drawing of the truss, so as to get their full allowance in the strain diagram. Strains in opposite directions, of course, counteract each other; the stress, therefore, to be exerted by the material need only be equal to the difference between the amounts of the opposing strains, and, of course, this stress will be directed against the larger strain.

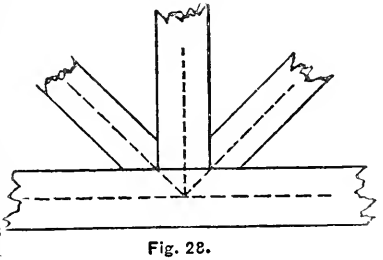


Fig. 28.

THE ARCH.

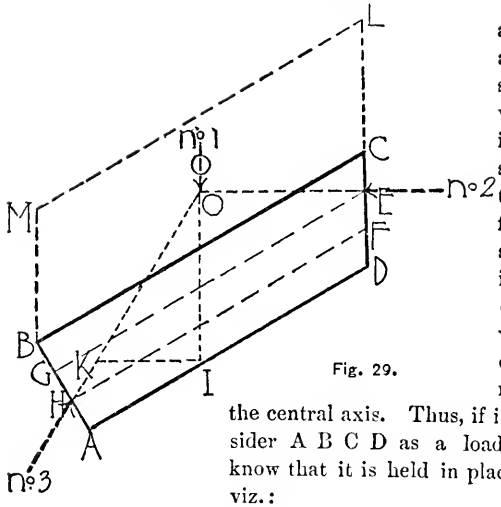


Fig. 29.

We consider an arch as a truss with a succession of straight pieces; we can calculate it graphically the same as any other truss, only we will find that the absence of central or inner members (struts and ties) will force the line of pressure, as a rule, far away from the central axis. Thus, if in Fig. 29, we consider A B C D as a loaded half-arch, we know that it is held in place by three forces, viz.:

1. The load B C L M which acts through its centre of gravity as indicated by arrow No. 1.

2. A horizontal force No. 2 at the crown C D, which keeps the arch from spreading to the right.

3. A force at the base B A (indicated by the arrow No. 3), which keeps the arch from spreading at the base. Now we know the direction and amount of No. 1, and can easily find Nos. 2 and 3. In an arch lightly loaded, No. 2 is always assumed to act at two-thirds way down C D, that is at F (where C E = E F = F D = $\frac{1}{3}$ C D). In an arch heavily loaded, No. 2 is always assumed to act one-third way down C D, that is at E; further the force No. 3 is always assumed to act through a point two-thirds way down B A, that is at H (where B G = G H = H A = $\frac{1}{3}$ B A). The reason for these assumptions need not be gone into here. Therefore to find forces Nos. 2 and 3 proceed as follows: If the arch is heavily loaded, draw No. 2 horizontally through E (C E being equal to $\frac{1}{3}$ C D), prolong No 2 till it intersects No. 1 at O, then draw O H (H A being equal to $\frac{1}{3}$ B A), which gives the direction of the resistance No. 3. We now have the three forces acting on the arch concentrated at the point O, and can easily find the amounts of each by using the parallelogram of forces. Make O I vertical and (at any scale) equal to whole load (or No. 1), draw I K horizontally, till it intersects O H at K; then scale I K,

and K O (at same scale as O I), which will give the amount of the forces Nos. 2 and 3. The line of pressure of this arch A B C D, is therefore not through the central axis, but along E O H (a curve drawn through E and H with the lines Nos. 2 and 3 as tangents is the real line of pressure).

Line of pressure not central.

Now let in Fig. 30, A B C E F D A represent a half-arch. We can examine A B C D same as before, and obtain I K = to force

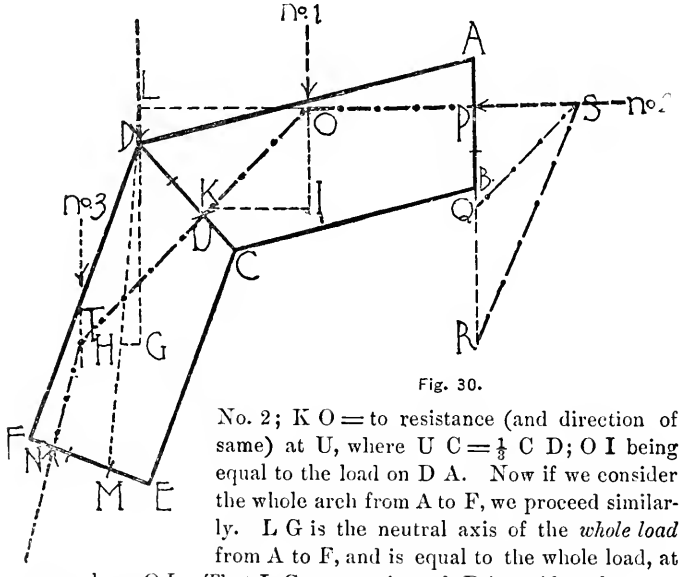


Fig. 30.

No. 2; K O = to resistance (and direction of same) at U, where U C = $\frac{1}{3}$ C D; O I being equal to the load on D A. Now if we consider the whole arch from A to F, we proceed similarly. L G is the neutral axis of the *whole load* from A to F, and is equal to the whole load, at same scale as O I. That L G passes through D is accidental.

Make E M = $\frac{1}{3}$ E F and draw L M; also G H horizontally till it intersects L M at H, then is G H the horizontal force or No. 2. We now have two different quantities for force No. 2, viz.: I K and G H, I K in this case being the larger. It is evident that if the whole half-arch is one homogeneous mass, that the greatest horizontal thrust of any one part, will be the horizontal thrust of the whole, we select therefore the larger force or I K as the amount of the horizontal thrust.

Now make S P = to I K and P Q = to No. 1, or load on A D and P R = to L G or whole load on F A, at any scale, then draw Q S and R S. Now at O we have the three forces concentrated, which act on the part of arch A B C D, viz.: Load No. 1 (= P Q), horizontal force No 2 (= S P) and resistance K O (= Q S). Now let No. 3 represent the vertical neutral axis of the part of whole load on F D, then

prolong K O until it intersects No. 3 at T; then at T we have the three forces acting on the part of arch E C D F, viz.: The load No. 3 (= Q R), the thrust from A B C D, viz.: O T (= S Q), and the resistance N T (= R S). To obtain N T draw through T a line parallel to R S, of course R S giving not only the direction, but also the amount of the resistance N T. The line of pressure of this arch therefore passes along P O, O T, T N. A curve drawn through points P, U and N — (that is, where the former lines intersect the joints A B, D C, F E) — and with lines P O, O T and T N as tangents is the real line of pressure. Of course the more parts we divide the arch into, the more points and tangents will we have, and the nearer will our line of pressure approach the real curve.

Now if this line of pressure would always pass through the exact centre or axis of the arch, the compression on each joint would of course be uniformly spread over the whole joint, and the amount of this compression on each square-inch of the joint would be equal to the amount of (line of) pressure at said joint, divided by the area of the cross-section of the arch in square inches, at the joint, but this rarely occurs, and as the position of the line of pressure varies from the central axis so will the strains on the cross section vary also.

Stress at intrados and extrados. Let the line A B in all the following figures represent the section of any joint of an arch (the thickness of arch being overlooked) C D the amount and actual position of line of pressure at said joint and the small arrows the stress or resistance of arch at the joint.

We see then that when C D is in the centre of A B, Fig. 31, the stress is uniform, that is the joint is, uniformly compressed, the amount of compression being equal to the average as above. As the line of pressure C D approaches one side, Fig. 32, the amount of compression on that side increases, while on the further side it decreases, until the line of pressure C D, reaches one-third way, Fig. 33 (that is, D B=$\frac{1}{3}$ A B). Then we see there is no compres-

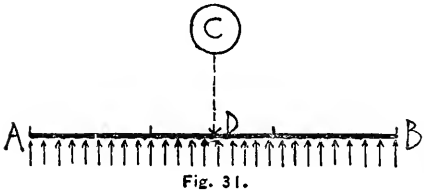


Fig. 31.

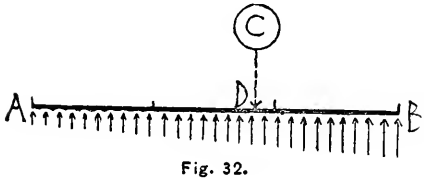


Fig. 32.

sion at A, but at B the compression is equal to just double the average as it was in Fig. 31. Now, as C D passes beyond the central third of A B, Fig. 34, the compression at the nearer side increases still further, while the further side begins to be subjected to stress in the opposite direction or tension, this action increasing of course the further C D is moved from the central third. This means that the edge of arch section at B would be subject to very severe crushing, while the other edge (at A) would tend to separate or open. When the line C D passes on to the edge B, the nearer two-thirds of arch joint will be in compression, and the further third in tension. As the line passes out of joint, and further and further away from B, less and less of the joint is in compression, while more and more is in tension, until the line of pressure C D gets so far away from the joint finally, that one-half of the joint would be in tension, and the other half in compression.

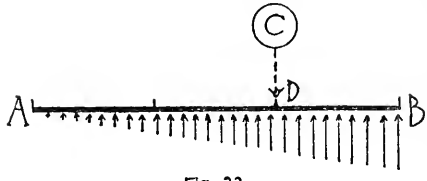


Fig. 33.

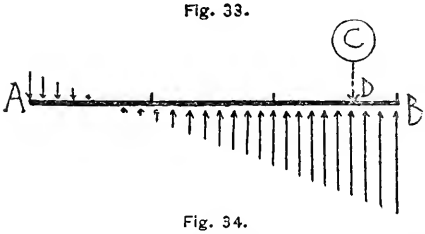


Fig. 34.

Tension means that the joint is tending to open upwards, and as arches are manifestly more fit to resist crushing of the joints than opening, it becomes apparent why it is dangerous to have the line of pressure far from the central axis. Still, too severe crushing strains must be avoided also, and hence the desirability of trying to get the line of pressure into the inner third of arch ring, if possible.

But the fact of the line of pressure coming outside of the inner third of arch ring, or even entirely outside of the arch, does not necessarily mean that the arch is unstable; in these cases, however, we must calculate the exact strains on the extreme fibres of the joint at both the inner and outer edges of the arch (intrados and extrados), and see to it that these strains do not exceed the *safe* stress for the material.

The formulæ to be used, are:

For the fibres at the edge nearest to the line of pressure

$$v = \frac{p}{a} + 6.\frac{x.p}{a.d} \qquad (44)$$

And for the fibres at the edge furthest from the line of pressure

$$v = \frac{p}{a} - 6.\frac{x.p}{a.d} \qquad (45)$$

Where $v =$ the stress in lbs., required to be exerted by the extreme edge fibres (at intrados and extrados).

Where $x =$ the distance of line of pressure from centre of joint in inches.

Where $a =$ the area of cross section of arch at the joint, in square inches.

Where $p =$ the total amount of pressure at the joint in lbs.

Where $d =$ the depth of arch ring at the joint in inches, measured from intrados to extrados.

When the result of the formulæ (44) and (45) is a positive quantity the stress v should not exceed $\left(\frac{c}{f}\right)$, that is the safe compressive stress of the material. When, however, the result of the formula (45) yields a negative quantity, the stress v should not exceed $\left(\frac{t}{f}\right)$, that is the safe tensile stress of the material, or mortar.

The whole subject of arches will be treated much more fully later on in the chapter on arches.

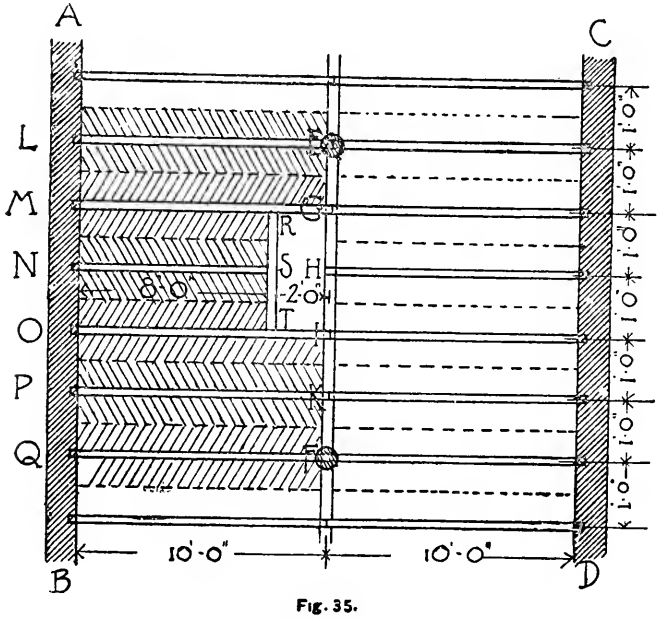


Fig. 35.

TO ASCERTAIN AMOUNT OF LOADS.

Let A B C D be a floor plan of a building, A B and C D are the walls, E and F the columns, with a girder between, the other lines being floor beams, all 12″ between centres; on the left side a well-hole is framed 2′ x 2′. Let the load assumed be 100 pounds per square foot of floor, which includes the weight of construction. Each of the right-hand beams, also the three left-hand beams E L, K P and F Q will each carry, of course, ten square feet of floor, or

Load on Beams.

10.100 = 1000 pounds each uniform load. Each will transfer one-half of this load to the girder and the other half to the wall. The tail beam S N will carry 8 square feet of floor, or

8.100 = 800 pounds uniform load. One-half of this load will be transferred to the wall, the other half to the header R T, which will therefore carry a load of 400 pounds at its centre, one-half of which will be transferred to each trimmer.

The trimmer beam G M carries a uniform load, one-half foot wide, its entire length, or fifty pounds a foot (on the off-side from well-hole), or

50.10 = 500 pounds uniform load, one-half of which is transferred to the girder and the other half to the wall. The trimmer also carries a similar load of fifty pounds a foot on the well-hole side, but only between M and R, which is eight feet long, or

50.8 = 400 pounds, the centre of this load is located, of course, half way between M and R, or four feet from support M, and six feet from support G, therefore M will carry (react)

$$\frac{6.400}{10} = 240 \text{ pounds and G will carry}$$

$$\frac{4.400}{10} = 160 \text{ pounds.}$$

See Formulæ (14) and (15).

We also have a load of 200 pounds at R, transferred from the header on to the trimmer; as R is two feet from G, and eight feet from M, we will find by the same formulæ, that G carries

$$\frac{8.200}{10} = 160 \text{ pounds and M carries}$$

$$\frac{2.200}{10} = 40 \text{ pounds.}$$

So that we find the loads which the trimmer transfers to G and M, as follows:

At M = 250 + 240 + 40 = 530 pounds.
" G = 250 + 160 + 160 = 570 pounds.

Load on Walls. The loads which trimmer O I transfers to wall and girder will, of course, be similar. We therefore find the total loading, as follows:

On the wall A B:

At L = 500 pounds.
" M = 530 pounds.
" N = 400 pounds.
" O = 530 pounds.
" P = 500 pounds.
" Q = 500 pounds.

Total on wall A B = 2960 pounds.

Load on Girder. On the wall C D we have six equal loads of 500 pounds each, a total of 3000 pounds.

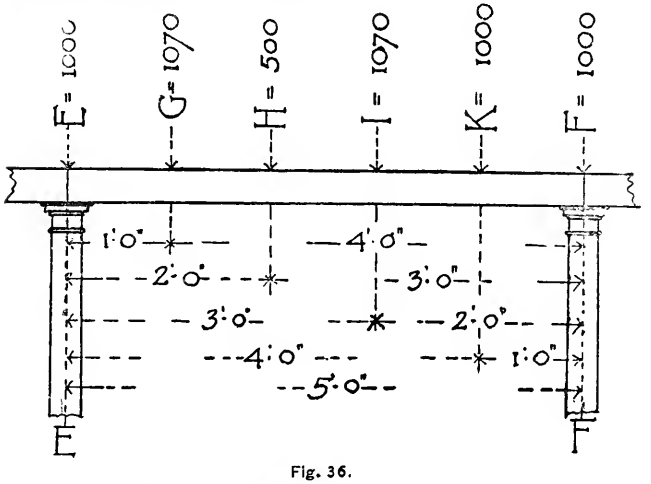


Fig. 36.

On the girder E F, we have:

At E from the left side 500 pounds, from the right 500 pounds. Total 1000 pounds.

At G from the left side 570 pounds, from the right 500 pounds. Total 1070 pounds.

At H from the left side nothing, from the right 500 pounds. Total 500 pounds.

At I from the left side 570 pounds, from the right 500 pounds. Total 1070 pounds.

At K from the left side 500 pounds, from the right 500 pounds. Total 1000 pounds.

At F from the left side 500 pounds, from the right 500 pounds. Total 1000 pounds.

Total on girder 5640 pounds.

As the girder is neither uniformly nor symmetrically loaded, we must calculate by Formulæ (16) and (17), the amount of each reaction, which will, of course, give the load coming on the columns E and F. (These columns will, of course carry additional loads, from the girders on opposite side, further, the weight of the column should be added, also whatever load comes on the column at floor above.)

Girder E F then transfers to columns,

At E $= 1000 + (\frac{4}{5}. 1070) + (\frac{3}{5}. 500) + (\frac{2}{5}. 1070) + (\frac{1}{5}. 1000) + (0. 1000) = 2784$ pounds.

At F $= 1000 + (\frac{4}{5}. 1000) + (\frac{3}{5}. 1070) + (\frac{2}{5}. 500) + (\frac{1}{5}. 1070) + (0. 1000) = 2856$ pounds.

As a check the loads at E and F must equal the whole load on the girder, and we have, in effect,

$$2784 + 2856 = 5640.$$

Now as a check on the whole calculation the load on the two columns and two walls should equal the whole load. The whole load being 20′ x 6′ x 100 pounds minus the well-hole 2′ x 2′ x 100 pounds, or $12000 - 400 = 11600$ pounds.

And we have in effect,

Load on A B	=	2960 pounds.
" C D	=	3000 pounds.
" two columns	=	5640 pounds.
Total loads	=	11600 pounds.

We therefore can calculate the strength of all the beams, headers and trimmers and girders, with loads on, as above given.

Load over Wall openings.

For the columns and walls, we must however add, the weight of walls and columns above, including all the loads coming on walls and columns above the point we are calculating for, also whatever load comes on the columns from the other sides. If there are openings in a wall, one-half the load over each opening goes to the pier each side of the opening, including, of course, all loads on the wall above the opening.

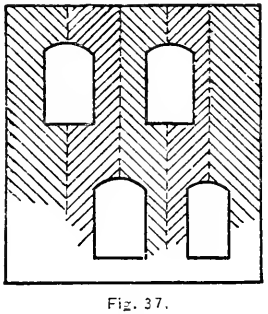


Fig. 37.

Thus in Figure 37, the weight of walls would be distributed, as

indicated by etched lines; where, however, the opening in the wall is very small compared to the mass of wall-space over, it would, of course, be absurd to consider all this load as on the arch, and practically, after the mortar has set, it would not be, but only an amount about equal to the part enclosed by dotted lines in Figure 38, the inclined lines being at an angle of 60° with the horizon. Where only part of the wall is calculated to be carried on the opening, the wooden centre should be left in until the mortar of the entire wall has set. In case of beams or lintels the wall should be built up until the intended amount of load is on them, leaving them free underneath; after the intended load is on them, they should be shored up, until the rest of wall is built and thoroughly set.

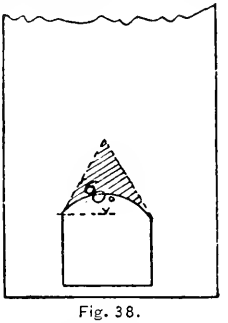


Fig. 38.

Wind Pressure and Snow. Wind-pressure on a roof is generally assumed at a certain load per square foot superficial measurement of roof, and added to the actual (dead) weight of roof; except in large roofs, or where one foot of truss rests on rollers, when it is important to assume the wind as a separate force, acting at right angles to incline of rafter.

The load of snow on roofs is generally omitted, when wind is allowed for, as, if the roof is very steep snow will not remain on it, while the wind pressure will be very severe; while, if the roof is flat there will be no wind pressure, the allowance for which will, of course, offset the load of snow.

If the roof should not be steep enough for snow to slide off, a heavy wind would probably blow the snow off.

In case of "continuous girders," that is, beams or girders supported at three or more points and passing over the intermediate supports without being broken, it is usual to allow more load on the central supports, than the formulæ (14) to (17) would give. This subject will be more fully dealt with in the chapter on beams and girders.

FATIGUE.

If a load or strain is applied to a material and then removed, the material is supposed to recover its first condition (provided it has not been strained beyond the limit of elasticity). This practically, however, is not the case, and it is found that a small load or strain often applied and removed will do more damage (fatigue the material more) than a larger one left on steadily. Most loads in buildings

are stationary or "dead" loads. But where there are "moving" loads, such as people moving, dancing, marching, etc., or machinery vibrating, goods being carted and dumped, etc., it is usual to assume larger loads than will ever be imposed; sometimes going so far as to double the actual intended load, or what amounts to the same thing, doubling (or increasing) the factor-of-safety, in that case retaining, of course, the actual intended load in the calculations. This is a matter in which the architect must exercise his judgment in each individual case.

Moving Loads.

CHAPTER II.

FOUNDATIONS.

Nature of Soils. THE nature of the soils usually met with on building sites are: rock, gravel, sand, clay, loamy earth, "made" ground and marsh (soft wet soil).

If the soil is hard and *practically* non-compressible, it is a good foundation and needs no treatment; otherwise it must be carefully prepared to resist the weight to be superimposed.

Stepping Courses. The base-courses of all foundation walls must be spread (or stepped out) sufficiently to so distribute the weight that there may be no appreciable settlement (compression) in the soil.

Two important laws must be observed:—

1. All base-courses must be so proportioned as to produce exactly the same pressure per square inch on the soil under all parts of building where the soil is the same. Where in the same building we meet with different kinds of soils, the base-courses must be so proportioned as to produce the same relative pressure per square inch on the different soils, as will produce an equal settlement (compression) in each.

2. Whenever possible, the base-course should be so spread that its neutral axis will correspond with the neutral axis of the superimposed weight; otherwise there will be danger of the foundation walls settling unevenly and tipping the walls above, producing unsightly or even dangerous cracks.

Example.

In a church the gable wall is 1′ 6″ *thick, and is loaded (including weight of all walls, floors and roofs coming on same) at the rate of* 52 *lbs. per square inch. The small piers are* 12″ x 12″ *and* 5′ *high, and carry a floor space equal to* 14′ x 10′. *What should be the size of base-courses, it being assumed that the soil will safely stand a pressure of* 30 *lbs. per square inch?*

If we were to consider the wall only, we should have the total pressure on the soil per running inch of wall, $18.52 = 936$ lbs.

Dividing this by 30 lbs., the safe pressure, we should need $\frac{936}{30} = 31{,}2''$ or say 32″ width of foundation, or we should step out *each side* of foundation wall an amount $\frac{32-18}{2} = 7''$ each side.

Now the load on pier, assuming the floor at 100 lbs. per square foot, would be $14 \times 10 \times 100 = 14000$ lbs. To this must be added the weight of the pier itself. There are 5 cubic feet of brickwork (weighing 112 pounds per foot) $= 5.112 = 560$ lbs., or, including base-course, a total load of say 15000 lbs. This is distributed over an average of 144 square inches; therefore pressure per square inch under pier.

$$\frac{15000}{144} = 104 \text{ or, say, } 100 \text{ lbs.}$$

We must therefore make the foundation under pier very much wider, in order to avoid unequal settlements. The safe pressure per square inch we assumed to be 30 lbs.; therefore the area required would be $= \frac{15000}{30} = 500$ square inches, or a square about 22″ x 22″.

We therefore shall have to step out *each side* of the pier an amount $\frac{22-12}{2} = 5''$.

The safe compressions for different soils are given in Table V, but in most cases it is a matter for experienced judgment or else experiment.

Testing Soils. It is usual to bore holes at intervals, considerably deeper than the walls are intended to go, at some spot where no pressure is to take place, thus enabling the architect to judge somewhat of the nature of the soil. If this is not sufficient, he takes a crowbar, and, running it down, his experienced touch should be able to tell whether the soil is solid or not. If this is not sufficient, a small boring-machine should be obtained, and samples of the soil, at different points of the lot, bottled for every one or two feet in depth. These can be taken to the office and examined at leisure. The boring should be continued if possible, until hard bottom is struck.

If the ground is soft, new made, or easily compressible, experiment as follows: Level the ground off, and lay down four blocks each, say 3″ x 3″; on these lay a stout platform. Alongside of platform plant a stick, with top level of platform marked on same. Now pile weight onto platform gradually, and let same stand. As soon as platform begins to sink appreciably below the mark on stick, you

have the practical ultimate resistance of the foundation; this divided by 36 gives the ultimate resistance of the foundation per square inch. One-tenth of this only should be considered as a safe load for a permanent building.

Drainage of soil. Drainage is essential to make a building healthy, but can hardly be gone into in these articles. Sometimes it is also necessary to keep the foundations from being undermined.

It is usual to lead off all surface or spring water by means of blind drains, built underground with stone, gravel, loose tile, agricultural-tile, half-tile, etc. To keep dampness out, walls are cemented and then asphalted, both on the outsides. If the wall is of brick, the cementing can be omitted. Damp-courses of slate or asphalt are built into walls horizontally, to keep dampness from rising by capillary attraction. Cellar bottoms are concreted and then asphalted; where there is pressure of water from

Damp-proofing.

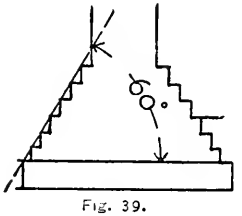


Fig. 39.

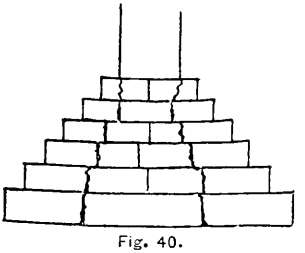


Fig. 40.

underneath, such as springs, tide-water, etc., the asphalt has to be sufficiently weighted down to resist same, either with brick paving or concrete.

Where there are water-courses they should be diverted from the foundations, but never dammed up. They can often be led into iron or other wells sunk for this purpose, and from there pumped into the building to be used to flush water-closets, or for manufacturing or other purposes. Clay, particularly in vertical or inclined layers, and sand are the foundations most dangerously affected by water as they are apt to be washed out.

Where a very wide base-course is required by the nature of the soil, it is usual to step out the wall above gradually; the angle of stepping should never be more acute than 60°, or, as shown in Figure 39. Care must also be taken that the stepped-out courses are sufficiently wide to project well in under each other and wall, to prevent same breaking through foundation, as indicated in Figure 40.

Where, on account of party lines or other buildings, the stepping

out of a foundation wall has to be done entirely at one side, the stepping should be even steeper than 60°, if possible; and particular attention must be paid to anchoring the walls together as soon and as thoroughly as possible, in order to avoid all danger of the foundation wall tipping outwardly.

Where a front or other wall is composed of isolated piers, it is well to combine all their foundations into one, and to step the piers down for this purpose, as shown in Figure 41. Where there is not sufficient depth for this purpose, inverted arches must be resorted to.

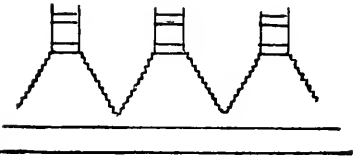


Fig. 41.

The manner of calculating the strength of inverted arches will be given under the article on arches. Inverted arches are not recommended, however (except where the foundation wall is by necessity very shallow), as it requires great care and good mechanics to build them well.

Inverted arches.

Two things must particularly be looked out for: 1, That the end arch has sufficient pier or other abutment; otherwise it will throw the pier out, as indicated in Figure 42. (This will form part of calculation of strength of arch.) Where there is danger of this, ironwork should be resorted to, to tie back the last pier.

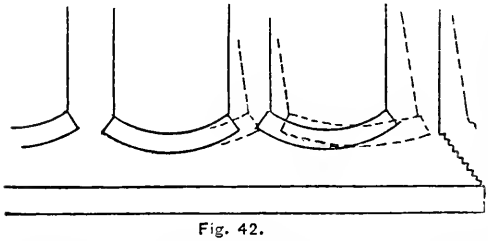


Fig. 42.

Size of Skew-back.

2. The skew-back of the arch should be sufficiently wide to take its proportionate share of load from the pier (that is, amount of the two skew-backs should be proportioned to balance of pier or centre part of pier, as the width of opening is to width of pier); otherwise the pier would be apt to crack and settle past arch, as shown in Figure 43.

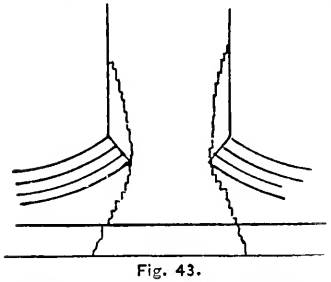


Fig. 43.

An easy way of getting the width of skew-back graphically is given below. In Figure 44,

draw A B horizontally at springing-line of inverted arch; bisect A C at F, and C B at E. Draw E O at random to vertical through F; then draw O C, and parallel to O C draw G D; then is C D the required skew-back.[1]

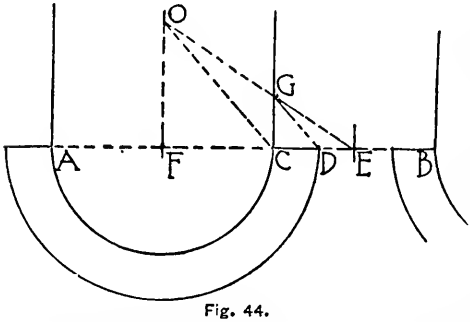


Fig. 44.

A good way to do is to give the arches wide skew-backs, and then to introduce a thick granite or blue-stone pier stone over them, as shown in Figure 45. This will force all down evenly and avoid cracks. The stone must be thick enough not to break at dotted lines, and should be carefully bedded.

Example.

A foundation pier carrying 150000 *lbs. is* 5′ *wide and* 3′ *broad The inverted arches are each* 24″ *deep. What thickness should the granite block have?*

We have here virtually a granite beam, 60″ long and 36″ broad, supported at two points (the centre lines of skew-backs) 36″ apart. The load is a uniform load of 150000 lbs. The safe modulus of rupture, according to Table V, for average granite is $\left(\frac{k}{f}\right) = 180$ lbs.

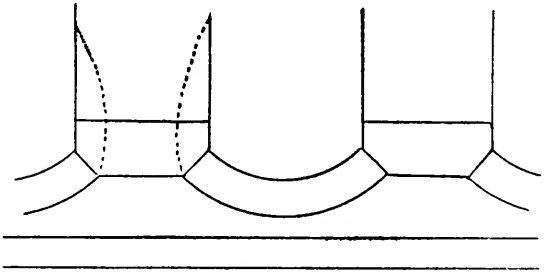


Fig. 45.

[1] In reality C D should be somewhat larger than the amount thus obtained; but this can be overlooked, except in cases where the pier approaches in width the width of opening. In such cases, however, stepping can generally be resorted to in place of inverted arches. Then, too, if the opening were very wide and the line of pressure came very much outside of central third of C D, it might be necessary to still further increase the width of skew-back, C D.

The bending moment on this beam, according to Formula (21) is

$$m = \frac{u.l}{8} = \frac{150\,000.36}{8} = 675\,000$$

The moment of resistance, r, is, from Formula (18)

$$r = \frac{m}{\left(\frac{k}{f}\right)} = \frac{675\,000}{180} = 3\,750$$

From Table I, No. 3, we find

$$r = \frac{bd^2}{6}, \text{ therefore}$$

$$\frac{bd^2}{6} = 3750 \text{ ; now, as } b = 36, \text{ transpose and we have}$$

$$d^2 = \frac{3750.6}{36} = 625.$$

Therefore $d = 25''$ or say $24''$. The size of granite block would have to be 5′ x 3′ x 2′.

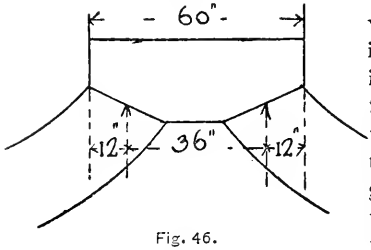


Fig. 46.

As this would be a very unwieldy block, it might be split in two lengthwise of pier; that is, two stones, each 5′ x 18″ x 2′ should be used, and clamped together. Before building piers, the arch should be allowed to get thoroughly set and hardened, to avoid any after shrinkage of the joints.

A parabolic arch is best. Next in order is a pointed arch, then a semi-circular, next elliptic, and poorest of all, a segmental arch, if it is very flat. But, as before mentioned, avoid inverted arches, if possible, on account of the difficulty of their proper execution.

Rock foundations. A rock foundation makes an excellent one, and needs little treatment, but is apt to be troublesome because of water. Remove all rotten rock and step off all slanting surfaces, to make

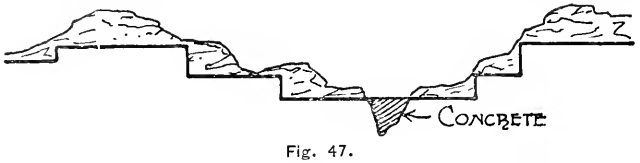


Fig. 47.

level beds, filling all crevices with concrete, as shown in Figure 47:

In no case build a wall on a slanting foundation.

Look out for springs and water in rock foundations. Where soft

soils are met in connection with rock, try and dig down to solid rock, or, if this is impossible, on account of the nature of the case or expense, dig as deep as possible and put in as wide a concrete base-course as possible. If the bad spot is but a small one, arch over from rock to rock, as shown in Figure 48:

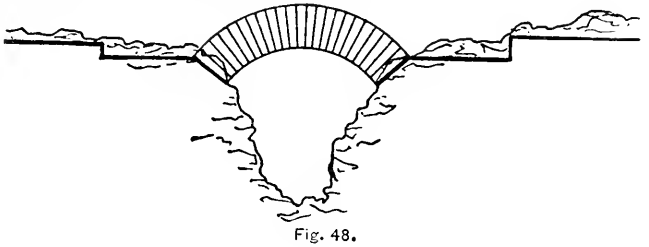


Fig. 48.

Sand, gravel and clay. Good hard sand or even quicksand makes an excellent foundation, if it can be kept from shifting and clear of water. To accomplish this purpose it is frequently "sheath-piled" each side of the base course.

Gravel and sand mixed make an excellent, if not the best foundation; it is practically incompressible, and the driest, most easily drained and healthiest soil to build on.

Clay is a good foundation, if in horizontal layers and of sufficient thickness to bear the superimposed weight. It is, however, a very treacherous material, and apt to swell and break up with water and frost. Clay in inclined or vertical layers cannot be trusted for important buildings, neither can loamy earth, made ground or marsh. If the base-course cannot be sufficiently spread to reduce the load to a minimum, pile-driving has to be resorted to. This is done in many different ways. If there is a layer of hard soil not far down, short piles are driven to reach down to same. These should be of sufficient diameter not to bend under their load; they should be calculated the same as columns. The tops should be well tied together and braced, to keep them from wobbling or spreading.

Short piles.

Example.

Georgia-pine piles of 16″ *diameter are driven through a layer of soft soil* 15′ *deep, until they rest on hard bottom. What will each pile safely carry?*

The pile evidently is a circular column 15′ long, of 16″ diameter, solid, and we should say with rounded ends, as, of course, its bear-

ings cannot be perfect. From Formula (3) we find, then, that the pile will safely carry a load.

$$w = \frac{a.\left(\frac{c}{f}\right)}{1 + \frac{l^2.n}{\rho^2}},$$ now from Table I, Section No. 7, and fifth column, we have

$$a = \frac{22}{7}.\ r^2 = \frac{22.8^2}{7} = 201.$$

From the same table we find, for Section No. 7, last column,

$$\rho^2 = \frac{r^2}{4} = \frac{8^2}{4} = 16.$$

From Table IV we find for Georgia pine, along fibres, $\left(\frac{c}{f}\right) = 750$.

And from Table II, for wood with rounded ends,

$n = 0{,}00067$, therefore:

$$w = \frac{201.750}{1 + \frac{180^2.\ 0{,}00067}{16}} = \frac{150750}{2{,}357} = 63958,$$

or say 30 tons to each pile.

Sand piles. Sometimes large holes are bored to the hard soil and filled with sand, making "sand piles." This, of course, can only be done where the intermediate ground is sufficiently firm to keep the sand from escaping laterally. Sometimes holes are dug down and filled in with concrete, or brick piers are built down; or large iron cylinders are sunk down and the space inside of them driven full of piles, or else excavated and filled in with concrete or other masonry, or even sand, well soaked and packed. If filled with sand, there should first be a layer of concrete, to keep the sand from possibly escaping at the bottom.

Where no hard soil can be struck, piles are driven over a large area, and numerous enough to consolidate the ground; they should not be closer than two feet in the clear each way, or they will cut up the ground too much. The danger here is that they may press the ground out laterally, or cause it to rise where not weighted. Sometimes, by sheath-piling each side, the ground can be sufficiently compressed between the piles, thereby being kept from escaping laterally.

Long piles. But by far the most usual way of driving piles is where they resist the load by means of the friction of their sides against the ground. In such cases it is usual to drive experimental piles, to ascertain just how much the pile descends at the last blow of the hammer or ram; also the amount of fall and weight of ram, and

then to compute the load the pile is capable of resisting: one-tenth of this might be considered safe.

The formula then is:—

$$w = \frac{r.f}{10.s} \qquad (46)$$

Where w = the safe load on each pile, in lbs.

" r = the weight of ram used, in lbs.

" f = the distance the ram falls, in inches.

" s = the set, in inches, or distance the pile is driven at the last blow.

Where there is the least doubt about the stability of the pile, use three-fourths w, and if the piles drive very unevenly, use only one-half w.

Some engineers prefer to assume a fixed rule for all piles. Professor Rankine allows 200 lbs. per square inch of area of head of pile. French engineers allow a pile to carry 50000 lbs., provided it does not sink perceptibly under a ram falling 4′ and weighing 1350 lbs., or does not sink half an inch under thirty blows. There are many other such rules, but the writer would recommend the use of the above formula, as it is based on each individual experiment, and is therefore manifestly safer.

Example.

An experimental pile is found to sink one-half inch under the last blow of a ram weighing 1500 *lbs., and falling* 12′. *What will each pile safely carry?*

According to formula (46), the safe load w would be

$w = \frac{r.f}{10.s}$ we have,

$r = 1500$ lbs.

$f = 12.12 = 144''$, and

$s = \frac{1}{2}''$, therefore

$w = \frac{1500.144}{10.\frac{1}{2}} = 43200$ lbs.

If several other piles should give about the same result, we would take the average of all, or else allow say 20 tons on each pile. If, however, some piles were found to sink considerably more than others, it would be better to allow but 10 tons or 15 tons, according to the amount of irregularity of the soil.

All cases of pile-driving require experience, judgment, and more or less experiment; in fact all foundations do.

All piles should be straight, solid timbers, free from projecting

branches or large knots. They can be of hemlock, spruce or white pine, but preferably, of course, of yellow pine or oak.

There is danger, where they are near the seashore, of their being destroyed by worms. To guard against this, the bark is sometimes shrunken on; that is, the tree is girdled (the bark cut all around near the root) before the tree is felled, and the sap ceasing to flow, the bark shrinks on very tightly.

Others prefer piles without bark, and char the piles, coat them with asphalt, or fill the pores with creosote. Copper sheets are the best (and the most expensive) covering.

Piles should be of sufficient size not to break in driving, and should, as a rule, be about 30′ long, and say 15″ to 18″ diameter at the top. They should not be driven closer than about 2′ 6″ in the clear, or they will be apt to break the ground all up. The feet should be shod with wrought-iron shoes, pointed, and the heads protected with wrought-iron bands, to keep them from splitting under the blows of the ram.

Sheath-piling. In sheath-piling it is usual to take boards (hemlock, spruce, white pine, yellow pine or oak) from 2″ to 6″ thick. Guide-piles are driven and cross pieces bolted to the insides of them. The intermediate piles are then driven between the guide piles, making a solid wooden wall each side, from 2″ to 6″ thick. Sometimes the sheath-piles are tongued and grooved. The feet of the piles are cut to a point, so as to drive more easily. The tops are covered with wrought-iron caps, which slip over them and are removed after the piles are driven.

Iron piles. Piles are sometimes made of iron; cast-iron being preferable, as it will stand longer under water. Screw-piles are made of iron, with large, screw-shaped flanges attached to the foot, and they are screwed down into the ground like a gimlet.

Sheath-piling is sometimes made of cast-iron plates with vertical strengthening ribs.

Where piles are driven under water, great care must be taken that they are *entirely immersed*, and at all times so. They should be cut off to a uniform level, below the lowest low-water mark. If they are alternately wet and dry, they will soon be destroyed by decay.

Base-courses over piles. After the piles are cut to a level, tenons are often cut on their tops, and these are made to fit mortises in heavy wooden girders which go over them, and on which the superstructure

The sizes given on this page are for heavy buildings; for very light work use piles of 8″ to 9″ diameter at tops, about 20 feet long and 16″ apart.

rests. This is usual for docks, ferry-houses, etc. For other buildings we frequently see concrete packed between and over their tops; this, however, is a very bad practice, as the concrete surrounding the tops is apt to decay them. It is better to cover the piles with 3″ x 12″ or similar planks (well lag-screwed to piles, where it is necessary to steady the latter) and then to build the concrete base course on these planks.[1]

Better yet, and the best method, is to get large-sized building-stones, with levelled beds, and to rest these directly on the piles. In this case care must be taken that piles come at least under each corner of the stone, or oftener, to keep it from tipping, and that the stone has a full bearing on each pile-head. On top of stone build the usual base-courses.

Piles should be as nearly uniform as possible (particularly in the case of short piles resting on hard ground), for otherwise their respective powers of resistance will vary very much.

Slip-joints. It is well to connect all very heavy parts of buildings (such as towers, chimneys, etc.) by vertical slip-joints with rest of building. The slip-joint should be carried through the foundation-walls and base-courses, as well as above.

Where there are very high chimneys or towers, or unbraced walls, the foundation must be spread sufficiently to overcome the leverage produced by wind. These points will be more fully explained in the chapter on "Walls and Piers."

Action of frost. All base-courses should be carried low enough to be below frost, which will penetrate from three to five feet deep in our latitudes. The reason of this is that the frost tends to swell or expand the ground (on account of its dampness) in all directions, and does it with so much force that it would be apt to lift the base-course bodily, causing cracks and possible failure above.

[1] The strength of planks is calculated the same as for beams laid on their flat sides.

CHAPTER III.

CELLAR AND RETAINING WALLS.

ANALYTICAL SOLUTION.

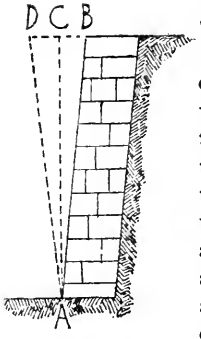


Fig. 49.

THE architect is sometimes called upon to build retaining-walls in connection with terraces, ornamental bridges, city reservoirs, or similar problems. Then, too, all cellar walls, where not adjoining other buildings, become retaining walls; hence the necessity to know how to ascertain their strength. Some writers distinguish between "face-walls" and "retaining-walls"; a face-wall being built in front of and against ground which has not been disturbed and is not likely to slide; a retaining-wall being a wall that has a filled-in backing. On this theory a face-wall would have a purely ornamental duty, and would receive no thrust, care being taken during excavation and building-operations not to allow damp or frost to get into the ground so as to prevent its rotting or losing its natural tenacity, and to drain off all surface or underground water. It seems to the writer, however, that the only walls that can *safely* be considered as "face-walls" are those built against rock, and that all walls built against other banks should be calculated as retaining-walls.

Most Economical Section. The cross-section of retaining-walls vary, according to circumstances, but the outside surface of wall is generally built with a "batter" (slope) *towards* the earth. The most economical wall is one where both the outside and back surfaces batter *towards* the earth. As one or both surfaces become nearly vertical the wall requires more material to do the same work, and

the most extravagant design of all is where the back face batters away from the earth; of course, the outside exposed surface of wall must either batter towards the earth (A B in Figure 49) or be vertical, (A C); it cannot batter away from the ground, otherwise the wall would overhang (as shown at A D). Where the courses of masonry are built at right angles to the outside surface the wall will be stronger than where they are all horizontal.

Thus, for the same amount of material in a wall, and same height, Figure 50 will do the most work, or be the strongest retaining-wall, Figure 51 the next strongest, Figure 52 the next, Figure 53 next,

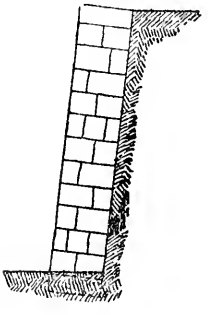


Fig. 50.

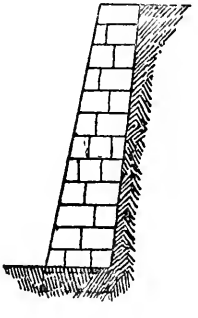


Fig. 51.

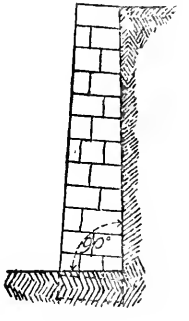


Fig. 52.

Figure 54 next, and Figure 55 the weakest. In Figure 50 and Figure 52 the joints are at right angles to the outside surface; in the

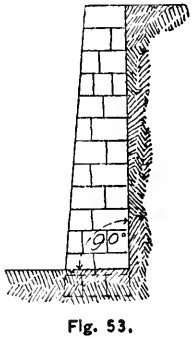


Fig. 53.

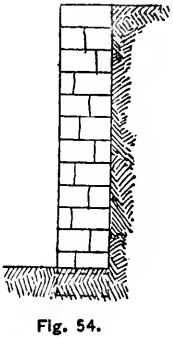


Fig. 54.

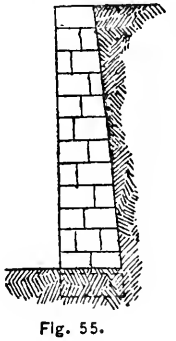


Fig. 55.

other figures they are all horizontal. For reservoirs, however, the shapes of Figures 53 or 55 are often employed.

To calculate the resistance of a retaining-wall proceed as follows:

Height of Line of Pressure. The central line or axis of the pressure O P or p of backing will be at one-third of the height of back surface, measured from the ground lines,[1] that is at O in Figure 56, where A O $= \frac{1}{3}$, A B.

The direction of the pressure-line (except for reservoirs) is usually assumed to form an angle of 57° with the back surface of wall, or

$\angle$ P O B = 57°.

For water it is assumed normal, that is, at right angles to the back surface of wall.

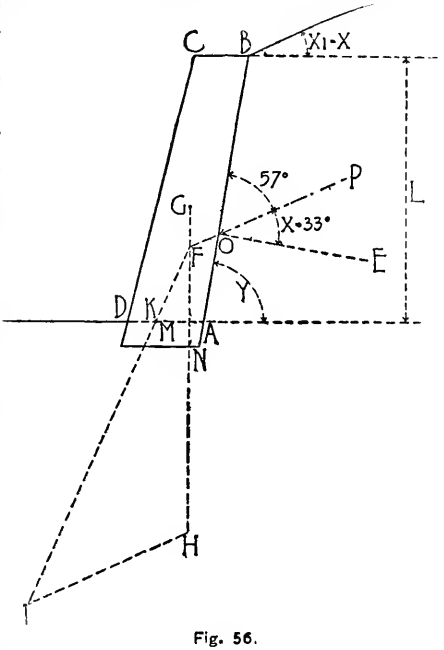


Fig. 56.

If it is desired, however, to be very exact, erect O E perpendicular to back surface, and make angle E O P, or ($\angle$ x) = the angle of friction of the filling-in or backing. This angle can be found from Table X.

Amount of pressure—General case. The amount (p) of the pressure P O is found from the following formulæ:

If the backing is filled in higher that the wall,[2]

Backing higher than Wall.

$$p = \frac{w.L^2}{2} \cdot \frac{\sin^2.(y-x)}{\sin^2.y.\sin.(y+x)} \qquad (47)$$

If the backing is filled in only to the top level of wall,

Backing level with Wall.

$$p = \frac{w.L^2}{2} \cdot \frac{\sin.x}{\sin(y+2x)} \cdot \qquad (48)$$

$$\left(\sqrt{\cot.x - \cot(y+2x)} - \sqrt{\cot.y - \cot(y+2x)}\right)$$

[1] Where the earth in front of the outside surface of wall C D is not packed very solidly below the grade line and against the wall, the total height of wall N B (including part underground) should be taken, in place of A B (the height above grade line).

[2] The top slope of backing in this case should never form an angle with the horizon, greater than the friction angle.

Where $p =$ the total amount of pressure, in pounds, per each running foot in length of wall.

Where $w =$ the weight, in pounds, per cubic foot of backing.

Where $L =$ the height of retaining wall above ground, in feet. See foot note 1, p. 98.

Where $y =$ the angle formed by the back surface of wall with the horizon.

Where $x =$ the angle of friction of the backing as per Table X.

TABLE X.[1]

Material.	Weight per cubic foot. w.	Angle of friction. x.
AVERAGE (except water)	120	33°
Very compact earth	115	65°
Dry clay	100	45°
Sharp pebbles	110	45°
Dry loam	100	40°
Sharp broken stones	100	38°
Dry rammed earth	110	37°
Dry sand	112	32°
Dry gravel	110	32°
Wet rammed earth	125	27°
Wet sand	125	24°
Wet gravel	125	24°
Round pebbles	110	23°
Wet loam	130	17°
Wet clay	125	17°
Salt water	64	0°
Rain water	62½	0°

Even those who do not understand trigonometry can use the above formulæ.

It will simply be necessary to add or subtract, etc., the numbers of degrees of the angles y and x, and then find from any table of *natural* sines, cosines, etc., the corresponding value for the amount of the new angle. The value, so found, can then be squared, multiplied, square root extracted, etc., same as any other arithmetical problem. Should the number of degrees of the new angle be more than 90°, subtract 90° from the angle and use the positive cosine of the *difference* in place of the sine of whole, or the tangent of the *difference* in place of the co-tangent of the whole; in the latter case the value of the tangent will be a negative one, and should have the negative sign prefixed.

Thus, if $x = 33°$ and $y = 50°$, formula (47) would become:

$$p = \frac{w.\,L^2}{2} \cdot \frac{\sin.^2\,(17)°}{\sin.^2\,50.°\,\sin.\,88°}$$

[1] Above table of friction angles is taken from Klasen's "*Hochbau und Brückenbau – Constructionen.*" As a rule it will do to assume the angle of friction at 33° and the weight of backing at 120 lbs. per cubic foot, except in the case of water.

The values of which, found in a table of natural sines, etc., is·

$$p = \frac{w.\,L^2}{2} \cdot \frac{0{,}2924^2}{0{,}7662 \cdot 0{,}9994}$$

$$= \frac{w.\,L^2}{2} \cdot 0{,}1458 \quad = 0{,}729.\, w.L^2$$

Similarly, in formula (48), we should have for the quantity:

$$\sqrt{\text{cot. } x - \text{cot. } (y + 2x)} = \sqrt{\text{cot. } 33° - \text{cot. } 116°}$$

$$= \sqrt{\text{cot. } 33° - [-\text{tg. } (116° - 90°)]}$$

$$= \sqrt{\text{cot. } 33° + \text{tg. } 26°}$$

$$= \sqrt{1{,}5399 + 0{,}4877}$$

$$= \sqrt{2{,}0276} \quad = 1{,}424$$

Average Case. As already mentioned, however, the angle of friction — (except for water when it is = 0°, that is, normal to the back surface of wall) — is usually assumed at 33°; this would reduce above formulæ to a very much more convenient form, viz.:

For the average angle of friction (33°)

If the backing is higher than the wall:

Backing higher than Wall.

$$p = \frac{w.\,L^2}{2} \cdot \frac{(10 - n.\,0{,}55)^2 \cdot \sqrt{144 + n^2}}{(10 + n.\,0{,}55) \cdot \quad 144} \qquad (49)$$

Or, if the backing is level with top of wall:

Backing level with Wall.

$$p = \frac{w.\,L^2}{2} \cdot \frac{\sqrt{144 + n^2}}{9 + n.1{,}7} \cdot \left(\sqrt{1{,}54 - \frac{n.\,0{,}4 - 11}{5 + n.\,0{,}9}} - \sqrt{\frac{n}{12} - \frac{n.\,0.4 - 11}{5 + n.\,0{,}9}} \right)^2 \qquad (50)$$

Where p, w and L same as for formulæ (47) and (48).

Where n = amount of slope or batter in inches (per foot height of wall) of rear surface of wall.

Thus, if the rear surface sloped *towards* the backing three inches (for each foot in height) we should have a positive quantity, or

$$n = +3.$$

If the rear surface sloped *away* from the backing three inches (per foot of height), n would become negative, or

$$n = -3.$$

When the rear surface of wall is vertical, there would be no slope, and we would have

Cellar Walls. $n = 0.$

The latter is the case generally for all cellar walls, which would still further simplify the formula, or, for *cellar walls* where weight of soil or backing varies materially from 120 pounds per cubic foot.

Cellar Walls—general case. $p = w.\ L^2.\ 0{,}138.$ (51)

For *cellar walls*, where the weight of soil or backing can be safely assumed to weigh 120 pounds per cubic foot

Cellar Walls—usual case. $p = 16\frac{2}{3}.\ L^2.$ (52)

Where p = the total amount of pressure, in pounds, per each running foot in length of wall.

Where w = the weight, in pounds, per cubic foot of backing.

Where L = the height, in feet, of ground line above cellar bottom.

For different slopes of the back surface of retaining walls (assuming friction angle at 33°) we should have the following table; + denoting slope *towards* backing, − denoting slope *away* from backing.

TABLE XI.

Slope of back surface of wall in inches per foot of height.	Value of p for backings of different weights per cubic foot.	Value of p for the average backing, assumed to weigh 120 lbs. per cubic foot.
+4″	$p = 0{,}072.\ w.\ L^2$	$p = 8\frac{2}{3}.\ L^2$
+3″	$p = 0{,}088.\ w.\ L^2$	$p = 11\ .\ L^2$
+2″	$p = 0{,}098.\ w.\ L^2$	$p = 12\ .\ L^2$
+1″	$p = 0{,}112.\ w.\ L^2$	$p = 13\frac{1}{2}.\ L^2$
0″	$p = 0{,}138.\ w.\ L^2$	$p = 16\frac{2}{3}.\ L^2$
−1″	$p = 0{,}157.\ w.\ L^2$	$p = 19\ .\ L^2$
−2″	$p = 0{,}185.\ w.\ L^2$	$p = 22\frac{1}{2}.\ L^2$
−3″	$p = 0{,}205.\ w.\ L^2$	$p = 24\frac{2}{3}.\ L^2$
−4″	$p = 0{,}258.\ w.\ L^2$	$p = 31\ .\ L^2$

Now having found the amount of pressure p from the most convenient formula, or from Table XI, and referring back to Figure 56, proceed as follows:

To find Curve of Pressure. Find the centre of gravity G of the mass A B C D,[1] from G draw the vertical axis G H, continue P O till it intersects G H at F. Make F H equal to the weight in pounds of the mass A B C D (one foot thick), at any convenient scale, and at same scale make H I = p and parallel to P O, then draw I F and it is the resultant of the pressure of the earth, and the resistance of the retaining wall. Its point of intersection K with the base D A is a point of the curve of pressure.

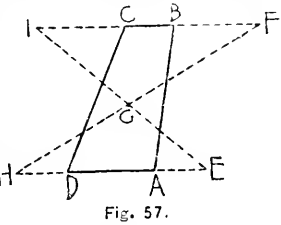


Fig. 57.

Amount of Stress at Joint. To find the exact amount of pressure on the joint D A use formula (44) for the edge of joint nearest to the point K or edge D, and formula (45) for the edge of joint farthest from the point K, or edge A.

[1] To find the centre of gravity of a trapezoid A B C D, Fig. 57, prolong C B until B F = C I = D A and prolong D A until A E = D H = C B, draw E I and H F and their point of intersection G is the centre of gravity of the whole.

Formula (44) was

$$v = \frac{p}{a} + 6.\frac{x.p}{a.d}$$

If M be the centre of D A, that is D M = M A = ½. D A, and remembering that the piece of wall we are calculating, is only one running foot (or one foot thick), we should have

For x = K M; expressed in inches.

For a = A D. 12; (A D expressed in inches).

For d = A D; in inches, and

For p = F I, in lbs., measured at same scale as F II and II I; or, the stress at D, (the nearer edge of joint) would be v, in pounds, per square inch,

$$v = \frac{\text{F I}}{\text{A D. 12}} + 6.\frac{\text{KM.FI}}{12.\text{AD}^2}$$

Remembering to measure all parts in inches *except* F I, which must be measured at same scale as was used to lay out F II and II I. Similarly we should obtain the stress at A in pounds per square inch.

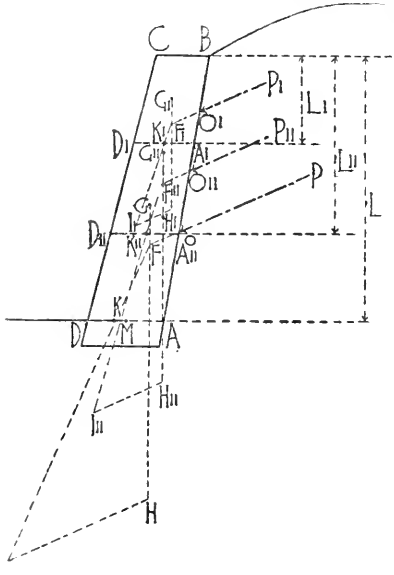


Fig. 58.

$$v = \frac{\text{F I}}{\text{AD. 12}} - 6.\frac{\text{KM.FI}}{12.\text{AD}^2}$$

v should not exceed the safe crushing strength of the material if positive; or if v is negative, the safe tensile strength of the mortar. If we find the wall too weak, we must enlarge A D, or if too strong, we can diminish it; in either case, finding the new centre of gravity G of the new mass A B C D and repeating the operation from that point; the pressure, of course, remaining the same so long as the slope of back surface remains unaltered. If the wall is a very high one, it should be divided into several sections in height, and each section examined separately, the base of each section being treated the same as if it were the joint at the ground line, and the *whole* mass of wall *in* the section and *above* the section being taken in each time.

Thus in Figure 58, when examining the part A_1 B C D_1 we should find O_1 P_1 for the part only, using L_1 as its height, the point O_1 being at one third the height of A_1 B or A_1 $O_1 = \frac{1}{3}$. A_1 B; G_1 would be the centre of gravity of A_1 B C D_1, while F_1 H_1 would be equal to the weight of its mass, one foot thick; this gives one point of the curve of pressure at K_1, with the amount of pressure $= F_1$ I_1, so that we can examine the pressures on the fibres at D_1 and A_1. Similarly when comparing the section A_{11} B C D_{11} we have the height L_{11}, and so find the amount of pressure O_{11} P_{11}, applied at O_{11}, where A_{11} $O_{11} = \frac{1}{3}$ A_{11} B; G_{11} is centre of gravity of A_{11} B C D_{11} while F_{11} H_{11} is equal to the weight of A_{11} B C D_{11}, one foot thick, and F_{11} I_{11} gives us the amount of pressure on the joint, and another point K_{11} of curve of pressure, so that we can examine the stress on the fibres at D_{11} and A_{11}. For the whole mass A B C D we, of course, proceed as before.

Reservoir Walls. For reservoirs the line of pressure O P is always at right angles to the back surface of the wall, so that we can simplify formula (50) and use for rain water:

$$p = 31\tfrac{1}{4}.\ L^2 \qquad (53)$$

For salt water:

$$p = 32.\ L^2 \qquad (54)$$

Where $p =$ the amount of pressure, in pounds, on one running foot in length of wall, and at one-third the height of water, measured from the bottom, and p taken normal to back surface of wall,

Where L = the depth of water in feet.

If Backing is Loaded. Where there is a superimposed weight on the backing of a retaining-wall proceed as follows:

In Figure 59 draw the angle C A D $= x$, the angle of friction of the material. Then take the amount of load, in pounds, coming on B C and one running foot of it in thickness (at right angles to B C), divide this by the area, in feet, of the triangle A B C and add the quotient to w, the weight of the backing per cubic foot, then proceed as before, inserting the sum w_1 in place of w in formulæ (47) to (51) and in Table XI, when calculating p; or $w_1 = w + \frac{2.z}{B.L}$ (55)

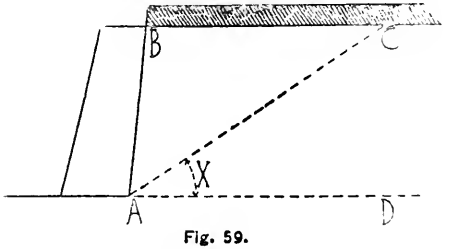


Fig. 59.

Where w_1 = the amount, in pounds, to be used in all the formulæ (47) to (51) and in Table XI, in place of w.

Where w = weight of soil or backing, in pounds, per cubic foot.

Where z = the total superimposed load on backing, in pounds, per running foot in length of wall.

Where B = length, in feet, of B C, as found in Figure 59.

Where L = the height of wall, in feet.

Where there is a superimposed load on the backing, the central line of pressure p should be assumed as striking the back surface of wall higher than one-third its height, the point selected, being at a height X from base; where X is found as per formula (56)

$$X = \frac{L.\ (w_1 - \frac{2}{3}\ w)}{2w_1 - w} \qquad (56)$$

Where X = the height, in feet, from base, at which pressure is applied, when there is a superimposed load on the backing.

Where L = the height, in feet, of wall.

Where w = the weight, in pounds, per cubic foot of backing.

Where w_1 = is found from formula (55)

When calculating the pressure against cellar walls, only the actual weight of the material of walls, floors and roof should be assumed as coming on the wall, and no addition should be made for wind nor for load on floors, as these cannot always be relied on to be on hand. The additional compression due to them should, however, be added afterwards.[1]

GRAPHICAL METHOD.

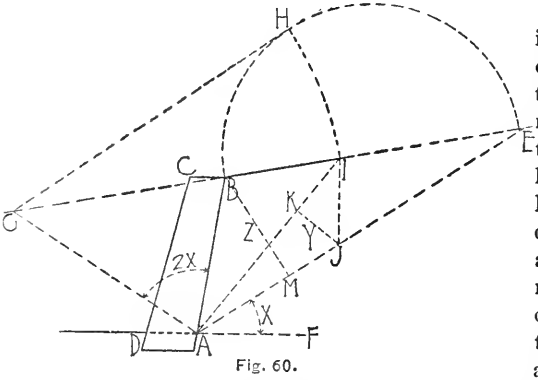


Fig. 60.

The graphical method of calculating retaining-walls is much easier than the analytical, being less liable to cause errors, and is recommended for office use, though the analytical method

[1] Where a wall is not to be kept braced until the superimposed wall, etc., is on it, these should of course be entirely omitted from the calculation, and the wall must be made heavy enough to stand alone.

might often serve as a check for detecting errors, when undertaking important work.

If A B C D is the section of a retaining wall and B I the top line of backing, draw angle F A M = x = the angle of friction, usually assumed at 33° (except for water); continue B I to its intersection at E with A M; over B E draw a semi-circle, with B E as diameter; make angle B A G = 2 x (usually 66°), continuing line A G till it intersects the continuation of B I at G; draw G H tangent to semi-circle over B E; make G I = G H; draw I A, also I J parallel to B A; draw J K at right angles to I A; also B M at right angles to A E. Now for the sake of clearness we will make a new drawing of the wall A B C D in Figure 61.

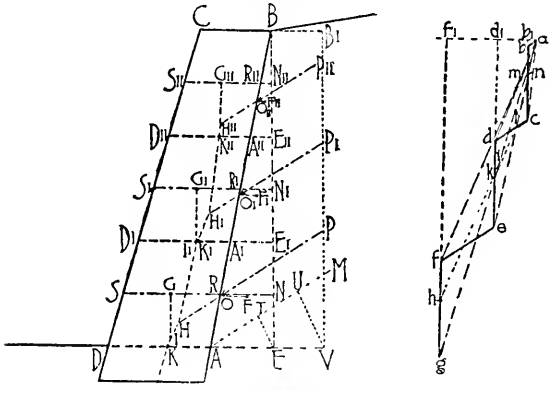
Calling B M = Z and K J = Y (both in Figure 60) make A E = Q (Figure 61) where Q is found from formula (57) following:—

$$Q = \frac{Y.Z.s}{L.m} \qquad (57)$$

Where Q = the length of A E in Figure 61, in feet,

Where Y = the length of K J in Figure 60, in feet,[1]

Where Z = the length of B M in Figure 60, in feet,



Figs. 61 and 62.

Where s = the weight of one cubic foot of backing, in lbs.

Where m = the weight of one cubic foot of wall, in lbs.

Where L = the height of backing, in feet, at wall.

[1] If the incline of line B I to the horizon is equal to the angle of friction, as is often the case, find A G as before and use this length in place of K J or Y, which, of course, it will be impossible to find, as A M and B I would be parallel and would have no point of intersection; of course, B I should never be steeper than A M, or else all of the soil steeper than the line of angle of friction would be apt to slide.

Sections must have equal heights. Draw E B, divide the wall into any number of sections of *equal height*, in this case we will say three sections, A A_I D_I D; A_I A_{II} D_{II} D_I and A_{II} B C D_{II}. Find the centres of gravity of the different parts, viz.: G, G_I and G_{II}, also F, F_I and F_{II}. Bisect D D_I at S, also D_I D_{II} at S_I and D_{II} C at S_{II}. Draw S N, S_I N_I and S_{II} N_{II} horizontally. Through G, G_I and G_{II} draw vertical axes, and through F, F_I and F_{II} horizontal axes, till they intersect A B at O, O_I and O_{II}. Draw O P, O_I P_I and O_{II} P_{II} parallel to M A, where angle M A E = x = angle of friction of soil, or backing. In strain diagram Figure 62 make $a\ b_I = R_{II}\ N_{II}$; also $b_I\ d_I = R_I\ N_I$ and $d_I\ f_I = R\ N$. From b_I, d_I and f_I draw the vertical lines. Now begin at a; draw $a\ b$ parallel to M A; make $b\ c = S_{II}\ R_{II}$; draw $c\ d$ parallel to M A; make $d\ e = S_I\ R_I$; draw $e\ f$ parallel to M A and make $f\ g = S\ R$. Draw $a\ c$, $a\ d$, $a\ e$, $a\ f$ and $a\ g$. Now returning to Figure 61, prolong P_{II} O_{II} till it intersects the vertical axis through G_{II} at H_{II}; draw H_{II} H_I parallel to $a\ c$ till it intersects P_I O_I at H_I; draw H_I I parallel to $a\ d$ till it intersects the vertical axis through G_I at I_I; draw I_I II parallel to $a\ e$ till it intersects P O at II; draw II I parallel to $a\ f$ till it intersects the vertical through G at I; draw I K parallel to $a\ g$. Then will points K, K_I and K_{II} be points of the curve of pressure. The amount of pressure at K_{II} will be $a\ c$, at K_I it will be $a\ e$, and at K it will be $a\ g$, from which, of course, the strains on the edges D, D_I and D_{II}, also A, A_I and A_{II} can be calculated by formulæ (44) and (45). To obtain scale, by which

Scale of strain diagram. to measure $a\ c$, $a\ e$ and $a\ g$, make $g\ h$, Figure 62 at any scale equal to the weight, in pounds, of the part of wall A A_I D_I D one foot thick, draw $h\ i$ parallel $f\ a$, then $g\ i$ measured at same scale as $g\ h$, is the amount of pressure, in pounds, at K. Similarly make $e\ k$ = weight of centre part, and $c\ m$ = weight of upper part, draw $k\ l$ parallel $d\ a$, and $m\ n$ parallel $b\ a$, then is $e\ l$ the pressure at K_I and $c\ n$ the pressure at K_{II}, both measured at same scale; or, a still more simple method would be to take the weight of A A_I D_I D, in pounds, and one foot thick, and divide this weight by the length of $g\ f$ in inches; the result being the number of pounds per inch to be used, when measuring lengths, etc., in Figure 62. The above graphical method is very convenient for high walls, where it is desirable to examine many joints, but *care must be taken to be sure to get the parts all of equal height*, otherwise, the result would be incorrect.

If backing loaded. In case of a superimposed weight find w_1, as directed in formula (55), make A T at any scale equal to w and A U = w_1, draw T E and parallel thereto U V, draw V B_I

parallel to E B and use V B_1 in place of E B, proceeding otherwise as before. The points O of application of pressure P O, will be slightly changed, particularly in the upper part, as they will be horizontally opposite the centres of gravity of the enlarged trapezoids, and in the upper case this point would be much higher, the figure now being a trapezoid, instead of a triangle as before.

Buttressed walls. Where a wall is made very thin and then buttressed at intervals, all calculations can be made the same as for walls of same thickness throughout, but the vertical axis through centre of gravity of wall should be shifted so as to pass through the centre of gravity of the whole mass, including buttresses; and the weight of thin part of wall should be increased proportionately to the amount of buttress, thus: If a 12″ wall is buttressed every 5 feet (apart) with 2′ x 2′ buttresses, proceed as follows:

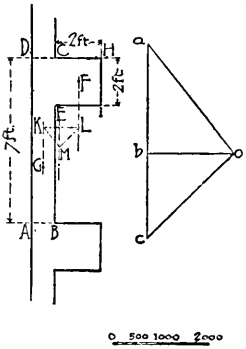


Fig. 63.

Find the centre of gravity G of the part of wall A B C D (in plan) Figure 63, also centre of gravity F of part E I H C, draw lines through F and G parallel to wall. Now make $a\ b$ parallel to wall and at any scale equal to weight or area of A B C D and $b\ c$ equal to that of E I H C. From any point o draw the lines $o\ a$, $o\ b$ and $o\ c$; now draw K L (anywhere between parallel lines F and G), but parallel to $b\ o$, and from L draw L M parallel to $o\ c$, and from K draw K M parallel to $a\ o$, a line through their point of intersection M drawn parallel to wall is the neutral axis of the whole mass. When drawing the vertical section of wall-part A B C D, Figure 64, therefore, instead of locating the neutral axis through the centre of wall it will be as far outside as M is from B C, in Figure 63; that is, at G H, Figure 64.

Fig. 64.

When considering the weight per cubic foot of wall, we add the proportionate share of buttress; now in Figure 63 there are 4 cubic feet of buttress to every 7 feet of wall, so that we must add to the usual weight w per cubic foot of wall $\frac{4}{7}\ w$, or $w.\ (1 + \frac{4}{7})$

To put this in a formula.

$$w_{11} = w\ (1 + \frac{A}{A_1}) \qquad (58)$$

Where w_1 = the weight per cubic foot, in pounds, to be used for buttressed walls, after finding the neutral axis of the whole mass.

Where w = the actual weight, in pounds, per cubic foot of the material.

Where A = the area in square feet of one buttress.

Where A_1 = the area in square feet of wall from side of one buttress to corresponding side of next buttress.

Walls with counterforts. Buttresses, however, will not be of very much value, unless they are placed quite close together. Buttresses on the back surface of a wall are of very little value, unless thoroughly bonded and anchored to walls; these latter are called counterforts. It is wiser and cheaper in most cases to use the additional masonry in thickening out the lower part of wall its entire length.

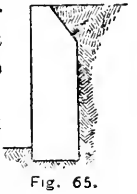


Fig. 65.

Resistance to frost. Where frost is to be resisted the back part of wall should be sloped, for the depth frost is likely to penetrate (from 3 to 4 feet in our climate), and finished smoothly with cement, and then asphalted, to allow the frozen earth to slide upwards, see Figure 65.

Example I.

Cellar wall to frame dwelling. *A two story and attic frame house has a 12″ brick foundation wall, the distance from cellar bottom to ground level being 6 feet. The angle of friction of ground to be assumed at 33° and the weight per cubic foot at 120 pounds. Is the wall safe?*

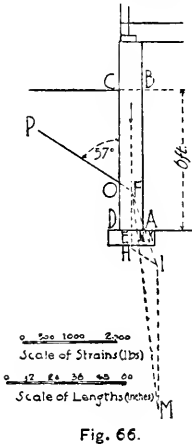


Fig. 66.

The weight of wall and superstructure must, of course, be taken at its minimum, when calculating its resistance to the ground, we shall, therefore, examine one of the sides on which no beams or rafters rest. The weight will consist, therefore, only of brick wall and frame wall over. We examine only one running foot of wall, and have

8 cubic feet of brick at 112 lbs. = 896 lbs
24 feet (high) of frame wall at 15 lbs. = 360 "
Total weight resisting pressure = 1256 "

The pressure itself will be according to formula (52)

$$p = 16\tfrac{2}{3}.\ L^2 = 16\tfrac{2}{3}.\ 36$$
$$= 600 \text{ lbs.}$$

Now make D O = $\frac{1}{3}$. D C. Make angle P O C = 57°; prolong P O till it intersects the

vertical neutral axis of wall[1] at F; make F H = 1256 pounds, at any scale, draw H I parallel to P O, and make H I = p = 600 pounds at same scale. Draw I F, then is its point of intersection K (with D A) a point of the curve of pressure, and F I (measured at same scale) is the amount of pressure p to be used in formulæ (44) and (45). By careful drawing we will find that K comes $\frac{1}{2}''$ beyond A (outside of A D), or $6\frac{1}{2}''$ from centre E of A D. F I we find measures 1660 units, therefore p = 1660 pounds.

To find the actual stress or resistance v of edge of fibres of brickwork at A use (44), viz.:

$$v = \frac{p}{a} + 6.\frac{x.p}{a.d}$$

and as p = 1660 and x = E K = $6\frac{1}{2}''$ and a = 12. 12 = 144 inches and d = A D = 12'' we have:

$$v = \frac{1660}{144} + 6.\frac{6\frac{1}{2}.1660}{144.12} = 11\frac{1}{2} + 37\frac{1}{2} = +49 \text{ pounds}$$

as this is a positive quantity it will be compression.

The resistance of edge-fibres at D will be according to formula (45)

$$v = \frac{1660}{144} - 6.\frac{6\frac{1}{2}.1660}{144.12} = 11\frac{1}{2} - 37\frac{1}{2} = -26 \text{ pounds}$$

as this is a negative quantity D will be subjected to tension; that is, there is a tendency for A B C D to tip over around the point A, the point D tending to rise. The amount of tension at D is more than ordinary brickwork will safely stand, according to Table V, still, as it would only amount to 26 pounds on the extreme edge-fibres and would diminish rapidly on the fibres nearer the centre, we can consider the wall safe, even if of but fairly good brickwork, particularly as the first-story beams and girders and the end and possible cross-walls, will all help to stiffen the wall. Had we taken a foot-slice of the wall under the side carrying the beams, we should have had an additional amount of weight resisting the pressure. If the beams were 18 ft. span, we should have three floors each 9 ft. long and with load weighing, say, 90 pounds per foot; to this must be added the roof, or about 13 ft. à 50 pounds, the additional load being:

Floors,	3. 9. 90 =	2430
Roof,	13. 50 =	650
Total		3080

Now make I M = 3080 pounds at same scale as F H, etc., draw M F and its point of intersection N with D A would be a point of

[1] It should really be the vertical neutral axis of the whole weight, which would be a trifle nearer to D C than centre of wall.

the curve of pressure, and F M, at same scale the amount of pressure on D A for the bearing walls of house; E N we find measures $2\frac{1}{2}''$, and F M measures 4600 units or pounds. The stress at A, then, would be:

$$v = \frac{4600}{144} + 6.\frac{2\frac{1}{2}.4600}{144.12} = +72 \text{ pounds.}$$

and the stress at D would be:

$$v = \frac{4600}{144} - 6.\frac{2\frac{1}{2}.4600}{144.12} = -8 \text{ pounds.}$$

There will, therefore, be absolutely no doubt about the safety of bearing walls.

Example II.

Cellar wall deeper than adjoining building. *A cellar wall A B C D is to be carried* 15 *feet below the level of adjoining cellar; for particular reasons the neighboring wall cannot be underpinned. It is desirable not to make the wall A B C D over* 2′ 4″ *thick. Would this be safe? The soil is wet loam.*

In the first place, before excavating we must sheath-pile along line C D, then as we excavate we must secure horizontal timbers along the sheath-piling and brace these from opposite side of excavation. The sheath-piling and horizontal timbers must be built in and left in wall. The braces will have to be built around and must not be removed until the whole weight is on the wall.

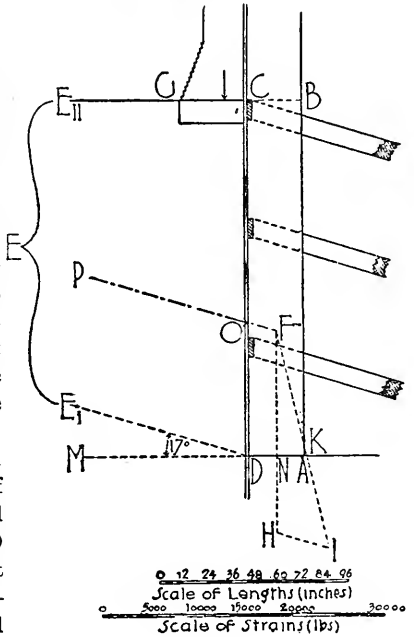


Fig. 67.

The weight of the wall C G, per running foot of length, including floors and roofs, we find to be 13000 pounds, but to this we must add the possible loads coming on floors, which we find to be 6000 pounds additional, or 19000 pounds total, possible maximum load. This load will

be distributed over the area of C D E. In calculating the weight of A B C D resisting the pressure, we must take, of course, only the minimum weight; that is, the actual weight of construction and omit all loads on floors, as these may not always be present. The weight of walls and unloaded floors coming on A B C D, and including the weight of A B C D itself, we find to be 21500 pounds per running foot. Now to find the pressure p, proceed as follows: Make angle E_1 D M = 17°, the angle of friction of wet loam (See Table X), and prolong D E_1 till it intersects C E_{11} at E. Now C E, we find, measures 52 feet; C D or L is 15 feet; then, instead of using w in formula (51), we must use w_1, as found from formula (55), viz.:

$$w_1 = w + \frac{2.\ 19000}{CE.\ \ L}$$

w for wet loam (Table X) is 130 pounds; therefore,

$$w_1 = 130 + \frac{2.\ 19000}{52.\ 15} = 130 + 48,\ 7 = 179.$$

Inserting this value for w in formula (51) we have:

$$p = 179.\ 15^2.\ 0,\ 138 = 5558.$$

The height X from D at which P O is applied is found from formula 56, and is:

$$X = \frac{15.\ (179 - \frac{2}{3}.\ 150)}{2.179 - 150} = \frac{15.\ (179 - 100)}{358 - 150} =$$

$5',697 = 5'\ 8\frac{1}{2}''$

Make D O = X = $5'\ 8\frac{1}{2}''$; draw P O parallel to E D till it intersects the vertical neutral axis of wall (centre line) at F; make F H (vertically) at any scale equal to 21500, draw H I parallel to P O and make H I = p = 5558 pounds at same scale, draw I F, then is its point of intersection with the prolongation of A D at K a point of the curve of pressure, and F I measured at same scale as F H is the amount of pressure on joint. The distance K from centre of joint N we find is $14\frac{1}{2}''$, F I measures 23800 units or pounds; the stress (v) at A, therefore, will be, formula (44):

$$v = \frac{23800}{28.12} + 6.\frac{14\frac{1}{2}.\ 23800}{28.12.28} = +\ 292$$

While the stress at D would be formula (45)

$$v = \frac{23800}{28.12} - 6.\frac{14\frac{1}{2}.23800}{28.12.28} = -\ 150$$

Or the edge at A would be subjected to a compression of 292 pounds, while the edge at D would be submitted to a tension of 150 pounds per square inch, both strains much beyond the safe limit of even the best masonry. The wall will, therefore, have to be thickened and a new calculation made.

Example III.

Wall to stage pit. *A stage-pit 30 feet deep is to be enclosed by a stone-wall, 3 feet thick at the top and increasing 4 inches in thickness for every 5 feet of depth. The wall, etc., coming over this wall weighs 25000 pounds per running foot, but cannot be included in the calculation, as peculiar circumstances will not allow braces to be kept against the cellar wall, until the superimposed weight is on it. The surrounding ground to be taken as the average, that is, 120 pounds weight per cubic foot, and with an angle of friction of 33°.*

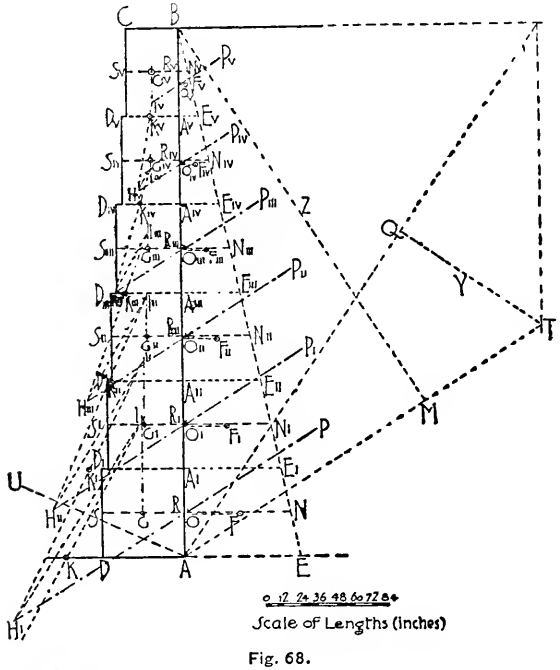


Fig. 68.

Find B M (= Z) and Q T (= Y) by making angles T A E = 33° and B A U = 66° and then proceeding as explained for Figure 60. We scale B M and Q T at same scale as height of wall A B is drawn, and find:

B M = Z = 25 ft. 6″ = 25½

Q T = Y = 9 ft. 8″ = 9⅔; assuming each cubic foot of wall to weigh 150 pounds we find Q from Formula (57)

$$Q = \frac{9\frac{2}{3}.\ 25\frac{1}{2}.120}{30.\ 150} = 6{,}573 = 6'\ 7''.$$

Make A E = Q = 6′ 7″ and draw B E.

At *equal heights*, that is, every 5 feet, in this case, draw the joint lines D E, D_i E_i, D_{ii} E_{ii}, etc. Find the centres of gravity F, F_i, F_{ii}, etc., of the six parts of A E B, (see foot-note, p. 101) and also the centres of gravity G, G_i, G_{ii}, etc., of the six parts of the wall itself, which, in the latter case, will be at the centre of each part.

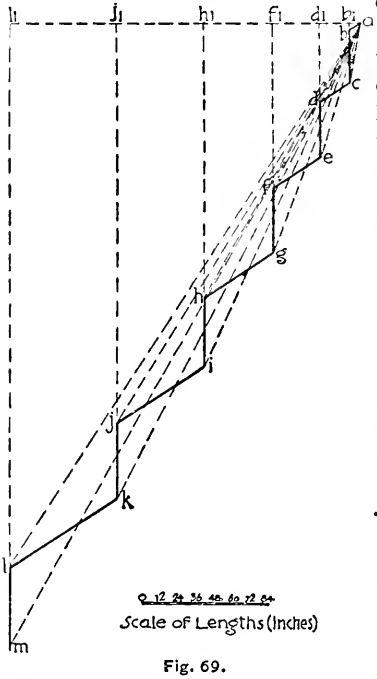


Fig. 69.

Horizontally, opposite the centres F, F_i, F_{ii}, etc., apply the pressures P O, P_i O_i, etc., against wall, and parallel to M A. Through centres G, G_i, G_{ii}, etc., draw vertical axes.

Draw the lines S N, S_i N_i, S_{ii} N_{ii}, etc., at half the vertical height of each section. Now in Figure 69 make $a\,b_i = R_v\,N_v$; $b_i\,d_i = R_{iv}\,N_{iv}$; $d_i\,f_i = R_{iii}\,N_{iii}$; $f_i\,h_i = R_{ii}\,N_{ii}$; $h_i\,j_i = R_i\,N_i$; and $j_i\,l_i = R\ N$. Draw the vertical lines through these points. Now begin at a, make $a\,b$ parallel to P O, make $b\,c = R_v\,S_v$; draw $c\,d$ parallel P O; make $d\,e = R_{iv}\,S_{iv}$; draw $e\,f$ parallel P O, make $f\,g = R_{iii}\,S_{iii}$, and similarly $g\,h$, $i\,j$ and $k\,l$ parallel P O, and $h\,i = R_{ii}\,S_{ii}$; $j\,k = R_i\,S_i$ and $l\,m = R\,S$. Draw from a lines to all the points c, d, e, f, g, etc. Now in Figure 68 begin at P_v O_v, prolong it till it intersects vertical axis G_v at I_v, draw I_v II_v parallel $a\,c$ till it intersects P_{iv} O_{iv} at II_v; draw II_v I_{iv} parallel to $a\,d$ till it intersects vertical axis G_{iv} at I_{iv}; draw I_{iv} II_{iv} parallel to $a\,e$ till it intersects P_{iii} O_{iii} at II_{iv} and similarly II_{iv} I_{iii} parallel $a\,f$; I_{iii} II_{iii} parallel $a\,g$; II_{iii} I_{ii} parallel $a\,h$; I_{ii} II_{ii} parallel to $a\,i$; II_{ii} I_i parallel to $a\,j$; I_i II_i parallel to $a\,k$; II_i I parallel to $a\,l$, and I K parallel to $a\,m$.

The points of intersection K, K_i, K_{ii}, etc., are points of the curve of pressure. To find the amount of the pressure at each point, find

weight per running foot of length of any part of wall, say, the bottom part (A A, D, D) the contents are 5′ high, 4′ 8″ wide, 1′ thick $= 5.4\frac{2}{3}.1$ $= 23\frac{1}{3}$ cubic feet à 150 pounds

$= 3500$ pounds.

Divide the weight by the length of $m\ l$ in inches, and we have the number of pounds per inch, by which to measure the pressures. As $m\ l$ measures 56 inches, each inch will represent $\frac{3500}{56} = 62\frac{1}{2}$ lbs.

Now let us examine any joint, say, $A_m\ D_m$; $I_m\ II_m$ which intersects $A_m\ D_m$ at K_m is parallel to $a\ g$. Now $a\ g$ scales 166 inches, therefore, pressure at $K_m = 166.\ 62\frac{1}{2} = 10375$ pounds. In measuring the distance of K_m from centre of joint in the following, remember that the width of $A_m\ D_m$ is 44 inches, the width of masonry above joint, and *not* 48″ (the width of masonry below). $A_m\ K_m$ scales 38″, therefore, distance x of K_m from centre of joint is $x = 38 - 22 = 16$″.

We have, then, from Formula (44)

stress at D_m; $v = \frac{10375}{44.12} + 6.\frac{16.10375}{44.12.44} = +63$ pounds.

and from formula (45)

stress at A_m; $v = \frac{10375}{44.12} - 6.\frac{16.10375}{44.12.44} = -23$ pounds.

The joint $A_m\ D_m$, therefore, would be more than safe.

Let us try the bottom joint A D similarly. I K is parallel to $a\ m$ now $a\ m$ scales 480″, therefore, the pressure at K is $p = 480.\ 62\frac{1}{2} =$ 30000 pounds.

Now K is distant 53 inches from centre of joint, therefore, stress at D is $v = \frac{30000}{56.12} + 6.\frac{53.30000}{56.12.56} = +298$ pounds.

and stress at A is $v = \frac{30000}{56.12} - 6.\frac{53.30000}{56.12.56} = -209$ pounds.

The wall would evidently have to be thickened at the base. If we could only brace the wall until the superimposed weight were on it, this might not be necessary. If we could do this we should lengthen $b\ c$ an amount of inches equal to the amount of this load divided by $62\frac{1}{2}$ (the number of pounds per inch), or $b\ c$ instead of being 36 inches long would be:

$$36 + \frac{25000}{62\frac{1}{2}} = 436 \text{ inches long.}$$

While this lengthening of $b\ c$ would make the lines of pressure $a\ c$, $a\ e$, $a\ f$, etc., very much longer, and consequently the actual pressure very much greater, it will also make them very much steeper and consequently bring this pressure so much nearer the centre of each joint,

that the pressure will distribute itself over the joint much more evenly, and the worst danger (from tension) will probably be entirely removed.

Example IV.

Reservoir Wall. *A stone reservoir wall is plumb on the outside, 2 feet wide at the top and 5 feet wide at the bottom; the wall is 21 feet high, and the possible depth of water 20 feet. Is the wall safe?*

Divide the wall into three parts in height; that is, D D_1 = D_1 D_{11} = D_{11} C. Find the weight of the parts from each joint to top, per running foot of length of wall, figuring the stonework at 150 pounds per cubic foot, and we have:

Weight of A_{11} B C D_{11} = 2975 pounds.

Weight of A_1 B C D_1 = 7140 pounds.

Weight of A B C D = 12495 pounds.

Find centres of gravity of the parts A B C D (at G), of A_1 B C D_1 (at G_1) and of A_{11} B C D_{11} (at G_{11}). Apply the pressures P O at $\frac{1}{3}$ height of A E; P_1 O_1 at $\frac{1}{3}$ height of A_1 E and P_{11} O_{11} at $\frac{1}{3}$ height of A_{11} E, where E top level of water.

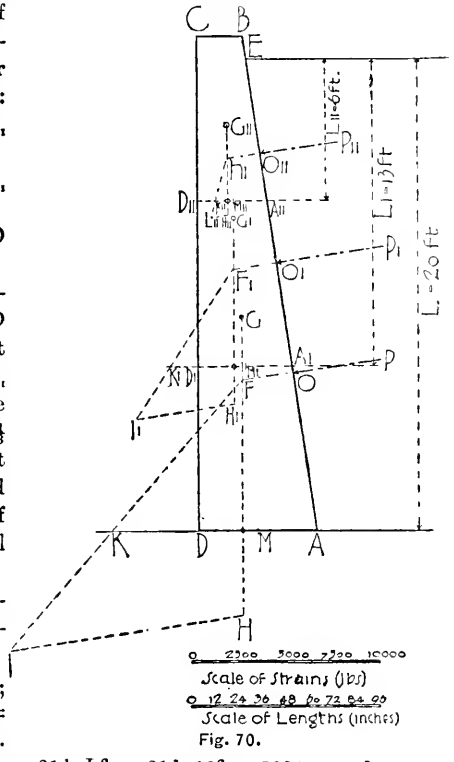


Fig. 70.

The amount of pressures will be from formula (53).

For part A_{11} E; P_{11} O_{11} = $31\frac{1}{4}$. L^2_{11} = $31\frac{1}{4}$. 6^2 = 1125 pounds.

For part A_1 E; P_1 O_1 = $31\frac{1}{4}$. L^2_1 = $31\frac{1}{4}$. 13^2 = 5281 pounds.

For part A E; P O = $31\frac{1}{4}$. L^2 = $31\frac{1}{4}$. 20^2 = 12500 pounds.

The pressures P O, P_1 O_1, etc., will be applied at right angles to A E, prolong these lines, till they intersect the vertical axes through

(the centres of gravity) G, G_1 and G_{II} at F, F_1 and F_{II}. Then make

$F_{II}\ II_{II} =$ 2975, weight of upper part.
$F_1\ II_1 =$ 7140, weight of A_1 B C D_1, and
F II = 12495, weight of A B C D.

Draw through II, II_1 and II_{II} the lines parallel to pressure lines making

$II_{II}\ I_{II} = P_{II}\ O_{II} =$ 1125
$II_1\ I_1 = P_1\ O_1 =$ 5281
II I = P O = 12500

Draw $I_{II}\ F_{II}$, $I_1\ F_1$ and I F, then will their lengths represent the amounts of pressure at points K_{II}, K_1 and K on the joints $A_{II}\ D_{II}$, $A_1\ D_1$ and A D.

$F_{II}\ I_{II}$ measures 3300 units or pounds.
$F_1\ I_1$ " 9500 " "
F I " 18800 " "

By scaling we find that

K_{II} is $10\frac{1}{2}$ inches from centre of $D_{II}\ A_{II}$
K_1 is 37 " " " $D_1\ A_1$
K is 74 " " " D A

The stresses to be exerted by the wall will, therefore, be

at D_{II}; $v = \frac{3300}{36.12} + 6.\frac{10\frac{1}{2}.3300}{36.12.36} = +21$ pounds.

at A_{II}; $v = \frac{3300}{36.12} - 6.\frac{10\frac{1}{2}.3300}{36.12.36} = -6$ pounds.

at D_1 ; $v = \frac{9500}{48.12} + 6.\frac{37.9500}{48.12.48} = +93$ pounds.

at A_1 ; $v = \frac{9500}{48.12} - 6.\frac{37.9500}{48.12.48} = -60$ pounds.

at D ; $v = \frac{18800}{60.12} + 6.\frac{74.18800}{60.12.60} = +219$ pounds.

at A ; $v = \frac{18800}{60.12} - 6.\frac{74.18800}{60.12.60} = -167$ pounds.

From the above it would appear that none of the joints are subject to excessive compression: further that joint $D_{II}A_{II}$ is more than safe, but that the joints $D_1\ A_1$ and D A are subject to such severe tension that they cannot be passed as safe. The wall should, therefore, be redesigned, making the upper joint lighter and the lower two joints much wider.

CHAPTER IV.

WALLS AND PIERS.

WALLS are usually built of brick or stone, which are sometimes, though rarely, laid up dry, but usually with mortar filling all the joints. The object of mortar is threefold:

Object of mortar. 1. To keep out wet and changes of temperature by filling all the crevices and joints.

2. To cement the whole into one mass, keeping the several parts from separating, and,

3. To form a sort of cushion, to distribute the crushing evenly, taking up any inequalities of the brick or stone, in their beds, which might fracture each other by bearing on one or two spots only.

To attain the first object, "grouting" is often resorted to. That is, the material is laid up with the joints only partly filled, and liquid cement-mortar is poured on till it runs into and fills all the joints. Theoretically this is often condemned, as it is apt to lead to careless and dirty work and the overlooking of the filling of some parts; but *practically* it makes the best work and is to be recommended, except, of course, in freezing weather, when as little water as possible should be used.

To attain the second object, of cementing the whole into one mass, it is necessary that the mortar should adhere firmly to all parts, and this necessitates soaking thoroughly the bricks or stones, as otherwise they will absorb the dampness from the mortar, which will crumble to dust and fail to set for want of water. Then, too, the brick and stone need washing, as any dust on them is apt to keep the mortar from clinching to them. In winter, of course, all wetting must be avoided, and as the mortar will not set so quickly, a little lime is added, to keep it warm and prevent freezing.

Thickness of joints. To attain the third object, the mortar joint must be made thick enough to take up any inequalities of the brick or stone. It is, therefore, impossible to set any standard for

joints, as the more irregular the beds of the brick or stone, the larger should be the joint. For general brickwork it will do to assume that the joints shall not average over one-quarter of an inch above irregularities. Specify, therefore, that, say, eight courses of brick laid up "in the wall" shall not exceed by more than two inches in height eight courses of brick laid up "dry." For front work it is usual to gauge the brick, to get them of exactly even width, and to lay them up with one-eighth inch joints, using, as a rule, "putty" mortar. While this makes the prettiest wall, it is the weakest, as the mortar has little strength, and the joint being so small it is impossible to bond the facing back, except every five or six courses in height. "Putty" mortar is made of lime, water and white lead, care being taken to avoid all sand or grit in the mortar or on the beds.

Quality of mortars.

The best mortar consists of English Portland cement and sharp, clean, coarse sand. The less sand the stronger the mortar.

Sand for all mortars should be free from earth, salt, or other impurities. It should be carefully screened, and for very important work should be washed. The coarser and sharper the sand the better the cement will stick to it. English Portland cement will stand as much as three or four parts of sand. Next to English come the German Portland cements, which are nearly as good. Then the American Portland, and lastly the Rosendale and Virginia cements. Good qualities of Rosendale cements will stand as much as two-and-a-half of sand. Of limes, the French lime of Teil is the strongest and most expensive. Good, hard-burned lime makes a fairly good mortar. It should be thoroughly slacked, as otherwise, if it should absorb any dampness afterwards, it will begin to burn and swell again. At least forty-eight hours should be allowed the lime for slacking, and it is very desirable to strain it to avoid unslacked lumps. Lime will take more sand than cement, and can be mixed with from two to four of sand, much depending on the quality of the sand, and particularly on the "fatness" of the lime. It is better to use plenty of sand (with lime) rather than too little; it is a matter, however, for practical judgment and experiment, and while the specification should call for but two parts of sand to one of lime, the architect should feel at liberty to allow more sand if thought desirable. Lime and Rosendale cement are often mixed in equal proportions, and from three to five parts of sand added; that is, one of lime, one of cement, and three to five of sand. It is advisable to specify that all parts shall be actually measured in barrels, to avoid such tricks, for instance, as hiring a

decrepit laborer to shovel cement or lime, while two or three of the strongest laborers are shovelling sand, it being called one of cement to two or three of sand. A little lime should be added, even to the very best mortars, in winter, to prevent their freezing.

Frozen walls. When a wall has been frozen, it should be taken down and re-built. Never build on ice, but use salt, if necessary, to thaw it; sweep off the salt-water, which is apt to rot the mortar, and then take off a few courses of brick before continuing the work. Protect walls from rain and frost in winter by using boards and tarpaulins. Some writers claim that it does no harm for a wall to freeze; this may be so, provided all parts freeze together and are kept frozen until set, and that they do not alternately freeze and thaw, which latter will undoubtedly rot the mortar.

Plaster-of-Paris makes a good mortar, but is expensive and cannot stand dampness. Cements or limes that will set under water are called hydraulic.

Quickness of setting is a very desirable point in cements. All cement-mortars, therefore, must be used perfectly fresh; any that has begun to set, or has frozen, should be condemned, though many contractors have a trick of cutting it up and using it over with fresh mortar. To keep dampness out of cellar-walls the outside should be plastered with a mortar of some good hydraulic cement, with not more than one part of sand to one part of cement; this cement should be scratched, roughened, and then the cement covered outside with a heavy coat of asphalt, put on hot and with the trowel. In brick walls, the coat of cement can be omitted and the joints raked out, the asphalt being applied directly against the brick. This asphalt should be made to form a tight joint, with the slate or asphalt damp-course, which is built through bottom of wall, to stop the rise of dampness from capillary attraction.

In ordinary rubble stonework the mortar should be as strong as possible, as this class of work depends entirely on the mortar for its strength.

For the strengths of different mortars, see Table V.

Some cements are apt to swell in setting, and should be avoided.

Smoke flues. Where flues or unplastered walls are built, the joints should be "*struck*," that is, scraped smooth with the trowel. No flues should be "pargetted"; that is, plastered over, as the smoke rots the mortar, particles fall, and the soot accumulating in the crevices is apt to set fire to the chimney. Joints of chimneys are liable to be eaten out from the same reason, and the loose por-

tions fall or are scraped out when the flues are cleaned, leaving dangerous cracks for fire to escape through. It is best, therefore, to line up chimneys inside with burned earthenware or fire-clay pipes. If iron pipes are used, cast-iron is preferable; wrought-iron, unless very thick, will soon be eaten away. Where walls are to be plastered, the joints are left as rough as possible, to form a good clinch.

Wall furrings. Outside walls are not plastered directly on the inside, unless hollow; otherwise, the dampness would strike through and the plaster not only be constantly damp, but it would ultimately fall off. Outside walls, unless hollow, are always "furred." In fireproof work, from one to four-inch thick blocks are used for this purpose. These blocks are sometimes cast of ashes, lime, etc., but are a very poor lot and not very lasting. Generally they are made of burnt clay, fire-clay or porous terra-cotta. The latter is the best, as, besides the advantages of being lighter, warmer and more damp-proof, it can be cut, sawed, nailed into, etc., and holds a nail or screw as firmly as wood. These blocks are laid up independently of the wall, but occasionally anchored to the same by iron anchors. The plastering is applied directly to the blocks.

In cheaper and non-fireproof work, furrings are made of vertical strips of wood about two inches wide, and from one to two inches thick, according to the regularity of the backing. For very fine work, sometimes, an independent four-inch frame is built inside of the stone-wall, and only anchored to same occasionally by iron anchors. Where there are inside blinds, a three or four inch furring is used (or a fireproof furring), and this is built on the floor beams, as far inside of the wall as the shutter-boxes demand. To the wooden furrings the laths are nailed. Furrings are set, as a rule, sixteen inches apart, the lath being four feet long; this affords four nailings to each lath. Sometimes the furrings are set twelve inches apart, affording five nailings. All ceilings are cross-furred every twelve inches, on account of stiffness, and the strips should not be less than one-and-three-eighths inches thick, to afford strength for nailing. Furring-strips take up considerable of the strain of settlements and shrinkage, and prevent cracks in plastering by distributing the strain to several strips. To still further help this object, the "heading-joints" of lath should not all be on the same strip, but should be frequently broken (say, every foot or two), and should then be on some other strip. Laths should be separated sufficiently (about three-eighths inch) to allow the plaster to be well worked through the joint and get a strong grip or "clinch" on the back of the laths. If a building is

properly built, theoretically correct in every respect, it should not show a single crack in plastering. Practically, however, this is impossible.

Shrinkage of joints.

But there never need be any fear of shrinkage or settlement, in a well-constructed building, where the foundations, joints and timbers are properly proportioned. The danger is never from the *amount* of settlements or shrinkage, but from the *inequality* of same in different parts of the building. Inequalities in settlements are avoided by properly proportioning the foundations. Inequalities in shrinkage of the joints, though quite as important, are frequently overlooked by the careless architect. He will build in the same building one wall of brick with many joints, another of stones of all heights and with few joints, and then put iron columns in the centre, making no allowance whatever for the difference in shrinkage. If he makes any, it is probably to call for the most exact setting of the columns, for the hardest and quickest-setting Portland cement for the stonework, and probably be content with lime for the brickwork. To avoid uneven shrinkages, allowances should be made for same. Brickwork will shrink, according to its quality, from one-sixteenth to one-eighth inch per story, ten to twelve feet high, and according to the total height of wall. The higher the wall, the greater the weight on the joints and the greater the shrinkage. Iron columns should, therefore, be made a trifle shorter than the story requires, the beams being set out of level, lower at the column. The plan should provide for the top of lowest column to be one-sixteenth or one-eighth inch low, while the top of highest column would be as many times one-sixteenth or one-eighth inch low as there were stories; or if there were eight stories, the top of bottom column for the very best brickwork would be, say, one-sixteenth inch low, and the top of highest column would be one-half inch low. Stone walls should have stone backings in courses as high as front stones, if possible; if not, the backing should be set in the hardest and quickest-setting cement.

Slip-joints.

Stone walls should be connected to brick walls by means of slip-joints. By this method the writer has built a city stone-front, some 150 feet high and over 50 feet wide, connected to brick walls at each side, without a single stone sill, or transom, or lintel cracking in the front. The slip-joint should carry through foundations and base courses where the pressure is not equal on all parts of the foundation. If for the sake of design, it is necessary to use long columns or pilasters, in connection with coursed stone backings, the columns or pilasters must either be strong enough to do the whole work of the wall, or else must be bedded in putty-

mortar with generous top and bottom joints, to allow for shrinkage of the more frequent joints behind them; otherwise, they are apt to be shattered. Such unconstructional designs had, however, better be avoided. In no case should a wall be built of part iron uprights and part masonry; one or the other must be strong enough to do the work alone; no reliance could be placed on their acting together. In frame walls, care should be taken to get the amount of "cross" timbering in inner and outer walls about equal, and to have as little of it as possible. Timber will shrink "*across*" the grain from one-fourth to one-half inch per foot. Where the outer walls are of masonry, and inner partitions or girders are of wood, great care must be taken that the shrinkage of each floor is taken up by itself. If the shrinkage of all beams and girders is transferred to the bottom, it makes a tremendous strain on the building and will ruin the plastering. To effect this, posts and columns should bear *directly* on each other, and the girders be attached to their sides or to brackets, but by no means should the girder run between the upper and lower posts or columns. If there are stud-partitions, the head pieces should be as thin as possible, and the studs to upper partitions should rest *directly* on the head of lower partitions.

Shrinkage of timber.

In masonry, all beds should be as nearly level as possible, to avoid unequal crushing. Particularly is this the case with cut stonework. If the front of the stone comes closer than the backing (which is foolishly done sometimes to make a small-looking joint), the face of the stone will surely split off. If the back of a joint is broken off carelessly, and small stones inserted in the back of a joint to form a support to larger stones, they will act as wedges, and the stone will crack up the centre of joint and wall. Stones should be bedded, therefore, perfectly level and solid, except the front of joint for about three-fourth inches back from the face, which should not be bedded solid, but with "putty"-mortar. Light-colored stones, particularly lime-stones, are apt to stain if brought in connection with cement-mortar. A good treatment for such stones is to coat the back, sides and beds with lime-mortar, or, if this is not efficacious, with plaster-of-Paris.

All beds level.

Cement stains.

All stones should be laid on their "natural beds"; that is, in the same position as taken from the quarry. This will bring the layers of each stone into horizontal positions, on top of each other, and avoid the "peeling" so frequently seen. Ashlar should be well anchored to the backing. The joints should be filled with putty-mortar, and should be sufficiently large to take up

Natural bed.

Size of stones. the shrinkage of the backing. Stones should not be so large as to risk the danger of their being improperly bedded and so breaking. Professor Rankine recommends for soft stones, such as sand and lime stones, which will crush with less than 5000 pounds pressure per square inch, that the length shall not exceed three times the depth, nor the breadth one-and-a-half times the depth. For hard stones, which will resist 5000 pounds' compression per square inch, he allows the length to be from four to five times the depth, and the breadth three times the depth. Stones are sometimes joined with "rebated" joints, or "dove-tail" joints, the latter particularly in circular work, such as domes or light-houses.

Sills bedded hollow. All sills in either stone or brick walls should be bedded at the ends only, and the centre part left hollow until the walls are thoroughly set and settled; otherwise, as the piers go down, the part between them, not being so much weighted, will refuse to set or settle equally with them, and will force up the centre of sill and break it. Where there are lintels across openings in one piece, with central mullion, the lintel should either be jointed on the mullion, or else the mullion bedded in putty at the top. Otherwise, the lintel will break; or, if it be very strong, the mullion will split; for, as the piers set or settle, the lintel tends to go down with them, and, meeting the mullion, must either force it down, or else break it, or break itself.

Slip-Joints. Walls of uneven height, even where of the same material, should be connected to each other by means of a slip-joint, so as to provide for the uneven shrinkage. Slip-joints must be so designed that while they allow independent vertical movement to each part, neither can separate from the other in any other direction. Figures 71 to 73 give a few examples.

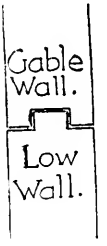


Fig. 71.

Figure 71 shows the plan of a gable-wall connected with a lower wall, by means of a slip-joint. Figure 72 shows the corner of a front stone-wall connected similarly with side-wall of brick. Figure 73 the corner of a tower or chimney connected with a lower wall. The joint must be built plumb from top to bottom. Where the higher wall sets over the tongue above lower wall, one or two inches must be left hollow over the tongue, to allow for settlement or shrinkage of the higher wall and to prevent its resting on the tongue and possibly cracking it off. Where iron anchors are used in connection with slip-joints, they should be so arranged as to allow free vertical movement.

Such joints and anchors must be designed with reference to each special case. In stepped-foundations (on shelving-rock, etc.), or in walls of uneven heights where slip-joints are impracticable, the foundations or walls should be built up to each successive level and be allowed to set thoroughly before building further. A hard and quick-setting cement should be used, and the joints made as small as possible. In no case should one wall of a building be carried up much higher than the others, where slip-joints are not to be used. When building on top of old work, clean same off thoroughly or the mortar will not take hold (clinch). In summer, soak the old work thoroughly. Where new work has to be built against old work, a slip-joint should be used, if possible, or else a straight joint should be used with slip-anchors, and after the new work has thoroughly set, bond-stones can be cut in. In such cases, the foundations should be spread as much as possible, to avoid serious settlements. In all work involving old and new walls combined, the quickest and hardest-setting cements should be used. Sometimes it is advisable not to load walls until they have set, unless all walls are loaded alike, as the uneven weights on green walls are apt to crack them. All walls should be well braced, and wooden centres left in till they have set thoroughly.

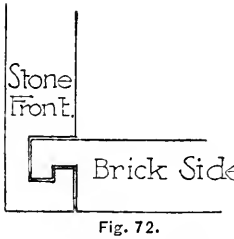


Fig. 72.

Old and new walls.

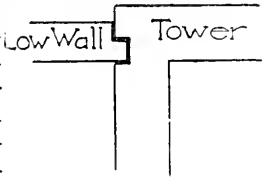


Fig. 73.

Timber in walls. Timber of any kind, in walls, should be avoided, if possible. It should only be used for temporary support, as it is liable to rot, shrink, burn out, or to absorb dampness and swell, in either case causing settlements or cracks, even if not endangering the wall. In no case bond a wall with timbers. Where it is necessary to nail into a wall, wooden plugs are sometimes driven into the joints; they are very bad, however, and liable to shake the wall in driving. Wooden strips, let in, weaken the wall just that much. Wooden nailing-blocks, though not much better, are frequently used. The block should be the full thickness of the bricks, plus the upper and lower joints. If

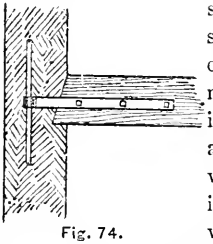


Fig. 74.

there is any mortar over or under the block, the nailing will jar it loose and the block fall out. The best arrangement to secure nailings is to build-in porous terra-cotta blocks or bricks.

Beam ends. Where ends of beams are built into the wall, they should always be cut off to a slant, as shown in Figure 74.

The anchors should be attached to the side, so as to allow the beam to fall out in case it is burned through. If the beams were not cut to a slant, the leverage produced by their weight, when burned through, would be apt to throw the wall; as it is, each beam can fall out easily and the wall, being corbelled over the beam-opening, remains standing. It is desirable to "build-in" the ends of wooden beams as little as possible, to prevent dry-rot; if it can be arranged to circulate air around their ends, it will help preserve them. Beams should always be levelled-up with good-sized pieces of slate, and not with wood-chips, which are liable to crush. The old-fashioned way of corbelling out to receive beams, leaving the wall intact, has much to commend it. A modern practice is to corbel out one course of brick, at each ceiling-level, just sufficient to take the projection of furring-strips; this will stop draughts in case of fire, also rats and mice from ascending. All slots for pipes, etc., should be bricked up solid around the pipes for about one foot at each ceiling-level for the same purposes.

Wooden lintels. Where wooden lintels are used in walls, there should always be a relieving-arch over them, so arranged that it would stand, even if the lintel were burned out or removed; the lintel should have as little bearing as possible, and be shaved off at the ends.

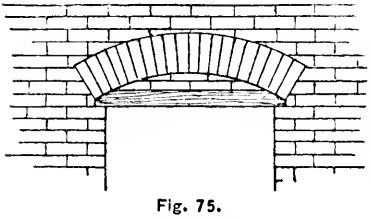


Fig. 75.

Figure 75 shows a wooden lintel correctly built-in. Figure 76 shows a very blundering way of building-in a wooden lintel, but one, nevertheless, frequently met with. It is obvious, however, in the latter case, that if the lintel were removed the abutment to the arch would sink and let the arch down. The relieving-arch, after it has set, should be strong enough to carry the wall, the lintel being then used for nailing only. The rule for lintels is to make their depth about one-tenth of the span. Arches are built of "row-locks" (that is, "headers,") or of "stretchers," or a combination of both, according

Bonded arches best.

to design. The strongest arch, however, is one which has a combination of both headers and stretchers; that is, one which is bonded on the face, and also bonded into the backing. Straight arches and arches built in circular walls should always be bonded into the backing, or if the design does not allow of this, they should be anchored back. "Straight" arches should be built with a slight "camber" up towards the centre, to allow for settlement and to satisfy the eye. About one-eighth inch rise at the centre for each foot of span is sufficient. Straight arches should never be built, as shown in Figure 77, and known as the French or Dutch arch, as there is absolutely no strength to them. Fireplaces are frequently arched over in this way, but the practice is a very bad one.

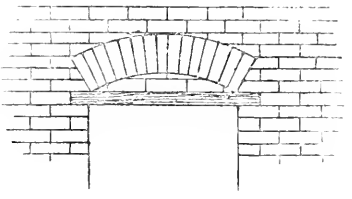


Fig. 76.

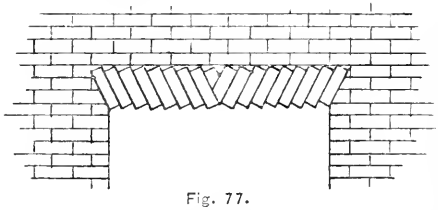
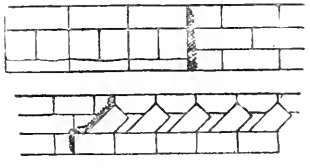


Fig. 77.

Brick facings. Brick facings, as laid up in this country, usually consist of all stretchers. Every fifth or sixth course is bonded into the backing, either by splitting the brick in two, as shown in Figure 78, and using short headers behind it, or by breaking off the rear corners, as shown in Figure 79, and using diagonally-laid bond-brick.



Figs. 78 and 79.

The latter course is the better, but there is no strength in either; particularly as, as a rule, the front brick are so much softer and weaker than those in the backing.

The English bond, in which a course of headers alternates with a course of stretchers, is much to be preferred; or, better yet, the Flemish bond, where in each course a header alternates with a stretcher. Of course, in both English and Flemish bond, if the front brick are thinner than the bricks used in the backing, larger and more unsightly joints will be necessary in front.

Regular work the best. It is best, as a rule, not to count on the front work for strength. We frequently see masons laying up brick walls by first laying a single course of headers or stretchers on the outside of the wall, and then one on the inside, and then filling the balance of wall with bats and all kind of rubbish. This makes a very poor wall. The specification should provide that no bats or broken brick will be allowed, leaving it to the architect's discretion to stop their use, if it is being overdone; of course, some few will have to be used. But, after all, the best wall is that one which is built the most regularly and with the most frequent bonds, and no architect should be talked out of good, regular work, as being *too theoretical*, by so-called "practical" men. The necessity for regularity and bond is easily illustrated by taking a lot of bricks of different sizes, or even toy blocks, and attempting to pile them up without regularity; or, even if piled regularly, without bond. It will quickly be seen that the most regular and most frequently-bonded pile will go the highest. By "bond" is meant alternating headers and stretchers with regularity, and so as to cover and break joints.

Use of bond-stones. The use of "bond-stones" *at intervals only* is bad; they should be carried through the whole surface (width and length) of wall and be of even thickness, or else be omitted. Using bond-stones in one place only tends to concentrate the compression on one part of the wall. Thus, bond-stones built under each other at regular intervals, as shown in Figure 80, are bad, as they give the pressure no chance to spread, but keep concentrating it back onto the part of wall immediately under bond-stones, whereas, in Figure 81, the pressure is allowed to spread gradually over a larger area of the wall. Where, however, a heavy girder, column or other weight comes on a wall, it is distributed by means of a large block, generally granite or stone, or sometimes by a large iron plate.

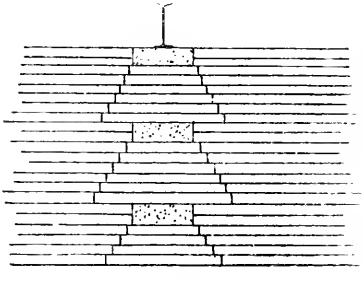


Fig. 80.

The block or plate should have sufficient area not to crush the brick-work directly under it. Where girder-ends are built into walls, it is also desirable to build a block *over* the girder-end as well as under the same. The upper block prevents any part of the wall

from resting on the girder or being affected by its shrinkage, if of wood; if the girder is of iron the upper block will wedge in the girder-end more firmly, and the girder will be able to carry more load. See page 57.

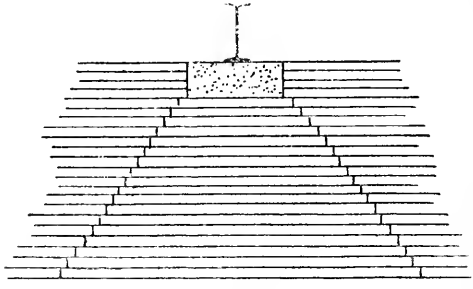


Fig. 81.

Anchors for girders and beams are usually made of iron and of such shape as to allow the girder or beam to fall out in case of fire. Anchors made of iron are not objectionable in inner walls, or where not exposed to dampness; all iron should be thoroughly painted, however, with red lead or metallic paint. Before painting, all rust should be scraped off. Don't believe the "practical" man who says the paint will stick better if you leave the rust on the iron; it will stick better to the rust, no doubt, but not to the iron. For outside work all iron should be galvanized; but it is better to use copper for anchors, dowels, clamps, etc. All copings should be clamped together, the clamps being counter-sunk. Slanting-work and tracery should be dowelled together. Where iron is let into stones, and run with lead or sulphur, the iron is apt to swell with rust or heat and burst the stone. Dampness in walls is one of the worst dangers, both on account of frost and decay. All exterior mouldings and sills should be projected and have "drip-moulds" cut underneath, to cause the water to drop or drip; this will prevent considerable dampness and keep the outside surface of the wall from becoming dirty, as the dust lodging on top of mouldings discolors the rain-water, and the latter, instead of streaking down the wall, will drop off.

Metal anchors.

Drip-moulds.

Hollow walls. Walls are frequently built hollow to prevent dampness, but this raises many objections. Shall the inner or outer wall be the thicker? If the outer wall, then all beams, etc., have to be that much longer, so as to rest on the stronger part; they are liable to transfer dampness, and then, too, the thicker, and, consequently, greater, part of wall is exposed to dampness. If the inner wall is thicker, the construction, so far as beams, etc., are concerned,

no doubt is better, but the outer part is apt to be destroyed by the frost. Then at windows and doorways both must be connected, and dampness is apt to get through. It is well to ventilate the air-space between walls, at the bottom from inside and at the top from outside. The bottom of spaces should be drained. Tops of arches, or lintels over openings, should be cemented and asphalted (in the air-space), to shed any dampness settling on them.

The outer and inner walls should be frequently anchored together. Iron anchors galvanized, or copper anchors are best; they should have a half-twist, as shown, to prevent water running along them.

Fig. 82.

Care must be taken to keep hollow walls free from hanging mortar, which will communicate moisture from one wall to the other.

But hollow walls are not nearly so good as solid walls with porous terra-cotta furrings.

Where walls are coped with stone, there should be damp-courses of slate or asphalt under same, and the back side should be flashed, to prevent dampness descending. If gutters are cut in stone cornices, they should be lined with metal, preferably copper, the outer edge being let into a raggle and run in with lead.

Underpinning. When a wall, already built, has to have its foundations carried down lower, it is called "underpinning the wall." Holes are made through the wall at intervals and through these (at right angles to the wall) are placed the "needles," that is, heavy timbers, which carry the weight of the wall. Where the needle comes in contact with the wall, small cross-beams are laid on its upper side, and wedged and filled with mortar, to get a larger and more even bearing against the wall. At the inner and outer ends of the needles heavy upright timbers are placed underneath, running down to the new, lower level. The foot or ground bearing of these timbers is formed by heavy planks crossing each other, to spread the weight over more ground; wedges are driven under the feet of the uprights, till the ends of the needle are forced up, and the centre of the needle shows a decided downward curve of deflection, indicating that the weight of wall is on the needle. Frequently jack-screws are used in place of wedges, to get the weight onto the uprights. As soon as the needles carry the weight of wall, the intermediate portions of wall are torn out and the excavating to the lower level begins. If the soil is loose, sheath-piling must be resorted to on each side of wall. Frequently the feet of the uprights are "cribbed," that is, sheath-piled all around, to hold the ground under

them together and keep it from compressing. The new wall is built up from the lower level between and around the needles. On top of the new wall two layers of dressed-stone are placed filling up between the old and new work. Between these stones iron wedges are driven in opposite pairs, one from the inside and one from the outside. These wedges must be evenly driven from both sides or the wall might tip. These wedges are driven until the weight of the wall is on them and off the needles.

This is readily seen, for the needles straighten out when relieved of the load. The jack-screws are now lowered or the wedges under the uprights eased up; the uprights taken away, needles removed, and the holes filled up. Underpinning operations must be slowly and carefully performed, as they are very risky. If there is any danger of a wall tipping during the operation, grooves are cut into the wall and "shores" or braces placed against it. The feet of the shores rest on cross-planks, same as uprights, and are wedged up to get a secure bearing of the top of the shore against the wall. Where the outside of a wall cannot be got at, "spring needles" are used from the inside. That is, the one end of the needle acts as a lever and supports the wall, while the other, inner end, is chained and anchored down to prevent its tipping up.

Strength of bricks. The strength of a wall depends, of course, largely on the material used. A good, hard-burned brick, well laid in cement-mortar, makes a very strong wall. To tell a good brick, first examine the color; if it is very light, an orange-red, the brick is apt to be soft. If the brick is easily carved with a knife, it is soft. If it can be crushed to powder easily, it is soft. If two bricks are struck together sharply, and the sound is dull, the bricks are poor; if the sound is clear, ringing, metallic, the bricks are good and hard. If a brick shows a neat fracture, it is a good sign; a ragged fracture is generally a poor sign. The fracture also shows the evenness of the burning and fineness of the material. A brick that chips and cannot be cut easily is a good brick. The darker the brick, the harder burned. This, of course, does not hold good for artificially-colored bricks. The straighter and more regular the brick, the softer it is (as a rule), as hard-burning is apt to warp a brick.

What has been said of the strength of bricks holds good of terra-cotta. The latter should be designed to be of same thickness, if possible, in all parts, and any hollows caused thereby must be filled-in solid. It is best to fill-in the hollows with bricks and mortar several

days in advance, and let the filling set, so as to be sure it will not swell up afterwards and burst the terra-cotta.

Strength of stones. To judge of the strength and durability of stones is a more difficult matter. If the stone be fractured, and presents, under a magnifying glass, a bright, clear, sharp surface, it is not likely to crumble from decay; if the surface is dull-appearing and looks earthy, it is likely to decay. Of course, samples can be tested for their crushing and tensile strengths, etc. And we can tell somewhat of the weathering qualities by observing similar stones in old buildings; much, however, depends whether the stones come from the same part of the quarry. Another test is to weigh different samples, when dry; immerse them in water for a given period, say, twenty-four hours, then weigh them again, and the sample absorbing the least amount of water (in proportion to its original weight) is, of course, the best stone.

Another test is to soak the stone in water for two or three days and put it out to freeze; if it does not chip or crack, it will probably weather well. Chemical tests are made sometimes, such as using sulphuric acid, to detect the presence of lime and magnesia; or, soaking the stones in a concentrated boiling solution of sulphate of soda; the stones are then exposed to the air, when the solution crystalizes in the pores and chips off particles of the stone, acting similarly to frost. The stones are weighed before and after the tests, the one showing the least proportional loss of weight being, of course, the better.

If stones are laid on their natural beds, however, little need be feared of the result, if the stone seems at all serviceable. The main dangers to walls are from wet and frost. Very heavy and oft-repeated vibrations may sometimes shake the mortar-joints, but this need not be seriously feared, in most cases; machinery may often cause sufficient vibrations to be unpleasant, or even to endanger wood or iron work, but hardly well-built masonry. Of course, the higher a building is, the greater will be the amount of vibrations and their strength. For this reason it is advisable to place the heaviest machinery on the lowest (ground) floor. The beds of such machinery should be as far

Machinery foundations. as possible from any foundations of walls, columns, etc., and the beds should be independent and isolated from all other masonry. Malo, in *Le Génie Civil*, recommends the use of asphalt for machinery-foundations, as they take up the vibrations and noise, and are as solid as masonry, if properly built. His claim seems well founded, and has been demonstrated practically;

the asphalt foundation not only preventing vibrations but stopping the sound. A wooden form is made, covered inside with well-greased paper; into this are placed slightly-conical shaped wooden bars and boxes, also covered with well-greased paper, which are secured in the places to be occupied by the bolts and bolt-heads, and arranged for easy withdrawal. A layer of melted asphalt a few inches thick is then poured into the mould; over this are dumped heated, perfectly clean, sharp, broken stones and pebbles, rammed solid, the pebbles filling all interstices; then more asphalt is poured in, then another layer of stones and pebbles, etc. It is claimed that this foundation becomes so solid that it will not yield enough to disarrange the smooth running of any machinery, while its slightly-elastic mortar, besides avoiding vibrations and noise, prolongs very much the durability and usefulness of the machinery. Two dangers must be guarded against; viz., the direct contact of oil or heat with the asphalt. Stationary drip-pans guard against the former, while a layer of rubber, wood, cement, or other non-conductive material would accomplish the latter object. Where noise from machinery is to be avoided, a layer about one inch thick, of hard rubber or soft wood, should be placed immediately under the engine-plate. If this layer were bedded in asphalt the precaution would be still more effective.

Quality of asphalts. In all cases where asphalt is used, that with the least proportion of bitumen should be preferred.

Seyssell asphalt, which comes from France, is undoubtedly the best, and next to this comes the Swiss or Neuchâtel asphalt.

Trinidad asphalt, which is much used in this country, is much inferior, being softer and containing a larger proportion of bitumen or tar — a great disadvantage in many cases.

Openings over each other. In all walls try to get all openings immediately over each other. A rule of every architect should be to make an elevation of *every* interior wall, as well as of the exterior walls, to see that openings come over each other.

Tower walls. It is foolish to make chimneys or tower-walls unnecessarily thick (and heavy), as they brace and tie themselves together at each corner, and, consequently, are much stronger than ordinary walls. Tower-walls, however, often require thickening all the way down, to allow for deep splays and jambs at the belfry openings. The chief danger in towers is at the piers on main floor, which are frequently whittled down to dangerous proportions, to make large door openings. In tall towers and chimneys the leverage from wind must be carefully considered.

Considering the importance of ascertaining the exact strength of walls, it is remarkable that so little attention has been paid to the subject by writers and experimenters. The only known rule to the writer is Rondelet's graphical rule, which is as follows:

Rondelet's rule. If A B (Figure 83) be the height of a wall, B C the length of wall without buttress or cross-wall (A B C being a right angle), then draw A C; make A D = to either $\frac{1}{8}$ or $\frac{1}{10}$ or $\frac{1}{12}$ of A B, according to the nature of wall and building it is intended for ($\frac{1}{12}$ for dwellings, $\frac{1}{10}$ for churches and fireproof buildings, and $\frac{1}{8}$ for warehouses); then make A E = A D, and draw E F parallel to A B; then is B F the required thickness of wall. The rule, however, in many cases, gives an absurd result. Gwilt's "*Encyclopædia of Architecture*" gives this rule, and many additional rules, for its modification. There are so many of them, and they are so complex, however, as to be utterly useless in practice. Most cities have the thickness of walls regulated by law, but, as a rule, these thicknesses give the minimum strength that will do, and where they do not regulate the amount and size of flues and openings, frequently allow dangerously-weak spots in the wall.

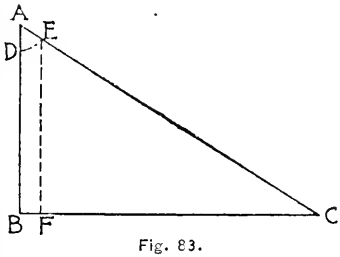


Fig. 83.

Formula for masonry. The writer prefers to use a formula, which he has constructed and based on Rankine's formula for long pillars, see Formula (3), and which allows for every condition of height, load, and shape and quality of masonry. In the case of piers, columns, towers or chimneys, whether square, round, rectangular, solid or hollow, the Formula (3) can be used, just as there given, inserting for ρ^2 its value, as given in the last column of Table I, using, of course, the numbered section corresponding to the cross-section of the pier, column or tower. By taking cross-sections at different points of the height, and using for l the height in inches from each such cross-section to the top, we will readily find how much to offset the wall. Care must be taken, where there are openings, to be sure to get the piers heavy enough to carry the additional load; the extra allowance for piers should be gotten by calculating the pier first as an isolated pier of the height of opening, and then by taking one of our cross-sections of the whole tower or chimney at the level where the openings are, and using whichever result required the greater strength.

Strength of piers, chimneys and towers.

As it would be awkward to use the height l in inches, we can modify the formula to use the height L in feet. Further, we know the value of n for brickwork, from Table II, and can insert this, too; we should then have:

For brick or rubble piers, chimneys and towers, of whatever shape:

$$w = \frac{a.\left(\frac{c}{f}\right)}{1 + 0{,}475.\left(\frac{L^2}{\varrho^2}\right)} \qquad (59)$$

Where w = the *safe total* load on pier, chimney or tower in pounds.

Where a = the area of cross-section of pier, etc., in square inches, at any point of height.

Where L = the height, in feet, from said point to top of masonry.

Where $\left(\frac{c}{f}\right)$ = the safe resistance to crushing, per square inch, as given in Table V. (See page 135.)

Where ϱ^2 = the square of the radius of gyration, of the cross-section, in inches, as given in Table I.

If it is preferred to use *feet and tons* (2000 lbs. each) we should have

$$W = \frac{A.\left(\frac{c}{f}\right)}{14 + 0{,}046.\left(\frac{L^2}{P^2}\right)} \qquad (60)$$

Where W = the *safe total* load on masonry, in tons, of 2000 lbs each.

Where A = the area of cross-section of masonry, at any point of height, in square feet.

Where L = the height, in feet, from said point to top.

Where $\left(\frac{c}{f}\right)$ = the safe resistance to crushing, in lbs., per square inch, as given in Table V. (See page 135.)

Where P^2 = the square of the radius of gyration, as given in last column of Table I, —*but all dimensions to be taken in feet.*

To obtain the load on masonry, include weight of all masonry, floors, roofs, etc., above the point and (if wind is not figured separately) add for wind 15 lbs. for each square foot of outside superficial area of *all* walls above the point.

Where towers, chimneys, or walls, etc., are isolated, that is not braced, and liable to be blown over by wind, the wind-pressure, must be looked into separately,

In regard to the use of $\left(\frac{c}{f}\right)$ the safe resistance, per square inch, of the material to crushing, it should be taken from Table V. So that for rubble-work we should use:

$$\left(\frac{c}{f}\right) = 100$$

And the same for poor quality brick, laid in lime mortar.

For fair brick in lime and cement (mixed) mortar, we should use:

$$\left(\frac{c}{f}\right) = 150$$

And for the best brickwork in cement mortar, we should use:

$$\left(\frac{c}{f}\right) = 200$$

If, however, a wall (or pier) is over 3 feet thick, and laid in good cement mortar, with the best hard-burned brick, and there are not many flues, etc., in the wall, we can safely use:

$$\left(\frac{c}{f}\right) = 250$$

If the wall (or pier) is over 3 feet thick, and there are no flues or openings, and the best brick and foreign Portland cements are used, it would be perfectly safe to use:

$$\left(\frac{c}{f}\right) = 300.$$

Example.

A tower 16 *feet square outside, carries a steeple weighing, including wind-pressure, some* 15 *tons. The belfry openings, on each side, are central and virtually equal to openings* 8 *feet wide by* 18 *feet high each. There are* 8 *feet of solid wall over openings. What should be the thickness of belfry piers? The masonry is ordinary rubble-work.*

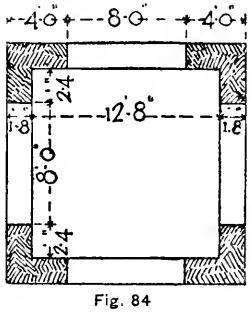


Fig. 84

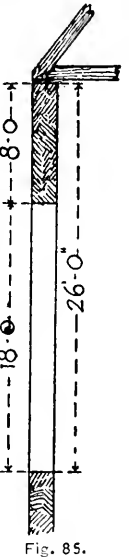


Fig. 85.

Tower Walls. In the first place we will try Formula (60) giving strength of whole tower at base of belfry piers. The load will be: $4.(26.16 - 18.8 - 26.1\frac{2}{3}) = 892$ superficial feet of masonry 20″ thick and weighing 250 lbs. per superficial foot = 223000 lbs. (see Figures

84 and 85) : add to this spire and we have at foot of belfry piers:

Actual load = 253090 lbs. or = 126 tons.

Now P² (the square of the radius of gyration) would be $P^2 = \frac{I}{A}$; the area $A = 16^2 - (12\frac{2}{3}^2 + 4.8.1\frac{2}{3}) = 43$; the moment of inertia $I = \frac{1}{12}.(16^4 - 12\frac{2}{3}^4 - 3\frac{1}{3}.8^3 - 8.16^3 + 8.12\frac{2}{3}^3) = 1799$.

Therefore $P^2 = \frac{1799}{43} = 41{,}8$. Now for rubble-work, Table V, $\left(\frac{c}{f}\right) = 100$; and, from Formula (60), the safe load would be:

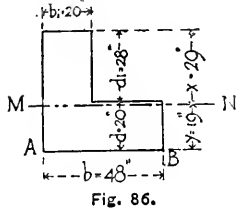


Fig. 86.

$$W = \frac{43.100}{14 + 0{,}046.\frac{26.26}{41{,}8}} = \frac{4300}{14{,}77}$$

= 291 tons; or more than strong enough.

Piers at opening. Now let us examine the strength of each pier by itself, Figure 86. In the first place we must find the distance y of the neutral axis M-N from say the line A B. This from Table I, Section No. 20 is:

$$y = \frac{\frac{48.20^2}{2} + 20.28.\left(20 + \frac{28}{2}\right)}{48.20 + 20.28} = 18{,}''8 \text{; or, say, } y = 19''.$$

Now $i = \frac{20.29^3 + 28.1^3 + 48.19^3}{3} = 272347$ (in inches) and $a = 20.48 + 20.28 = 1520$ square inches, therefore $\varrho^2 = \frac{i}{a} = 179$ (in inches).

The length of each pier is 18 feet, or

$L = 18$.

Therefore, from Formula (59) we have the safe load:

$$W = \frac{1520.100}{1 + 0{,}475.\frac{18.18}{179}} = 81940 \text{ pounds,}$$

or say the safe load on each pier would be 41 tons.

The actual load we know is $\frac{126}{4} = 31\frac{1}{2}$ tons, or the pier is more than safe.

Thickness of walls. Now let us see how far down it would be safe to carry the 20″ walls. We use formula (60) and have from Section Number 4, of Table 1:

$$P^2 = \frac{16^2 + 12\frac{2}{3}^2}{12} = 34\frac{2}{3} \text{ (in feet).}$$

The area would be

$$A = 16^2 - 12\tfrac{2}{3}^2 = 96 \text{ square feet.}$$

The load for each additional foot under belfry would be then:

$$96.150 = 14400 \text{ lbs., or } 7{,}2 \text{ tons.}$$

The whole load from top down for each additional foot would be, in tons:

$$W_1 = 126 + (L - 26).7{,}2 = 7{,}2.L - 61$$

While the safe load from Formula (60) would be:

$$W = \frac{96.100}{14 + 0{,}046.\frac{L^2}{34\frac{2}{3}}}$$

Now trying this for a point 50 feet below spire, we should have the actual load:

$$W_1 = 7{,}2.50 - 61 = 299 \text{ tons.}$$

and the safe load:

$$W = \frac{96.100}{14 + 0{,}046.\frac{50.50}{34\frac{2}{3}}} = 554 \text{ tons, or, we can go still}$$

lower with the 20″ work. For 70 feet below spire, we should have actual load:

$$W_1 = 7{,}2.70 - 61 = 443 \text{ to}$$

while the safe load:

$$W = \frac{96.100}{14 \times 0{,}046.\frac{70.70}{34\frac{2}{3}}} = 468 \text{ tons, or, 70 feet would be}$$

about the limit of the 20″ work.

If we now thicken the walls to 24″, we should have

$$A = 112 \text{ square feet.}$$

P^2 from Section 4, Table I, $= 33\frac{1}{3}$ (in feet).

The weight per foot would be $112.150 = 16800$ lbs. (or 8,4 tons) additional for every foot in height of 24″ work.

Therefore the actual load would be,

$$443 + (L - 70).8{,}4 \text{ or}$$

$$W_1 = L.8{,}4 - 145.$$

Now, for $L = 80$ feet, we should have the actual load:

$$W_1 = 527 \text{ tons, while the safe load would be:}$$

$$W = \frac{112.100}{14 + 0{,}046.\frac{80.80}{33\frac{1}{3}}} = 491 \text{ tons.}$$

This, though a little less than the actual load, might be passed. Rubble stone work, however, should not be built to such height, good

brickwork in cement would be better, as it can be built lighter; for $\left(\frac{c}{f}\right) = 200$, would give larger results, and brickwork weighs less, besides; then, too, we have the additional advantage of saving considerable weight on the foundations.

Thickening the walls of a tower or chimney on the inside does not strengthen them nearly so much as the same material applied to the outside would, either by offsetting the wall outside, or by building piers and buttresses.

It is mainly for this reason, and also to keep the flue uniform, that chimneys have their outside dimensions increased towards the bottom.

Example.

Calculation of chimneys. *A circular brick chimney is to be built 150 feet high, the flue entering about 6 feet from the base; the horse-power of boilers is 1980 IP. What size should the chimney be?*

The formula for size of flue is:

$$A = \frac{0.3 . IP + 10}{\sqrt{L}} \qquad (61)$$

Where A = the area of flue, in square feet.

Where L = the length of vertical flue in feet.

Where IP = the total horse-power of boilers.

Size of flue. A circular flue will always give a better draught than any other form, and the nearer the flue is to the circle the better will its shape be.

In our case the flue is circular, so that we will have

$$A = \frac{22}{7} . R^2 \text{ (see Table I, Sec. No. 7) or}$$

$$R = \sqrt{\frac{7.A}{22}}$$

Inserting the value of A from formula (61) we have:

$$R = \sqrt{\frac{7}{22} . \frac{0,3.1980 + 10}{\sqrt{144}}} = \sqrt{\frac{7}{22} . 50,3} = 4$$

or the radius of flue will be 4 feet (diameter 8 feet).

Now making the walls at top of chimney 8″ thick and adopting the rule of an outside batter of about $\frac{1}{4}$″ to the foot, or say 4″ every 15 feet, we get a section as shown in Figure 87.

Let us examine the strength of the chimney at the five levels A, B, C, D and E.

The thickness of the base of each part is marked on the right-hand section, and the average thickness of the section of the part on the left-hand side.

Take the part above A; the average area is $(\frac{22}{7}.5^2$ —flue area) or, 78—50 = 28 square feet.

This multiplied by the height of the part and the weight of one cubic foot of brickwork (112 lbs.) gives the weight of the whole, or actual load.

$W_1 = 28.30.112 =$ 94080 lbs., or 47 tons.

The area of the base at A would be:

$$A = \left(\frac{22}{7}.5\tfrac{1}{3}^2 - \text{flue area}\right);$$ or A = 89 — 50 = 39 square feet.

The height of the part is $L = 30$.

The square of the radius of gyration, in feet, is:

$$P^2 = \frac{5\frac{1}{3}^2 + 4^2}{4} = 11{,}11$$

Inserting these values in Formula (60) the safe load at A would be:

$$W = \frac{39.200}{14 + 0{,}046.\frac{30.30}{11{,}11}}$$

= 440 tons, or about nine times the actual load. Now, in examining the joint B we must remember to take the whole load of brickwork to the top as well as whole length L to top (or 60 feet).

The load on B we find is:

$W_1 = 131$ tons, while the safe load is:

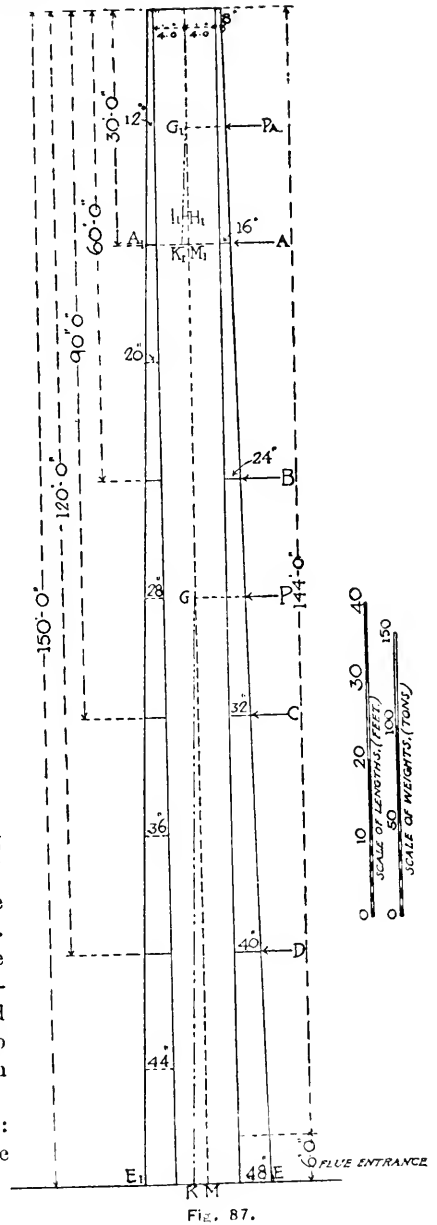


Fig. 87.

$$W = \frac{63.200}{14 + 0{,}046.\frac{60.60}{13}} = 472 \text{ tons.}$$

Similarly, we should find on C the load:

$W_1 = 259$ tons, while the safe load is:

$$W = \frac{89.200}{14 + 0{,}046.\frac{90.90}{15{,}11}} = 461 \text{ tons.}$$

On D we should find the load:

$W_1 = 432$ tons, while the safe load is:

$$W = \frac{119.200}{14 + 0{,}046.\frac{120.120}{17{,}44}} = 448 \text{ tons.}$$

Below D the wall is considerably over three feet thick, and is solid, therefore we can use $\left(\frac{c}{f}\right) = 300$, provided good Portland cement is used and best brick, which should, of course, be the case at the base of such a high chimney. We should have then the load on E:

$W_1 = 657$ tons, while the safe load is:

$$W = \frac{151.300}{14 + 0{,}046.\frac{150.150}{20}} = 690 \text{ tons.}$$

The chimney is, therefore, more than amply safe at all points, the bottom being left too strong to provide for the entrance of flue, which will, of course, weaken it considerably. We might thin the upper parts, but the bricks saved would not amount to very much and the offsets would make very ugly spots, and be bad places for water to lodge. If the chimney had been square it would have been much stronger, though it would have taken considerably more material to build it.

Tops of Chimney Flues.

It is generally best to build the flue of a chimney plumb from top to bottom, and, of course, of same area throughout. Sometimes the flue is gradually enlarged towards the top for some five to ten feet in height, which is not objectionable, and the writer has obtained good results thereby; some writers, though, claim the flue should be diminished at the top, which, however, the writer has never cared to try. Galvanized iron bands should be placed around the chimney at intervals, particularly around the top part, which is exposed very much to the disintegrating effects of the weather and the acids contained in the smoke. No smoke flue should ever be pargetted (plastered) inside, as the acids in the smoke will eat up the lime, crack the plaster, and cause it to fall. The

crevices will fill with soot and be liable to catch fire. The mortar-joints of flues should be of cement, or, better yet, of fire-clay, and should be carefully struck, to avoid being eaten out by the acids.

Calculation of Walls.—Bulging. Where walls are long, without buttresses or cross-walls, such as gable-walls, side-walls of building, etc., we can take a slice of the wall, one running foot in length, and consider it as forced to yield (bulge) inwardly or outwardly, so that for ρ^2 we should use:

$\rho^2 = \frac{d^2}{12}$; where d the thickness of wall in inches. The area or a would then be, in square inches, $a = 12.d$.

Inserting these values in formula (59) we have for

BRICK OR STONE WALLS.

$$w = \frac{d.\left(\frac{c}{f}\right)}{0{,}0833 + 0{,}475.\frac{L^2}{d^2}} \qquad (62)$$

Where w = the safe load, in lbs., on each running foot of wall (d'' thick).

Where d = the thickness, in inches, of the wall at any point of its height.

Where L = the height, in feet, from said point to top of wall.

Where $\left(\frac{c}{f}\right)$ = the *safe* resistance to crushing, in lbs., per square inch, as given in Table V. (See page 135.)

If it is preferred to use tons and feet, we insert in formula (60): for A = D, where D the thickness of wall, in feet, and we have:

$P^2 = \frac{D^2}{12}$; therefore

$$W = \frac{D.\left(\frac{c}{f}\right)}{14 + 0{,}552.\frac{L^2}{D^2}} \qquad (63)$$

Where W = the *safe* load, in tons, of 2000 lbs., on each running foot of wall (D feet thick).

Where D = the thickness of wall, in feet, at any point of its height.

Where L = the height, in feet, from said point to the top of wall.

Where $\left(\frac{c}{f}\right)$ = the safe resistance to crushing of the material, in lbs., per square inch, as found in Table V. (See page 135.)

Anchored Walls. Where a wall is thoroughly anchored to each tier

of floor beams, so that it cannot possibly bulge, except between floor-beams, use the height of story (that is, height between anchored

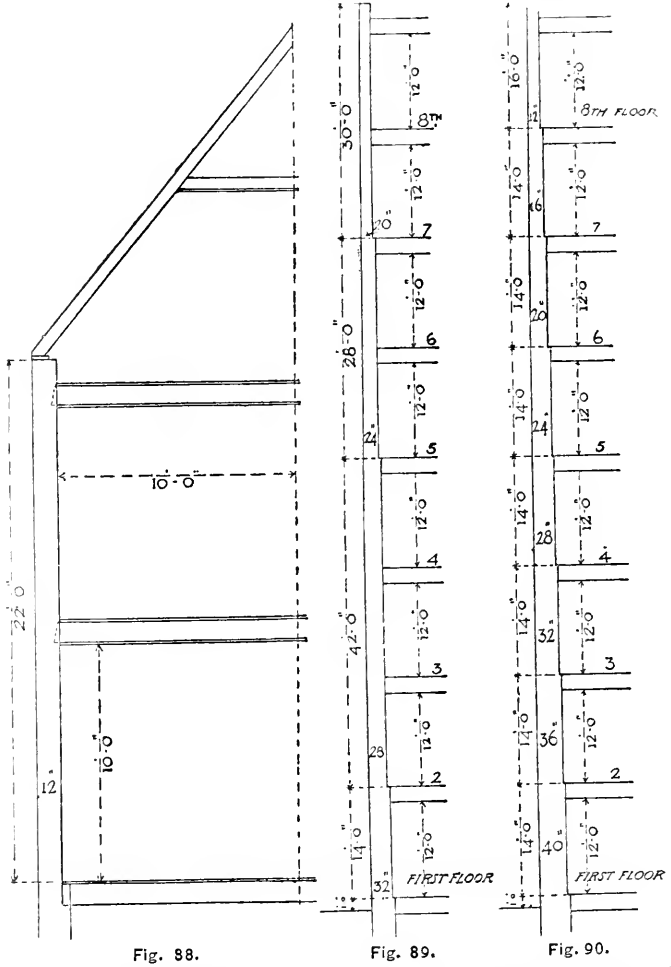


Fig. 88. Fig. 89. Fig. 90.

beams in feet) in place of L and calculate d or D for the bottom of wall at each story.

The load on a wall consists of the wall itself, from the point at which the thickness is being calculated to the top, plus the weight of

one foot in width by half the span of all the floors, roofs, partitions, etc. Where there are openings in a wall, add to pier the proportionate weight which would come over opening; that is, if we find the load per running foot on a wall to be 20000 lbs., and the wall consists of four-foot piers and three-foot openings alternating, the piers will, of course, carry not only 20000 lbs. per running foot, but the 60000 lbs. coming over each opening additional, and as there are four feet of pier we must add to each foot $\frac{60000}{4} = 15000$ lbs.; we therefore calculate the pier part of wall to carry 35000 lbs. per running foot. The actual load on the wall must not exceed the safe load as found by the formula (62) or (63).

Example.

Wall of Country House. *A two-story-and-attic dwelling has brick walls 12 inches thick; the walls carry two tiers of beams of 20 feet span; is the wall strong enough? The brickwork is good and laid in cement mortar.*

We will calculate the thickness required at first story beam level, Figure 88.

The load is, per running foot of wall:

Wall	$= 22.112 =$ 2464 lbs.
Wind	$= 22.\ 15 =$ 330 lbs.
Second floor	$= 10.\ 90 =$ 900 lbs.
Attic floor	$= 10.\ 70 =$ 700 lbs.
Slate roof (incl. wind and snow)	$= 10.\ 50 =$ 500 lbs.
Total load	$=$ 4894 lbs.

For the quality of brick described we should take from Table V:

$$\left(\frac{c}{f}\right) = 200 \text{ lbs.}$$

The height between floors is 10 feet, or

$$L = 10,$$

therefore, using formula (62) we have:

$$w = \frac{d.\left(\frac{c}{f}\right)}{0{,}0833 + 0{,}475.\frac{L^2}{d^2}} = \frac{12.200}{0{,}0833 + 0{,}475.\frac{10.10}{12.12}} = 5807 \text{ lbs.}$$

So that the wall is amply strong. If the wall were pierced to the extent of one-quarter with openings, the weight per running foot would be increased to 6525 lbs. Over 700 lbs. more than the safe load, still the wall, even then, would be safe enough, as we have allowed some 330 lbs. for wind, which would rarely, if ever, be so strong; and further, some 1200 lbs. for loads on floors, also a very

ample allowance; and even if the two ever did exist together it would only run the compression $\left(\frac{c}{f}\right)$ up to 225 lbs. per inch, and for a *temporary* stress this can be safely allowed.

The writer would state here, that the only fault he finds with formulæ (59), (60), (62) and (63), is that their results are apt to give an excess of strength; still it is better to be in fault on the safe side and be sure.

Example.

Walls of City Warehouse. *The brick walls of a warehouse are* 115 *feet high, the* 8 *stories are each* 14 *feet high from floor to floor, or* 12 *feet in the clear. The load on floors per square foot, including the fire-proof construction, will average* 300 *lbs. What size should the walls be? The span of beams is* 26 *feet on an average.* (See Fig. 89, page 142.)

According to the New York Building Law, the required thicknesses would be: first story, 32″; second, third, and fourth stories, 28″; fifth and sixth stories, 24″; seventh and eighth stories, 20″.

At the seventh story level we have a load, as follows, for each running foot of wall:

Wall	$= 30.1\frac{2}{3}.112 =$	5600
Wind	$= 30.30 =$	900
Roof	$= 13.120 =$	1560
Eighth floor	$= 13.300 =$	3900
Total		$= 11960$ lbs., or 6 tons.

The safe load on a 20″ wall 12 feet high, from formula (63) is:

$$W = \frac{1\frac{2}{3}.200}{14 + 0{,}552.\frac{12.12}{1\frac{2}{3}.1\frac{2}{3}}} = \frac{333}{14 + 0{,}552.51{,}84} = 7{,}811 \text{ tons, or } 15622 \text{ lbs.}$$

If one-quarter of the wall were used up for openings, slots, flues, etc., the load on the balance would be 8 tons per running foot, which is still safe, according to our formula.

At the fifth-story level the load would be:

Load above seventh floor		$= 11960$
Wall	$= 28.2.112 =$	6272
Wind	$= 28.30 =$	840
Sixth and seventh floors	$= 2.13.300 =$	7800
Total		$= 26872$ lbs. or $13\frac{1}{2}$ tons.

The safe load on a 24″ wall, 12 feet high, from formula (63) is:

$$W = \frac{2.200}{14 + 0{,}552.\frac{12.12}{2.2}} = \frac{400}{14 + 0{,}552.36} = 11{,}809 \text{ tons},$$

or 23618 lbs.

This is about 10 per cent less than the load, and can be passed as safe, but if there were many flues, openings, etc., in wall, it should be thickened.

At the second-story level the load would be:

Load above fifth floor		= 26872
Wall	= 42. $2\frac{1}{3}$.112	= 10976
Wind	= 42. 30	= 1260
Third, fourth and fifth floors	= 3.13.300	= 11700
Total		= 50808, or

25 tons.

The safe-load on a 28″ wall, 12 feet high, from formula (63) is:

$$W = \frac{2\frac{1}{3}.200}{14 + 0{,}552.\frac{12.12}{2\frac{1}{3}.2\frac{1}{3}}} = \frac{467}{14 + 0{,}552.26{,}45} = 16{,}33 \text{ tons},$$

or 32660 lbs.

Or, the wall would be dangerously weak at the second-floor level.

At the first-floor level the load would be:

Load above second floor		= 50808
Wall	= 14. $2\frac{2}{3}$.112	= 4181
Wind	= 14.30	= 420
Second floor	= 13.300	= 3900
Total		= 59309, or $29\frac{1}{2}$ tons.

The safe-load on a 32″ wall, 12 feet high, from formula (63) is:

$$W = \frac{2\frac{2}{3}.200}{14 + 0{,}552.\frac{12.12}{2\frac{2}{3}.2\frac{2}{3}}} = \frac{533}{14 + 0{,}552.20{,}25} = 21{,}169 \text{ tons},$$

or 42338 lbs.

The wall would, therefore, be weak at this point, too.

Now while the conditions we have assumed, an eight-story warehouse with all floors heavily loaded, would be very unusual, it answers to show how impossible it is to cover every case by a law, not based on the conditions of load, etc. In reality the arrangements of walls, as required by the law, are foolish. Unnecessary weight is piled on top of the wall by making the top 20″ thick, which wall has nothing to do but to carry the roof. (If the span of beams were increased to 31 feet or more the law compels this top wall to be 24″ thick, if 41 feet, it would have to be 28″ thick, an evident waste of material.) It

would be much better to make the top walls lighter, and add to the bottom; in this case, the writer would suggest that the eighth story be 12″; the seventh story 16″; the sixth story, 20″; the fifth story, 24″; the fourth story, 28″; the third story, 32″; the second story, 36″, and the first story 40″, see Figure 90. (Page 142.)

This would represent *but* $4\frac{2}{3}$ cubic feet of additional brickwork for every running foot of wall; or, if we make the first-story wall 36″ too, as hereafter suggested, the amount of material would be exactly the same as required by the law, and yet the wall would be much better proportioned and stronger as a whole. For we should find (for L = 12 feet),

Actual load at eighth-floor level,	3832
Safe load on a 12″ wall from Formula (62)	4298
Actual load at seventh-floor level,	10243
Safe load on a 16″ wall from Formula (62)	9135
Actual load at sixth-floor level,	17176
Safe load on a 20″ wall from Formula (62)	15729
Actual load at fifth-floor level,	24632
Safe load on a 24″ wall from Formula (62)	23750
Actual load at fourth-floor level,	32611
Safe load on a 28″ wall from Formula (62)	32825
Actual load at third-floor level,	41112
Safe load on a 32″ wall from Formula (62)	42638
Actual load at second-floor level,	50136
Safe load on a 36″ wall from Formula (62)	52902
Actual load at first-floor level,	59683
Safe load on a 40″ wall from Formula (62)	63492

The first-story wall could safely be made 36″ if the brickwork is good, and there are not many flues, etc., in walls, for then we could use $\left(\frac{c}{f}\right) = 250$, which would give a safe load on a 36″ wall = 65127 lbs., or more than enough.

The above table shows how very closely the Formula (62) would agree with a practical and common-sense arrangement of exactly the same amount of material, as required by the law.

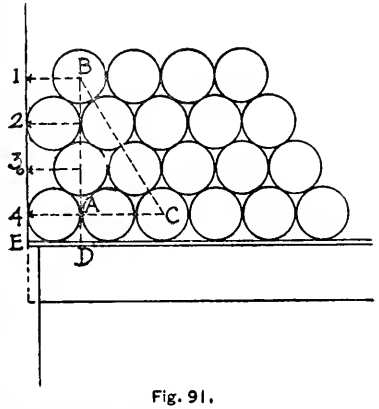


Fig. 91.

Thrust of barrels. Now, if the upper floor were laden with barrels, there might be some danger of these thrusting out the wall. We will suppose an extreme case, four layers of flour barrels packed against the wall, leaving a 5-foot aisle in the centre. We should have 20 barrels in each row (Fig. 91), weighing in all $20.196 = 3920$ lbs. These could not well be placed closer than 3 feet from end to end, or, say, 1307 lbs., per running foot of wall; of this amount only one-half will thrust against wall, or, say, 650 pounds. The radius of the barrel is about 20″. If Figure 92 represents three of the barrels, and we make A B $= w_1 = \frac{1}{2}$ the load of the flour barrels, per running foot of walls, it is evident that D B will represent the horizontal thrust on wall, per running foot. As D B is the radius, and as we know that A D $= 2$ D B or $=$ 2 radii, we can easily find A B, for:

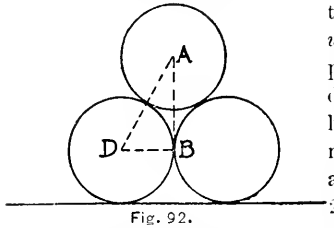


Fig. 92.

A D^2 — D B^2 = A B^2 or 4. D B^2 — D $B^2 = w_1^2$ or

$$D B^2 = \frac{w_1^2}{3} \text{ or } D B = \frac{w_1}{\sqrt{3}} = \frac{w_1}{1{,}73} = 0{,}578.w_1$$

Or, $h = 0{,}578.\ w_1$ (64)

Where $h =$ the horizontal thrust, in lbs., against each running foot of wall, $w_1 =$ one-half the total load, in lbs., of barrels coming on one foot of floor in width, and half the span. In our case we should have:

$$h = 0{,}578.650 = 375 \text{ lbs.}$$

Now, to find the height at which this thrust would be applied, we see, from Figure 91, that at point 1 the thrust would be from one line of barrels; at point 2, from two lines; at point 3, from three lines, etc.; therefore, the average thrust will be at the centre of gravity of the triangle A B C, this we know would be at one-third the height A B from its base A C.

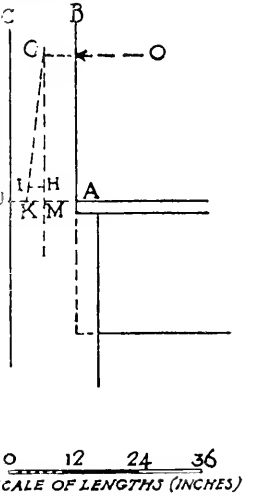


Fig. 93.

Now B C is equal to $6r$ or six radii of the barrels; further, A $C = 3r$, therefore:

A $B^2 = 36.r^2 - 9r^2 = 25.r^2$, and A $B = 5r$; therefore, $\frac{AB}{3} = 1\frac{2}{3}r$.

To this must be added the radius A D (below A) so that the central point of thrust, O in this case, would be above the beam a distance

$$y = 2\tfrac{2}{3}.r.$$

Where y = the height, in inches, above floor at which the average thrust takes place. Where r = radius of barrels in inches.

Our radius is 10″, therefore:

$$y = 26\tfrac{2}{3}''$$

Now, in Figure 93 let A B C D be the 12″ wall, A the floor level, G M the central axis of wall, and A O = $26\frac{2}{3}$; draw O G horizontally; make G H at any scale equal to the permanent load on A D, which, in this case, would be the former load less the wind and snow allowances on wall and roof, or

$$3832 - (16.30 + 13.30) = 2962, \text{ or, say } 3000 \text{ lbs.}$$

Therefore, make G H = 3000 lbs., at any scale; draw H I = h = 375 lbs., at same scale, and draw and prolong G I till it intersects D A at K. The pressure at K will be p = G I = 3023.

We find the distance M K measures

$$M\,K = x = 3\tfrac{1}{3}''.$$

Therefore, from formula (44) the stress at D will be:

$$v = \frac{3023}{144} + 6.\frac{3\frac{1}{3}.3023}{12.144} = +56 \text{ lbs. (or compression)}$$

While at A the stress would be, from formula (45):

$$v = \frac{3023}{144} - 6.\frac{3\frac{1}{3}.3023}{12.144} = -14 \text{ lbs. (or tension), so that}$$

the wall would be safe.

The writer has given this example so fully because, in a recent case, where an old building fell in New York, it was claimed that the walls had been thrust outwardly by flour barrels piled against them.

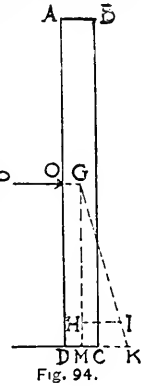


Fig. 94.

Narrow Piers. Where piers between openings are narrower than they are thick, calculate them, as for isolated piers, using for d (in place of thickness of wall) the width of pier between openings; and in place of L the height of opening. The load on the pier will consist, besides its own weight, of all walls, girders, floors, etc., coming on the wall above, from centre to centre of openings.

Wind-pressure. To calculate wind-pressure, assume it to be normal to the wall, then if A B C D, Figure 94, be the section of the whole wall above ground (there being no beams or braces against wall).

Make O D $= \frac{1}{2}$. A D $= \frac{L}{2}$; draw G H, the vertical neutral axis of the whole mass of wall, make G H, at any convenient scale, equal to the whole weight of wall; draw H I horizontally equal to the total amount of wind-pressure.

This wind-pressure on vertical surfaces is usually assumed as being equal to 30 lbs. per square foot of the surface, provided the surface is flat and normal, that is, at right angles to the wind. If the wind strikes the surface at an angle of 45° the pressure can be assumed at 15 lbs. per foot.

It will readily be seen, therefore, that the greatest danger from wind, to rectangular, or square towers, or chimneys, is when the wind strikes at right angles to the widest side, and not at right angles to the diagonal. In the latter case the exposed surface is larger, but the pressure is much smaller, and then, too, the resistance of such a structure diagonally is much greater than directly across its smaller side.

In circular structures multiply the average outside diameter by the height, to obtain the area, and assume the pressure at 15 lbs. per square foot. In the examples already given we have used 15 lbs., where the building was low, or where the allowance was made on all sides at once. Where the wall was high and supposed to be normal to wind we used 30 lbs. Referring again to Figure 94, continue by drawing G I, and prolonging it till it intersects D C, or its prolongation at K. Use formulæ (44) and (45) to calculate the actual pressure on the wall at D C, remembering that, $x =$ M K; where M the centre of D C, also that,

$p =$ G I; measured at same scale as G H.

Remember to use and measure everything uniformly, that is, all feet and tons, or else all inches and pounds. The wind-pressure on an isolated chimney or tower is calculated similarly, except that the neutral axis is central between the walls, instead of being on the wall itself; the following example will fully illustrate this.

Example.

Wind-pressure on Chimney. *Is the chimney, Figure* 87, *safe against wind-pressure?*

We need examine the joints A and E only, for if these are safe the intermediate ones certainly will be safe too, where the thickening of walls is so symmetrical as it is here.

The load on A we know is 47 tons, while that on E is 657 tons.

Now the wind-pressure down to A is:

$$P_a = 10.30.15 = 4500 \text{ lbs., or } = 2\tfrac{1}{4} \text{ tons.}$$

On base joint E, the wind-pressure is:

$$P = 12\tfrac{2}{3}.150.15 = 28500 \text{ lbs., or } = 14\tfrac{1}{4} \text{ tons.}$$

We can readily see that the wind can have no appreciable effect, but continue for the sake of illustration. Draw P_a horizontally at half the height of top part A till it intersects the central axis G_1; make $G_1 H_1$ at any convenient scale $= 47$ tons, the load of the top part; draw $H_1 I_1$ horizontally, and (at same scale) $= 2\frac{1}{4}$ tons $=$ the wind-pressure on top part; draw $G I_1$; then will this represent the total pressure (from load and wind) at K_1 on joint $A A_1$.

Use Formula (44) to get the stress at A_1 where $x = K_1 M_1 = 9''$, or $\frac{3}{4}$ feet; and $p = G_1 I_1$, which we find scales but little over 47 tons; and Formula (45) for stress at A. For d the width of joint we have of course the diameter of base, or $10\frac{2}{3}$ feet. Therefore

$$\text{Stress at } A_1 = \frac{47}{39} + 6.\frac{47.\frac{3}{4}}{39.10\frac{2}{3}} = +1{,}71 \text{ tons, per square foot, or } \frac{1{,}71.2000}{144} = +24 \text{ lbs. (compression), per square inch, and}$$

$$\text{stress at A} = \frac{47}{39} - 6.\frac{47.\frac{3}{4}}{39.10\frac{2}{3}} = +0{,}7 \text{ tons, per square foot, or } \frac{0{,}7.2000}{144} = +10 \text{ lbs. (compression), per square inch.}$$

To find the pressure on base $E E_1$; draw P G horizontally at half the whole height; make G M $= 657$ tons (or the whole load)[1] and draw M K horizontally, and $= 14\frac{1}{4}$ tons (or the whole wind-pressure). Draw G K and (if necessary) prolong till it intersects $E E_1$ at K. From formulæ (44) and (45) we get the stresses at E_1 and E: p being $=$ G K $= 658$ tons; and $x =$ M K $= 20''$, or $1\frac{2}{3}$ feet. For d the width of base we have the total diameter, or 16 feet. Therefore stress at

$$E_1 = \frac{658}{151} + 6.\frac{658.1\frac{2}{3}}{151.16} = +7 \text{ tons per sq. ft., or } \frac{7.2000}{144} = +97 \text{ lbs. (compression), per square inch, and stress at E} = \frac{658}{151} - 6.\frac{658.1\frac{2}{3}}{151.16} =$$

$+1\frac{2}{3}$ tons (compression), per square foot, or $\frac{1\frac{2}{3}.2000}{144} = +23$ lbs. (compression) per square inch.

There is, therefore, absolutely no danger from wind.

Strength of Corbels. Corbels carrying overhanging parts of the walls, etc., should be calculated in two ways, first, to see

[1] Scale of weights, Fig. 87, applies to $G_1 H_1$, etc., but not to G M.

whether the corbel itself is strong enough. We consider the corbel as a lever, and use either Formula (25), (26) or (27); according to how the overhang is distributed on the corbel, usually it will be (25). Secondly, to avoid crushing the wall immediately under the corbel, or possible tipping of the wall. Where there is danger of the latter, long iron beams or stone-blocks must be used *on top* of the back or wall side of corbel, so as to bring the weight of more of the wall to bear on the back of corbel.

To avoid the former (crushing under corbel) find the neutral axis G H of the whole mass, above corbel, Figure 95; continue G H till it intersects A B at K, and use Formulæ (44) and (45).

If M be the centre of A B, then use $x =$ K M, and $p =$ weight of corbel and mass above; remembering to use and measure all parts alike; that is, either, all tons and feet, or all pounds and inches.

Example.

Calculation of corbel. *A brick tower has pilasters 30″ wide, projecting 16″. On one side the tower is engaged and for reasons of planning the pilasters cannot be carried down, but must be supported on granite corbels at the main roof level, which is 36 feet below top of tower. The wall is 24″ under corbel, and averages from there to top 16″, offsets being on both sides, so that it is central over 24″ wall. What thickness should the granite corbel be?*

If we make the pilaster hollow, of 8″ walls, we will save weight on the corbel, though we lose the advantage of bonding into wall all way up. Still, we will make it hollow.

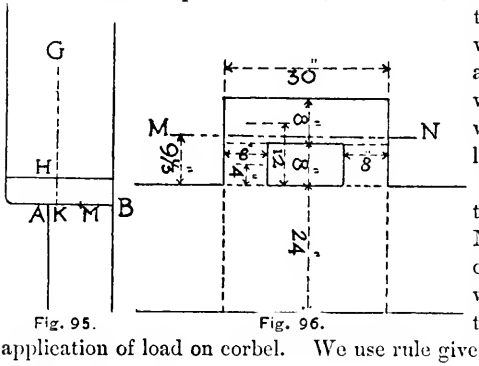


Fig. 95. Fig. 96.

First, find the distance of neutral axis M – N (Figure 96), of pilaster from 24″ wall, which will give the central point of application of load on corbel. We use rule given in the first article, and have

$$x = \frac{8.30.12 + 8.8.4 + 8.8.4}{8.30 + 8.8 + 8.8} = 9\tfrac{1}{3}''$$

The weight of pilaster (overlooking corbel) will be 6624 lbs., or $3\frac{1}{3}$ tons.

Assuming that only 30″ in length of the wall will come on back part of corbel, the weight on this part would be 8640 lbs., or $4\frac{1}{3}$ tons.

The bending moment of 6624 lbs. at $9\frac{1}{3}$″ from support on a lever is (see Formula 27):

$$m = 6624.9\tfrac{1}{3} = 61824 \text{ (lbs. inch).}$$

From Formula (18) we know that:

$$\frac{m}{\left(\frac{k}{f}\right)} = r$$

From Table I, Section Number 3, we have:

$$r = \frac{b.d^2}{6} = \frac{30.d^2}{6} = 5.d^2$$

And from Table V, we have for average granite:

$$\left(\frac{k}{f}\right) = 180$$

Inserting these values in Formula (18) we have:

$$\frac{61824}{180} = 5.d^2, \text{ or } d^2 = \frac{61824}{900} = 68{,}7, \text{ therefore, } d = 8\tfrac{1}{3}''.$$

We should make the block 10″ deep, however, to work better with brickwork. This would give at the wall a shearing area $= 10.30 =$ 300 square inches, or $\frac{6624}{300} = 22$ lbs., per square inch, which granite will certainly stand; still, it would be better to corbel out brickwork under granite, which will materially stiffen and strengthen the block.

Pressure under corbel. To find the crushing strain on brick wall under corbel, find the central axis of both loads by same rule as we find centre of gravity; that is, its distance A K from rear of wall will be (Figure 97)

$$\frac{4\frac{1}{3}.12 + 3\frac{1}{3}.33\frac{1}{3}}{4\frac{1}{3} + 3\frac{1}{3}} = 22\tfrac{1}{7}''$$

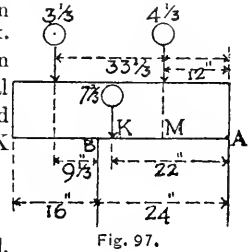


Fig. 97.

or, say, 22″, the pressure at this point will, of course, be $= 4\frac{1}{3} + 3\frac{1}{3} = 7\frac{2}{3}$ tons, or $p = 15340$ lbs.

The area will be $= 24.30 = 720$ square inches, while K M measures 10″; we have, then, from Formulæ (44) and (45) stress at

$$\text{B} = \frac{15340}{720} + 6.\frac{15340.10}{720.24} = +\ 74 \text{ lbs. (compression), and}$$

stress at $\text{A} = \frac{15340}{720} - 6.\frac{15340.10}{720.24} = -\ 31$ lbs. (tension).

There would seem to be, therefore, some tendency to tipping, still

we can pass it as safe, particularly as much more than 30″ of the wall will bear on the rear of corbel.

If the wind could play against inside wall of tower, it might help to upset the corbel, but as this is impossible, its only effect could be *against* the pilaster, which would materially help the corbel against tipping.

CHAPTER V.

ARCHES.

THE manner of laying arches has been described in the previous chapter, while in the first chapter was given the theory for calculating their strength; all that will be necessary, therefore, in this chapter will be a few practical examples. Before giving these, however, it will be of great assistance if we first explain the method of obtaining *graphically* the neutral axis of several surfaces, for which the arithmetical method has already been given (p. 7.).

Neutral axis found graphically.

To find the vertical neutral axis of two plane surfaces A B E F and B C D E, proceed as follows: Find the centres of gravity G_{II} of the former surface and G_I of the latter surface. Through these centres draw G_{II} H_{II} and G_I H_I vertically.

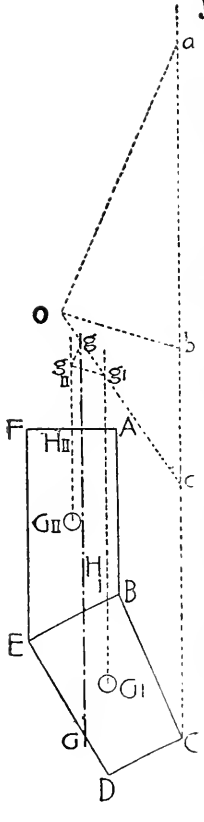


Fig. 98.

Draw a vertical line *a c* anywhere, and make *ab* at any scale equal to surface A B E F, and at same scale make *b c* = B C D E. From any point *o*, draw *o a*, *ob* and *oc*. From the point of intersection g_I of *o c* with H_I G_I, draw g_I g_{II} parallel *o b* till it intersects H_{II} G_{II} at g_{II}; from g_{II} draw g_{II} *g* parallel with *o a* till it intersects *o c* at *g*; through *g* draw *g* G vertically and this is the desired vertical neutral axis of the whole mass.

Where there are many parts the same method is used.

We will assume a segmental arch divided into five equal parts. Calling part A B C D = No. I; part D C E F = No. II, etc., and the vertical neutral axis of each part, No. I, No. II, etc.

Draw $a\ f$ vertically and at any convenient scale, make

$a\ b$ = No. I or A B C D
$b\ c$ = No. II or D C E F,
$c\ d$ = No. III
$d\ e$ = No. IV and
$e\ f$ = No. V

From any point o draw oa, ob, oc, od, oe and of.

From point of intersection 1 of vertical axis No. I and oa, draw 1 g parallel with $o\ b$ till it intersects vertical axis No. II at g: then draw $g\ h$ parallel with $o\ c$ till it intersects vertical axis No. III at h; similarly draw $h\ i$ parallel with $o\ d$ to axis No. IV and $i\ j$ parallel with $o\ e$ to axis No. V, and, finally, draw j 5 parallel with $o\ f$ till it intersects $a\ o$ (or its prolongation) at 5. A vertical axis through 5 is the vertical neutral axis of the whole mass. Prolong $j\ i$ till it intersects $o\ a$ at 4, $i\ h$ till it intersects $o\ a$ at 3, and $h\ g$ till it intersects $o\ a$ at 2.

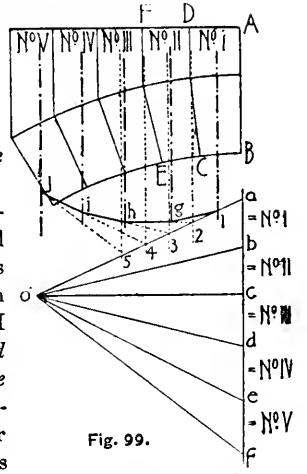


Fig. 99.

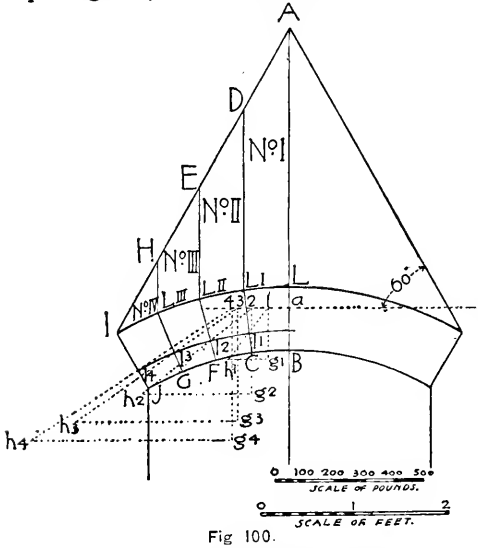


Fig 100.

A vertical axis through 4 will then be the vertical neutral axis of the mass of Nos. I, II, III and IV; a vertical axis through 3 will be the vertical neutral axis of the mass of Nos. I, II and III; and through 2 the vertical neutral axis of the mass of Nos. I and II. Of course the axis through 1 is the vertical neutral axis of No. I.

We will now take a few practical examples.

Example I.

In a solid brick wall an opening 3 feet wide is bricked over with an 8-inch arch. Is this strong enough? [1]

The thickness of the wall or arch, of course, does not matter, *where the wall is solid*, and we need only assume the wall and arch to be one foot thick. If the wall were thicker, the arch would be correspondingly thicker and stronger, so that in all cases where a load is evenly distributed over an arch we will consider both always as one foot thick. If a wall is hollow, or there are uneven loads, we can either take the full actual thickness of the arch, or we can proportion to one foot of thickness of the arch its proportionate share of the load.

Arch in solid wall.

In our example we assume everything as one foot thick. The load coming on the half-arch B J I L Fig. 100, will be enclosed by the lines A L and I A at 60° with the horizon. We divide the arch into, say, four equal voussoirs B C = C F = F G = G J. (The manner of dividing might, of course, have been arbitrary as well as equal, had we preferred.) Draw the radiating lines through C, F and G, and from their upper points draw the vertical lines to D, E and H.

Voussoirs arbitrary.

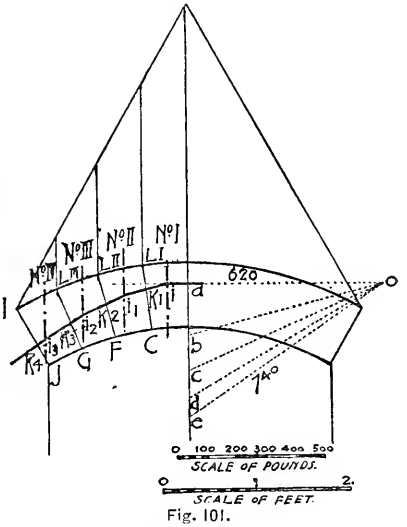


Fig. 101.

Now find the weight of each slice, *remembering* always to include the weight of the voussoir in each slice. We have, then, approximately,

No. I (A B C D) = 3′ x $\frac{1}{2}$′ x 112 pounds = 168 pounds
No. II (D C F E) = 2$\frac{1}{8}$′ x $\frac{1}{2}$′ x 112 " = 119 "
No. III (E F G H) = 1$\frac{1}{2}$′ x $\frac{7}{16}$′ x 112 " = 73 "
No. IV (H G J I) = $\frac{7}{8}$′ x $\frac{7}{16}$′ x 112 " = 43 "
Total = 403 "

[1] For convenience, most of the figures are duplicated: the first, showing manner of obtaining the horizontal thrust; the second, the manner of obtaining the line of pressure.

As the arch is evidently heavily loaded at the centre, we assume the point a at one-third the height of B L from the top, or L $a = \frac{\text{L B}}{3} = 2\frac{2}{3}''$ and draw the horizontal line a 4.

As previously explained, find the neutral axes:

Horizontal pressures.

1 g_1 of part No. I,
2 g_2 of parts No. I plus No. II,
3 g_3 of parts Nos. I, II and III, and
4 g_4 of the whole half arch.

Now make at any scale 1 g_1 = 168 units = No. I; similarly at same scale 2 g_2 = 168 + 119 units = 287 = No I and No. II, and

3 g_3 = 287 + 73 units = 360 = No. I + No. II + No. III.

4 g_4 = 360 + 43 units = 403 = weight of half arch and its load.

Now make: C $l_1 = \frac{1}{3}$ C $L_1 = 2\frac{2}{3}''$.

Similarly, F $l_2 = \frac{1}{3}$ F $L_2 = 2\frac{2}{3}''$; also, G $l_3 = \frac{1}{3}$ G $L_3 = 2\frac{2}{3}''$,

And, J $l_4 = \frac{1}{3}$ J I $= 2\frac{2}{3}''$.

Through g_1, g_2, g_3 and g_4 draw horizontal lines, and draw the lines 1 l_1, 2 l_2, 3 l_3 and 4 l_4 till they intersect the horizontal lines at h_1, h_2, h_3 and h_4; then will $g_1 h_1$ measured at same scale as 1 g_1 represent the horizontal thrust of A B C D; $g_2 h_2$ the horizontal thrust of A B F E; $g_3 h_3$ the horizontal thrust of A B G H and $g_4 h_4$ the horizontal thrust of the half arch and its load. In this case it happens that the latter is the greatest, so that we select it as our horizontal pressure, and make (in Fig. 101) $a\,o = g_1 h_4 = 620$ pounds, at any convenient scale.

Now (in Fig. 101) make

$a\,b$	= 168	pounds	= No. I.
$b\,c$	= 119	"	= No. II.
$c\,d$	= 73	"	= No. III.
$d\,e$	= 43	"	= No. IV.

Draw ob, oc, od and oe.

Curve of pressure.

Now begin at a, draw a 1 parallel with $o\,a$ till it intersects axis No. I at 1; from 1 draw 1 i_1 parallel with $o\,b$ till it intersects axis No. II at i_1; from i_1 draw $i_1\,i_2$ parallel with $o\,c$ till it intersects axis No. III at i_2; from i_2 draw $i_2\,i_3$ parallel with $o\,d$ till it intersects axis No. IV at i_3; from i_3 draw $i_3\,K_4$ parallel with $o\,e$. A curve through the points a, K_1, K_2, K_3 and K_4 (where the former lines intersect the voussoir joint lines) and tangent to the line a 1 $i_1\,i_2\,i_3$ K_4 would be the real curve of pressure. The amount of pressure on joint C L_1 would be concentrated at K_1 and would be equal to $o\,b$ (measured at same scale as $a\,b$, etc.). The pressure on joint F L_2 would be concentrated at K_2 and be equal to $o\,c$. The pressure on

joint G L_3 would be concentrated at K_3 and equal to $o\ d$. The pressure on the skew-back joint J I would be concentrated at K_4 and be equal to $o\ e$. The latter joint evidently suffers the most, for not only has it got the greatest pressure to bear, but the curve of pressure is farther from the centre than at any other joint. We need calculate this joint only, therefore, for if it is safe, the others certainly are so, too. By scale we find that J K_4 measures $2\frac{1}{2}''$, or K_4 is ($x =$) $1\frac{1}{2}''$ from the centre of joint; we find further that $o\ e$ scales 740 units, therefore ($p =$) $o\ e =$ 740 pounds.

Stress at extrados and intrados.

The width of joint is, of course, 8″, and the area = 8 x 12 = 96 square inches; therefore, from Formula (44)

$$\text{Stress at edge J} = \frac{740}{96} + 6.\ \frac{740.\ 1\frac{1}{2}}{96.\ 8} = +16{,}26$$

and from (45)

$$\text{Stress at edge I} = \frac{740}{96} - 6.\ \frac{740.\ 1\frac{1}{2}}{96.\ 8} = -0{,}85$$

Or the edge J would be subject to the slight compression of $16\frac{1}{4}$ pounds, and edge at I to a tension of a little less than one pound per square inch. The arch, therefore, is more than safe.

Example II.

A four-inch rowlock brick arch is built between two iron beams, of five feet span, the radius of arch being five feet. The arch is loaded at the rate of 150 pounds per square foot. Is it safe?

In this case we will divide the top of arch A D into five equal sections and assume that each section carries 75 pounds — (which, of course, is not quite correct). We find the horizontal pressures (Fig. 102) $g_1 h_1$, $g_2 h_2$, etc., as before, and find

Brick floor-arch.

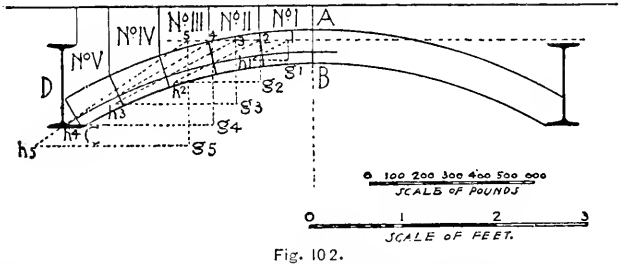


Fig. 102.

again $g_5 h_5$ the largest and equal to 575 pounds. We now make (Fig. 103) at any convenient scale, $o\ a = g_5 h_5 = 575$ pounds, and $a\ b =$

$b\,c = c\,d = d\,e = e\,f = 75$ pounds and draw $o\,a$, $o\,b$, $o\,c$, etc. We now find the broken line $a\ 1\ i_1\ i_2\ i_3\ i_4\ K_5$ where:

$a\ 1$ is parallel with	$o\,a$	$1\ i_1$ is parallel with	$o\,b$
$i_1\ i_2$ "	" $o\,c$	$i_2\ i_3$ "	" $o\,d$
$i_3\ i_4$ "	" $o\,e$	$i_4\ K_5$ "	" $o\,f$

In this case again evidently the greatest stress is on the skew-back joint C D, for it not only has the greatest pressure $o\,f$, but the curve of

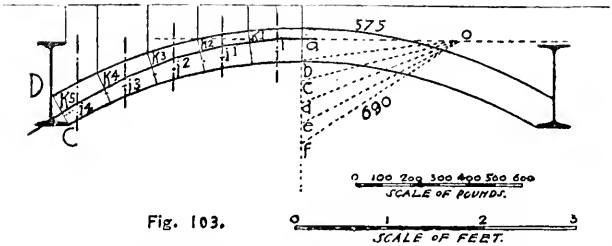


Fig. 103.

pressure passes farther from the centre than at any other joint. We find that C K_5 scales $1\frac{1}{4}$ inches, therefore the distance of K_5 from the centre is $(x =)\ \frac{3}{4}''$. We scale $o\,f$ and find it scales 690 units, or $(p\ ) = 690$ pounds. The joint is 4" wide and its area $= 4 \times 12 = 48$ square inches.

From Formulæ (44) and (45) we have then:

$$\text{Stress at C} = \frac{690}{48} + 6.\frac{690.\ \frac{3}{4}}{48.\ 4} = +\ 30{,}6 \text{ pounds and}$$

$$\text{Stress at D} = \frac{690}{48} - 6.\frac{690.\ \frac{3}{4}}{48.\ 4} = -\ 1{,}8 \text{ pounds.}$$

The arch, therefore, is perfectly safe.

Example III.

Two iron beams, five feet apart, same as before, but filled with a straight 7" hollow fire-clay arch. The load per foot to be assumed at 140 pounds. Is the arch safe?

Fireproof floor-arch.

Of the 350 pounds on the half arch we will assume 80 pounds to come on each of the blocks and 30

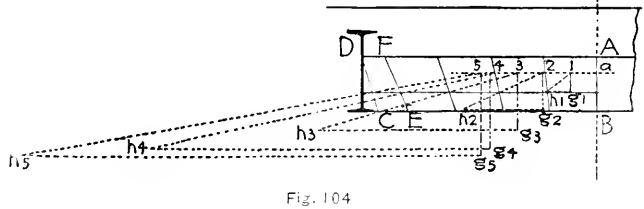


Fig. 104

pounds on the skew-back. We then (in Fig. 104) find, as before the horizontal pressures, $g_1 h_1$, $g_2 h_2$, etc. Again we find the largest pressure to be $g_5 h_5$, and as it scales 2040 units, we make (in Fig. 105) at any convenient scale and place $o\,a = g_5 h_5 = 2040$ pounds. We also make $a\,b = b\,c = c\,d = d\,e = 80$ units and $e\,f = 30$ units.

Draw *oa*, *ob*, *oc*, etc. Drawing the lines parallel thereto, beginning at *a* we get the line $a\,1\,i_1\,i_2\,i_3\,i_4\,K_5$, same as before Imagining a joint

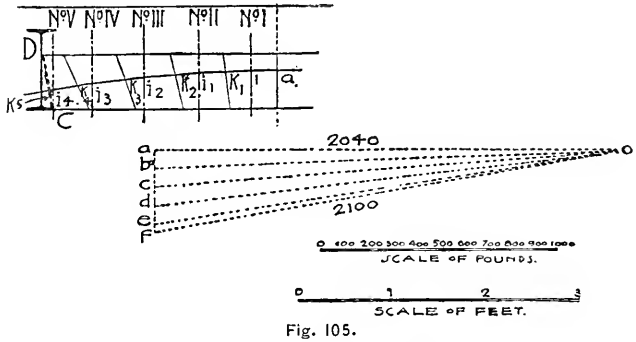


Fig. 105.

at C D this would evidently be the joint with greatest stress, for the same reasons mentioned before. We find C K_5 scales $2\frac{5}{8}''$, and as C D scales $7\frac{1}{4}''$ the point K_5 is distant from the centre of joint.

$$(x =)\ 3\tfrac{5}{8} - 2\tfrac{5}{8} = 1''$$

as $o\,f$ scales 2100 units or pounds, and the joint is $7\frac{1}{4}''$ deep with area $= 7\frac{1}{4}.\ 12 = 87$ square inches, we have:

$$\text{Stress at C} = \frac{2100}{87} + 6.\frac{2100.\ 1}{87.\ 7\frac{1}{4}} = +\ 44{,}14 \text{ pounds and}$$

$$\text{Stress at D} = \frac{2100}{87} - 6.\frac{2100.\ 1}{87.\ 7\frac{1}{4}} = +\ 4{,}14 \text{ pounds.}$$

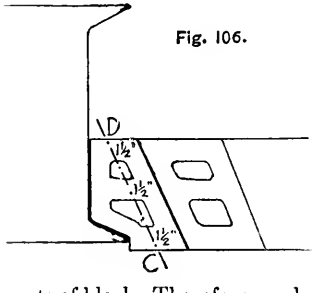


Fig. 106.

The arch, therefore, would seem perfectly safe. But the blocks are not solid; let us assume a section through the skew-back joint C D to be as per Fig. 106. We should have in Formulæ (44) and (45) x, p, and the depth of joint same as before, but for the area we should use $a = 3.1\frac{1}{2}.12 = 54$ square inches, or only the area of solid parts of block. Therefore we should have:

$$\text{Stress at C} = \frac{2100}{54} + 6.\frac{2100.1}{54.\ 7\frac{1}{4}} = +\ 71 \text{ pounds, and}$$

$$\text{Stress at D} = \frac{2100}{54} - 6.\frac{2100.1}{54.\ 7\frac{1}{4}} = +\ 6{,}71 \text{ pounds.}$$

There need, therefore, be no doubt about the safety of the arch.

Example IV.

Over a 20-inch brick arch of 8 feet clear span is a centre pier 16′ wide, carrying some two tons weight. On each side of pier is a window opening 2½ feet wide, and beyond, piers similarly loaded. Is the arch safe?

Arch in front wall concentrated loads.

We divide the half arch into five equal voussoirs. The amounts and neutral axes of the different voussoirs, and loads coming over each, are indicated in circles and by arrows; thus, on the top voussoir E B (Fig. 107) we have a load of 2100 pounds, another of 62 pounds, and the weight of voussoir or 228 pounds. The neutral axis of the three is the vertical through G_I (Fig. 108). Again on voussoir E F (Fig. 107) we

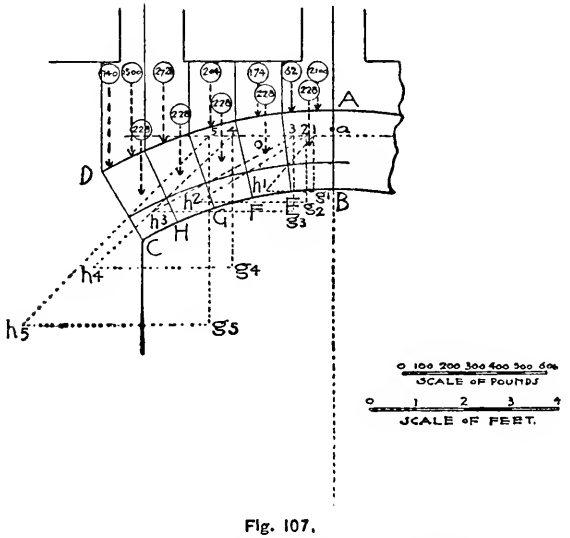


Fig. 107.

have the load 174 pounds, and weight of voussoir 228 pounds; the vertical neutral axis of the two being through G_{II} (Fig. 108). Similarly we get the neutral axes G_{III}, G_{IV} and G_V (Fig. 108) for each of the other voussoirs. Now remembering that 1 g_1 (Fig. 107) is the

neutral axis of and equal to the voussoir B E and its load; 2 g_2 the neutral axis of and equal to the sum of the voussoirs B E and E F and their loads, etc., we find the horizontal thrusts $g_1 h_1$, $g_2 h_2$, $g_3 h_3$, etc. The last $g_5 h_5$ is again the largest, and we find it scales 7850 units or pounds.

The arch being heavily loaded we selected a at one-third from the top of A B. We now make (Fig. 108) $a\,o = 7850$ pounds or units at any scale; and at same scale make $a\,b = 2390$ pounds; $b\,c = 402$ pounds; $c\,d = 432$ pounds; $d\,e = 2956$ pounds, and $e\,f = 1868$ pounds. Draw $o\,b$, $o\,c$, $o\,d$, etc. Now draw as before a 1 parallel with $o\,a$ to axis G_1; also 1 i_1 parallel with $o\,b$ to axis G_{II}; $i_1\,i_2$ parallel with $o\,c$ to G_{III}, etc. We then again have the points a, K_1, K_2, K_3, K_4

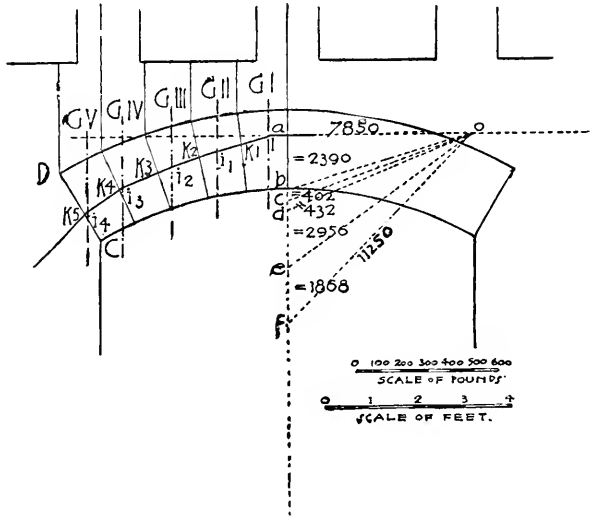


Fig. 108.

and K_5 of the curve of pressure. As K_5 is the point farthest from the centre of arch-ring and at the same time sustains the greatest pressure ($o\,f$) we need examine but the joint C D; for if this is safe so are the others. We insert, then, in Formulæ (44) and (45) for

$p = o\,f = 11250$ pounds, and as K_5 C measures $6\frac{1}{2}''$,

$x = 10'' - 6\frac{1}{2}'' = 3\frac{1}{2}''$; also as the joint is $20''$ wide,

$a = 12.\ 20 = 240$ square inches.

Therefore

$$\text{Stress at C} = \frac{11250}{240} + 6.\frac{11250.\ 3\frac{1}{2}}{240.\ 20} = +\ 96 \text{ pounds, and}$$

$$\text{Stress at D} = \frac{11250}{240} - 6.\frac{11250.\ 3\frac{1}{2}}{240.\ 20} = -\ 3 \text{ pounds.}$$

The arch is, therefore, safe.

Example V.

A 12-inch brick semi-circular arch has 12 foot span. A solid brick wall is built over the arch to a level with one foot above the keystone. The abutment piers are 5 feet high to the spring of arch and are each 3 feet wide, including, of course, the width of skew-backs. Are the arch and piers safe?

Arch in fence-wall with abutment.

As before, we will assume arch, pier, and wall over arch, each one foot thick. We will divide the load over arch into seven equally wide slices. This will make uneven voussoirs, but this does not matter, as our joint lines (and voussoirs) are only imaginary anyhow, and not necessarily of the shape of the actual voussoirs, which in brick would, of course, be represented by each single brick. The amount of the sums of each voussoir and its load, and the vertical neutral axes of the different sets are given by the arrows and lines G_1, G_2, G_3, etc. (in Fig. 110). When considering the safety of the abutment we treat it exactly the same as the voussoirs (and loads) of the arch; that is, we

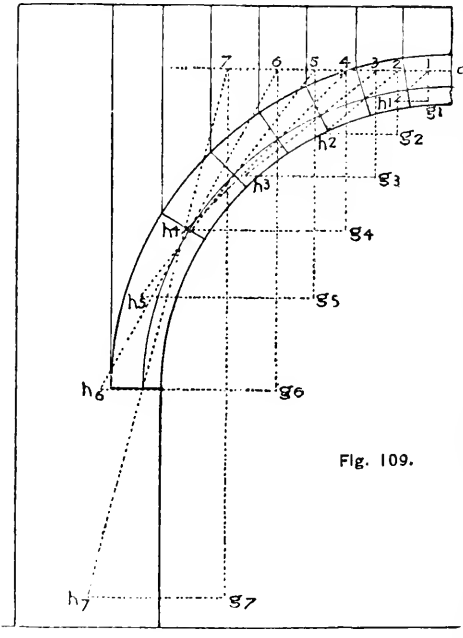


Fig. 109.

take the whole weight of the abutment, viz., C D E F I H C and find its neutral axis G_8.

Returning now to the arch, we go through the same process as before. We find the horizontal pressures (Fig. 109) $g_1 h_1$, $g_2 h_2$, etc. In this case we find that the last pressure $g_7 h_7$ is not as large as $g_6 h_6$; therefore we adopt the latter; it scales 1425 units or pounds. We now make (Fig. 110) $a\ o = 1425$ pounds; and $a\ b = 251$ pounds; $b\ c = 280$ pounds; $c\ d = 373$ pounds, etc.; $g\ h$ is equal to the last section of arch or 1782 pounds. We continue, however, and make $h\ i = 4600$ pounds = the weight of abutment. Draw $o\ a$, $o\ b$, $o\ c$, etc., to $o\ i$. Then get the tangents to the curve of pressure, as before, viz.: $a\ 1\ i_1\ i_2\ i_3\ i_4\ i_5\ i_6\ K_7$; we now continue $i_6\ K_7$, which is parallel with $o\ h$ till it intersects the vertical axis G_8 of the abutment at i_7 and from thence we draw $i_7\ K_8$ parallel with $o\ i$.

Thrust on abutment. We will now examine the base joint I H of pier. I K_8 scales $10\frac{1}{4}''$, and as the pier is 36″ wide, K_8 is $7\frac{3}{4}''$ from the centre of joint. The area is $a = 12.36 = 432$ and the pressure is $p = o\ i = 9100$ pounds.

Therefore,

$$\text{Stress at I} = \frac{9100}{432} + 6.\frac{9100.7\frac{3}{4}}{432.36} = +48 \text{ pounds},$$

and

$$\text{Stress at II} = \frac{9100}{432} - 6.\frac{9100.7\frac{3}{4}}{432.36} = -6 \text{ pounds}.$$

There is, therefore, a slight tendency for the pier to revolve around the point I, raising itself at II; still the tendency is so small, only 6 pounds per square inch, that we can safely pass the pier, so far as danger from thrust is concerned.

Joint C D at the spring of the arch looks rather dangerous, however, as $i_6\ i_7$ cuts it so near its edge D. Let us examine it. D K_7 measures $1\frac{1}{2}''$, therefore K_7 is $4\frac{1}{2}''$ from the centre of joint, which is 12″ wide. The area is, of course, $a = 12.\ 12 = 144$ and the pressure $p = o\ h = 4600$ pounds. Therefore,

$$\text{Stress at D} = \frac{4600}{144} + 6.\frac{4600.4\frac{1}{2}}{144.12} = +104 \text{ pounds},$$

and

$$\text{Stress at C} = \frac{4600}{144} - 6.\frac{4600.4\frac{1}{2}}{144.12} = -40 \text{ pounds}.$$

It is evident, therefore, that the arch itself is not safe, and it should be designed deeper; that is, the joints should be made deeper (say, 16″), and a new calculation made.

Tie-rods to arches.

If, instead of an abutment-pier, we had used an iron tie-rod, its sectional area would have to be sufficient to resist a tension equal to the greatest horizontal thrust *o a;*

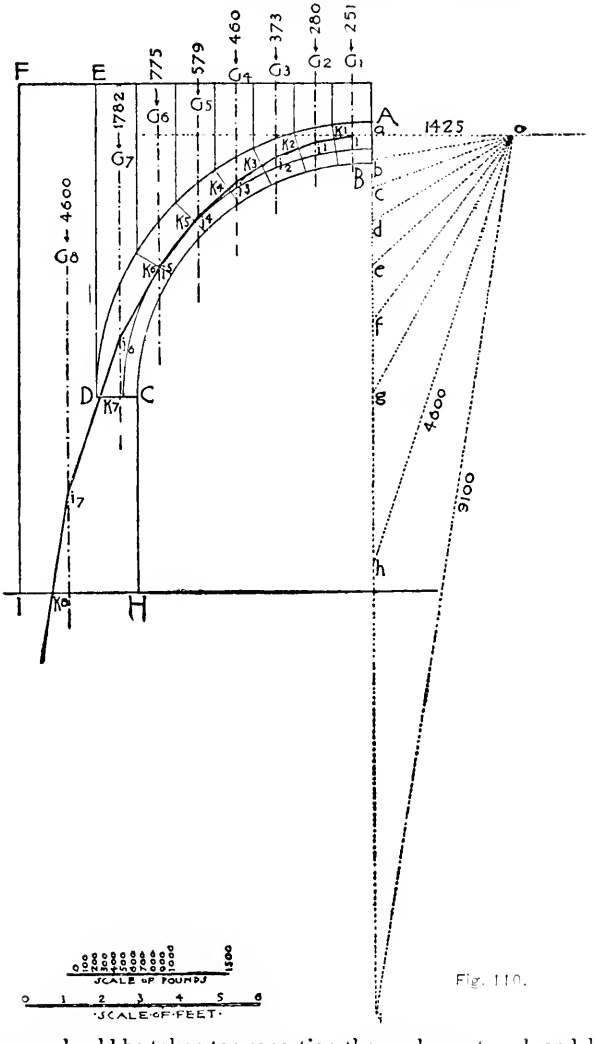


Fig. 110.

and care should be taken to proportion the washers at each end, large

enough that they may have sufficient bearing-surface so as not to crush the material of the skew-backs.

Thus, in Example III, Fig. 105, if we should place the iron tie-rods to the beams 5 feet apart, they would resist a tension equal to five times the horizontal thrust $o\ a$, which, of course, was calculated for 1 foot only, or

$$t = 5.\ 2040 = 10200 \text{ pounds.}$$

The safe resistance of wrought-iron to tension is from Table IV, 12000 pounds per square inch; we need, therefore:

$$\frac{10200}{12000} = 0{,}85 \text{ square inches}$$

of area in the rod; or the rod should be $1\frac{1}{16}''$ diameter. A $1''$ or even $\frac{7}{8}''$ rod would probably be strong enough, however, as such small iron is apt to be better welded, and, consequently, stronger, and the load on the arch would probably be a "dead" one.

As the end of rod will bear directly against the iron beam, the washers need have but about $\frac{1}{4}''$ bearing all around the end of the rod, so that the nut would probably be large enough, and no washer be needed.

Example VI.

A pier 28″ *wide and* 10′ *high supports two abutting semi-circular arches; the right one a* 20″ *brick arch of* 8′ *span; the left one an* 8″ *brick arch of* 3′ *span. The loads on the arches are indicated in the Figure* 111. *Is the pier safe?*

Uneven arches with central pier. The loads are so heavy compared to the weight of the voussoirs, that we will neglect the latter, in this case, and consider the vertical neutral axis of and the amount of each load as covering the voussoirs also; except in the case of the lower two voussoirs, where the axes are considerably affected. We find the curve of pressure of each arch as before.

For the large arch we would have the curve through a and i, for the small one through a_1 and i_1; the points i and i_1 being the intersections of the curves with the last vertical neutral axis of each arch.

Now from i draw $i\ x$ parallel with $o\ f$, and from i_1 draw $i_1\ x$ (backwards), but parallel with $o_1 f_1$ till the two lines intersect at x. Now make $f\ g$ parallel with and $= o_1 f_1$, and draw $g\ h$ vertically $= 2600$ pounds $=$ the weight of the pier from the springing line to the base (1′ thick). Draw $o\ g$ and $o\ h$. Now returning to x, draw $x\ y$ parallel with $g\ o$ till it intersects the neutral axis of the pier at y, and from y draw $y\ z$ parallel with $o\ h$ till it intersects the base joint C D of

pier (or its prolongation) at K_1. Continue also $x\ y$ till it intersects the springing joint A B and K. Now, then, to get the stresses at

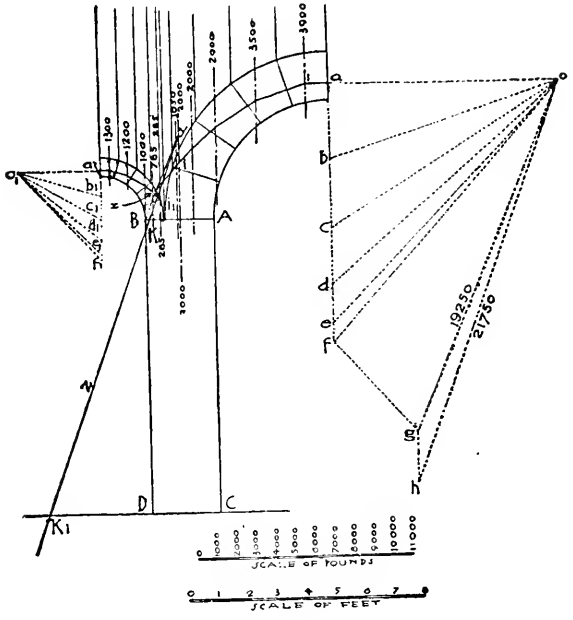


Fig. 111.

joint A B we know that the width of joint is 28″, therefore $a = 28.12 = 336$; further $p = o\ g = 19250$ pounds, and as $B\ K$ scales one inch, K is distant 13″ from the centre of joint, therefore

$$\text{Stress at B} = \frac{19250}{336} + 6.\ \frac{19250.\ 13}{336.\ 28} = +\ 217 \text{ pounds.}$$

and

$$\text{Stress at A} = \frac{19250}{336} - 6.\ \frac{19250.\ 13}{336.\ 28} = -\ 102 \text{ pounds.}$$

Thrust on central pier.

The arch, therefore, cannot safely carry such heavy loads. The pier we shall naturally expect to find still more unsafe, and in effect have, remembering that joint D C is 28″ wide, therefore, area 336 square inches, and as K_1 distant 54″ from centre of joint, and $p = o\ h = 21750$ pounds.

$$\text{Stress at D} = \frac{21750}{336} + 6.\ \frac{21750.\ 54}{336.\ 28} = +\ 813 \text{ pounds.}$$

and

$$\text{Stress at C} = \frac{21750}{336} - 6.\frac{21750.54}{336.28} = -684 \text{ pounds.}$$

Relief through iron-work. The construction, therefore, must be radically changed, if the loads cannot be altered. If the arches are needed as ornamental features, they should be constructed to carry their own weight only, and iron-work overhead should carry the loads, and bear either nearer to, or directly over, the piers, as farther trials and calculations might call for. If this is done the wall should be left hollow under all but the ends of iron-work until it gets its "set"; that is, until it has taken its full load and deflection; and then the wall should be pointed with soft "putty" mortar.

To avoid cracks.

Example VII.

The foundations of a building rest on brick piers 6′ *wide and* 18′ *apart. The piers are joined by* 32″ *brick inverted arches and tied together* 8′ *above the spring of the arch. Piers and arches are* 3′ *thick. Load on central piers is* 72 *tons; on end pier* 60 *tons. Is this construction safe?*

Inverted arches. We will first examine the inner or left pier. The pier being 3′ thick and the load 72 tons, each 1′ of thickness will, of course, carry $\frac{72}{3} = 24$ tons. The width from centre to centre of piers is just 24′, so that each running foot of wall under

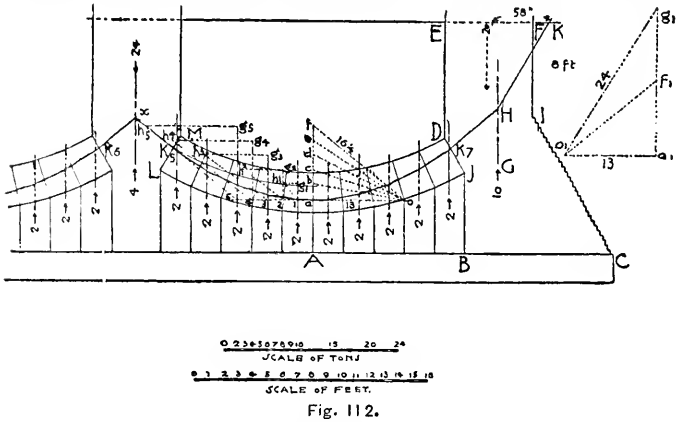


Fig. 112.

arch will receive a pressure of one ton. Now all we need to do is to imagine this pressure as the load on the arch. We can either draw

the arch upside down with a load of one ton per foot, or we can make the drawing with the arch in correct position and the weights pressing upward, as shown in Fig. 112. We divide the load into five equal slices, each about 2′ wide, therefore = 2 tons each. We make $a\ b = 2$ tons; $b\ c = 2$ tons, etc., and find the horizontal pressures $g_1\ h_1$, $g_2\ h_2$, etc., same as before. Again, $g_3\ h_3$ is the largest, measuring 13 units or tons. Now make $o\ a = 13$ tons, and draw $o\ b$, $o\ c$, etc. Construct the line $a\ 1\ K_5$ or curve of pressure same as before. Joint L M is evidently the most strained one. We find M K_5 measures 11″, and as the arch is 32″ (= M L), of course, K_5 is 5″ from the centre of joint. The area of joint is $a = 32.\ 12 = 384$. We scale $o\ f$, the pressure at K_5, and find that it measures $16\frac{1}{2}$ units, or 33000 pounds, therefore

Strength of arch.

$$\text{Stress at M} = \frac{33000}{384} + 6.\frac{33000.\ 5}{384.\ 32} = +168 \text{ pounds}$$

$$\text{Stress at L} = \frac{33000}{384} - 6.\frac{33000.\ 5}{384.\ 32} = +6 \text{ pounds.}$$

The arch, therefore, is perfectly safe. Of course, the left pier is safe, for being an inner pier the resistance $K_6\ x$ of the adjoining arch to the left will just counter-balance the thrust of our arch, or $K_5\ x$. But at the end (right) pier this is different. We, of course, proportion the length of foundation A C, to get same pressure as on rest of wall under arch. The end pier carries 60 tons, or $\frac{60}{3} = 20$ tons per foot thick. The pressure per running foot on wall under arch we found to be one ton, therefore A C should be 20 feet long. The half-arch will take the pressure of 10 feet (from A to B) or 10 tons, and the balance (10 tons) will come on B C. This will act through its central axis G H, which, at the arch skew-back, will be half-way between the end of arch J and outside pier-line I F. This will, of course, deflect somewhat the abutment (or last pressure) line K_7 H of the arch. At any convenient place draw $a_1\ f_1$ vertically equal 10 tons, and $a_1\ o_1$ horizontally equal 13 tons, the already known horizontal pressure. Then K_7 H is, of course, parallel with and equal $o_1\ f_1$. Now make $f_1\ g_1 =$ 10 tons and draw $o_1\ g_1$. Now on the pier draw H K parallel with $o_1\ g_1$; H being the point of intersection of G H and K_7 H. As the pier is tied back 8′ above the arch, we take our joint-line at E F, being 8′ above D. We find, by scale, that K is 58″ from the centre of E F, the latter being 72″ wide. The area is $a = 72.\ 12 = 864$. And as $o_1\ g_1$ scales 24 units, the pressure is, of course, 24 tons or 48000 pounds,

Central pier.

Thrust on end pier.

therefore:

$$\text{Stress at F} = \frac{48000}{864} + 6.\frac{48000.58}{864.72} = +315 \text{ pounds.}$$

and

$$\text{Stress at E} = \frac{48000}{864} - 6.\frac{48000.58}{864.72} = -207 \text{ pounds.}$$

Use of buttress. There is, therefore, no doubt of the insecurity of the end pier. Two courses for safety are now open. Either we can build a buttress sufficiently heavy to resist the thrust of the end arch, or we can tie the pier back. The former case is easily calculated; we simply include the mass of the buttress in the resistance and shift the axis G H to the centre of gravity of the area of pier and buttress up to joint E F. Of course, the buttress should be carried up to the joint-line E F, but it can taper away from there.

Use of tie-rods. If we tie back with iron we need sufficient area to resist a tension equal to the horizontal pressure *a o*, which in this case is 13 tons or 26000 pounds per foot thick of wall. As the wall is 3′ thick, the total horizontal thrust is 3. 26000 = 78000 pounds. The tensional stress of wrought-iron being 12000 pounds per square inch, we need $\frac{78000}{12000} = 6\frac{1}{2}$ square inches area, or, say, two wrought-iron straps, 4″ x $\frac{7}{8}$″, one each side of pier. By this method the inner or left arch becomes practically the end arch. For the last two piers and the right arch become one solid mass; and not only is their entire weight thrown against the second or inner arch, but the centre of gravity of the whole mass shifts to the centre line of end arch, or in our case 9′ *inside* of the end pier; so that there is no possible doubt of the strength of the abutment. There is one element of weakness, however, in the small bearing the pier has on the skew-back of the arch, the danger being of the pier cracking upwards and settling past and under the arch. This can be avoided, as already explained, by building a large bond-stone across the entire pier, forming skew-backs for the arch to bear against.

Example VIII.

A semi-circular dome, circular in plan, is 40′ inside diameter. The shell is 5′ thick at the spring and 2′ at the crown. The dome is of cut-stone. Will it stand?

Calculation of dome. We draw a section (Fig. 114) of the half-dome and treat it exactly the same as any other arch. The only difference is in the assumption of the weights. Instead of assuming the arch 1′ thick, we take with each voussoir its entire weight around

one-half of the dome. Thus, in our case, we divide the section into six imaginary voussoirs. The weight on each voussoir will act through its centre of gravity. Now weight No. I will be equal to the area of the top voussoir multiplied by the circumference of a semi-circle with A B as radius. Similarly, No. II will be equal to the area of the second voussoir multiplied by the circumference of a semi-circle with A C as radius; No. III will be equal to the area of voussoir 3, multiplied by the circumference of a semi-circle with A D as radius, etc. The vertical neutral axes Nos. I, II, III, etc., act, of course, through the centres of gravity of their respective voussoirs. Now the top voussoir measures 5′ 6″ by an average thickness of 2′ 1″ or $5\frac{1}{2} . 2\frac{1}{12} = 11{,}46$ or, say, $11\frac{1}{2}$ square feet. As A B (the radius) measures 2′ 9″, the circumference of its semi-circle would be 8′,6. Taking the weight of the stone at 160 pounds per cubic foot, we should then have the weight of No. I = 11,5. 8, 6. 160 = 15824 pounds, or, say, 8 tons.

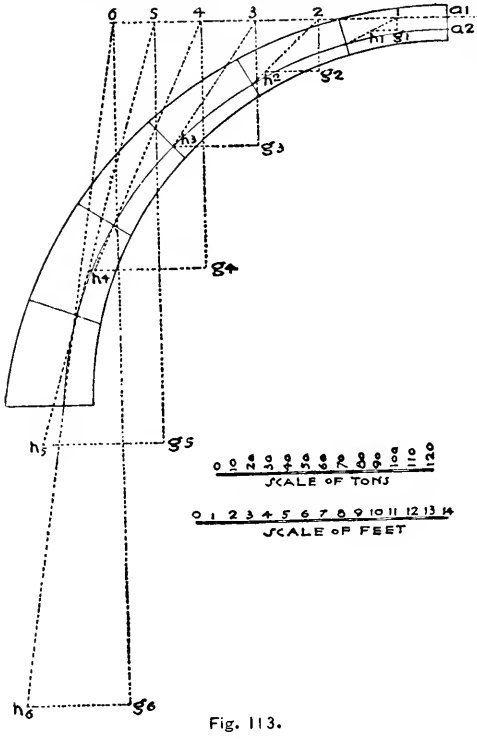


Fig. 113.

Similarly we should have:

No. II = 12,4. 25,1. 160 =	49798	pounds, or, say,	25	tons.
No. III = 14,2. 40. 160 =	90880	" "	45	"
No. IV = 17,4. 52,7. 160 =	146716	" "	73	"
No. V = 21,2. 62,8. 160 =	213017	" "	106	"
No. VI = 28,7. 69,1. 160 =	317307	" "	158	"

We now make $a\ b = 8$ tons, $b\ c = 25$ tons, $c\ d = 45$ tons, $d\ e = 73$ tons, $e\ f = 106$ tons and $f\ g = 158$ tons. We find the horizontal pressures $g_1\ h_1$, $g_2\ h_2$, etc. (in Fig. 113), same as before. In this case

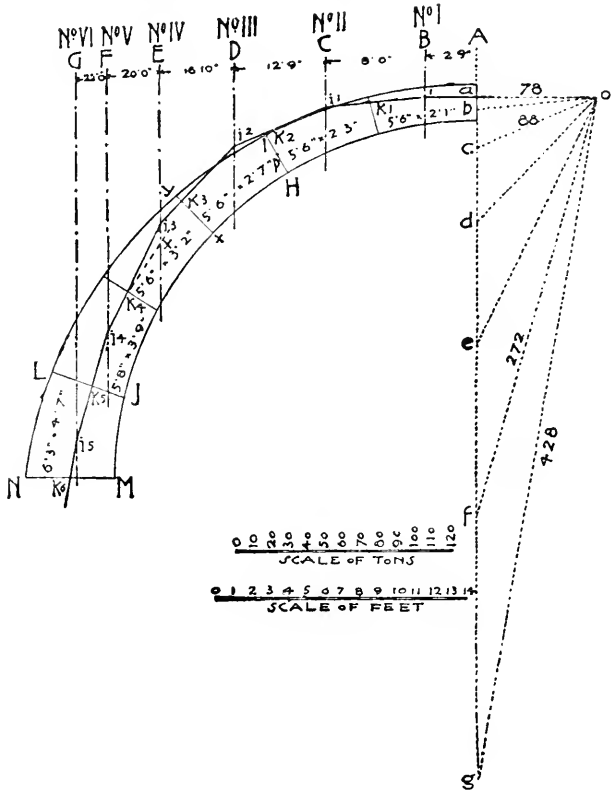


Fig. 114.

we find that the largest pressure is not the last one, but $g_5\ h_5$ which measures 78 units; we therefore select the latter and (in Fig. 114) make $a\ o = g_5\ h_5 = 78$ tons. Draw $o\ b$, $o\ c$, $o\ d$, etc., and construct the line $a\ 1\ i_1\ i_2\ i_3\ i_4\ i_5\ K_6$, same as before. In this case we cannot tell at a glance which is the most strained voussoir joint, for at joint H I the pressure is not very great, but the line is farthest from the centre of joint. Again, while joint J L has not so much pressure as

the bottom joint M N, the line is farther from the centre. We must, therefore, examine all three joints.

We will take H I first. The width of joint is 2′ 5″ or 29″. The pressure is $o\,c$, which scales 88 tons or 176000 pounds. The distance of K_2 from the centre of joint P is 16″. The area of the joint is, of course, the full area of the joint around one-half of the dome, or equal to H I multiplied by the circumference of a semi-circle with the distance of P from $a\,g$ as radius. The latter is 10′ 6″, therefore area $= 2\frac{5}{12}.\ 33 = 80$ square feet or 11520 square inches, therefore:

$$\text{Stress at I} = \frac{176000}{11520} + 6.\ \frac{176000.\ 16}{11520.\ 29} = +\ 65{,}9$$

$$\text{Stress at II} = \frac{176000}{11520} - 6.\ \frac{176000.\ 16}{11520.\ 29} = -\ 35{,}3.$$

For joint J L we should have: the width of joint $= 50''$. The pressure $= o\,f = 272$ tons or 544000 pounds. The distance of K_5 from the centre of joint is 8″, while for the area we have $50.\ 66\frac{2}{3}.\ 12 = 40000$ square inches, therefore:

$$\text{Stress at J} = \frac{544000}{40000} + 6.\ \frac{540000.\ 8}{40000.\ 50} = +\ 26{,}6 \text{ pounds,}$$

and

$$\text{Stress at L} = \frac{544000}{40000} - 6.\ \frac{540000.\ 8}{40000.\ 50} = +\ 0{,}5 \text{ pound.}$$

For the bottom joint M N we should have the width of joint $= 60''$. The pressure $= o\,g = 428$ tons or 856000 pounds. The distance of K_6 from the centre of joint is 4″, while for the area we have $60.\ 70\frac{2}{3}.\ 12 = 50880$ square inches, therefore:

$$\text{Stress at M} = \frac{856000}{50880} + 6.\ \frac{856000.\ 4}{50880.\ 60} = +\ 23{,}5 \text{ pounds,}$$

and

$$\text{Stress at N} = \frac{856000}{50880} - 6.\ \frac{856000.\ 4}{50880.\ 60} = +\ 10 \text{ pounds.}$$

The arch, therefore, would seem perfectly safe except at the joint H I, where there is a tendency of the joint to open at H. Had we, however, remembered that this is an arch, lightly loaded, and started our line at the lower third of the crown joint, instead of at the upper third, the line would have been quite different and undoubtedly safe.

The dangerous point K_2 in that case would be much nearer the centre of joint, while the other lines and joints would not vary enough to call for a new calculation, and we can safely pass the arch. One thing must be noted, however, in making the new figure, and that is that the horizontal pressures $g_1\,h_1$, $g_2\,h_2$, etc. (in Fig. 113), would have

to be changed, too, and would become somewhat larger than before, as the line a_1 6 would now drop to the level of a_2. A trial will show that the largest would again be $g_5 h_5$, and would scale 80 units or tons, which should be used (in Fig. 114) in place of $a\ o$.

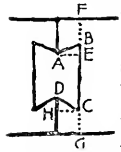


Fig. 115.

Thrust of dome. As regards abutments, there usually are none in a dome; it becomes necessary, therefore, to take up the horizontal thrust, either by metal bands around the entire dome, or by dovetailing the joints of each horizontal course. We must, of course, take the horizontal thrust existing at each joint. Thus, if we were considering the second joint H I the horizontal thrust would be $g_2 h_2$; or, if we were considering the fifth joint J L the horizontal thrust would be $g_5 h_5$. For the lower joint M N we might take its own horizontal thrust $g_6 h_6$, which is smaller than $g_5 h_5$, provided we take care of the joint J L separately; if not, we should take the larger thrust.

Metal bands. If we use a metal band its area manifestly should be strong enough to resist *one-half* the thrust, as there will be a section in tension at each end of the semi-circle, or

$$a = \frac{h}{2.\left(\frac{t}{f}\right)} \qquad (65)$$

Where a = area, in square inches, of metal bands around domes, at any joint.

Where h = the horizontal thrust at joint, in pounds.

Where $\left(\frac{t}{f}\right)$ = the safe resistance of the metal to tension, per square inch.

In our case, then, for the two lower joints, if the bands are wrought-iron, we should have, as $h = 80$ tons $= 160000$ pounds.

$$a = \frac{160000}{2.12000} = 6\tfrac{2}{3}$$

Or we would use a band, say, $5'' \times 1\tfrac{1}{3}''$.

Strength of dowels. If dovetailed dowels of stone are used, as shown in Fig. 115, there should be one, of course, in every vertical joint. The dowels should be large enough not to tear apart at A D, nor to shear off at A E, nor to crush A B. Similarly, care should be taken that the area of C G + B F is sufficient to resist the tension, and of H C to resist the shearing.

The strain on A D will, of course, be tension and equal to one-half the horizontal thrust; the same formula holds good, therefore, as for

metal bands, except, of course, that we use for $\left(\frac{t}{f}\right)$ its value for whatever stone we select. Supposing the dome to be built of average marble, we should have from Table V,

$$\left(\frac{t}{f}\right) = 70, \text{ therefore,}$$

$$a = \frac{160000}{2.\ 70} = 1143$$

Now if the dome is built in courses 2′ 6″ or 30″ high, the width of dovetail at its neck A D would need to be:

$$\text{A D} = \frac{1143}{30} = 38 \text{ inches.}$$

As this would evidently not have sufficient area at G C and B F we must make A D smaller, and shall be compelled to use either a metal dowel, or some stronger stone. By reference to Table V we find that for bluestone,

$$\left(\frac{t}{f}\right) = 140, \text{ therefore:}$$

$$a = \frac{160000}{2.\ 140} = 571 \text{ and,}$$

$$\text{A D} = \frac{571}{30} = 19 \text{ inches.}$$

This would almost do for the lower joint, which is 60″ wide, for if we made: G C + B F = 38″ and

A D = 19″ it would leave

60 — 38 + 19 = 3″ or, say, 1½″ splay; that is, D H = 1½″. Had we used iron, or even slate, however, there would be no trouble.

The shearing strain on either A E or H C will, of course, be equal to one-quarter of the horizontal thrust or

$$b = \frac{h}{4.\ d.\left(\frac{q}{f}\right)} \qquad (66)$$

Where b = the width of one-half the dowel, in inches (A E)

Where h = the horizontal thrust at the joint, in pounds

Where d = the height of the course, in inches.

Where $\left(\frac{q}{f}\right)$ = the safe resistance to shearing, per square inch, of either dowel or stone voussoir (whichever is weaker).

When the shearing stress of a stone is not known, we can take the tension instead. Thus in our case as the marble is the weaker, we

should use $\left(\frac{g}{f}\right) = 70$, therefore

$$A\ E = H\ C = \frac{160000}{4.\ 30.\ 70} = 19 \text{ inches.}$$

The compression on A B need not be figured, of course, for while the strain is the same as on A E, the area of A B is somewhat larger, and all stones resist compression better than shearing.

Cambered plank arch. When a plank is "cambered," sprung up, and the ends securely confined, it becomes much stronger transversely than when lying flat. The reason is very simple, as it now acts as an arch, and forces the abutments to do part of its work. Such a plank can be calculated the same as any other arch.

Spanish tile arches. Quite a curiosity in construction, somewhat in the above line, has been recently introduced in New York by a Spanish architect. He builds floor-arches but 3″ thick, of 3 successive layers of 1″-thick tiles, up to 20′ span, and more. His arches have withstood safely test-loads of 700 pounds a square foot. The secret of the strength of his arches consists in their following closely the curve of pressure, thus avoiding tension in the voussoirs, as far as possible. But even were this to exist, it could not open a joint without bodily tearing off several tiles and opening many joints, as shown in Fig. 116, owing to the fact that each course is thoroughly bonded and breaks joints with the course below; besides this, each upper layer is attached to its lower layer by Portland-cement mortar. Specimens of these tiles have been tested for the writer and were found to be as follows:

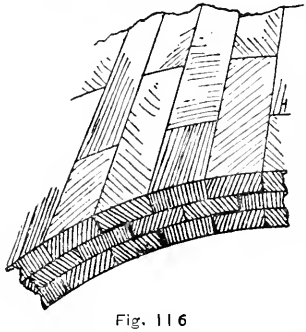


Fig. 116

Compression (12 days old) $c = 2911$ pounds

or say $\left(\frac{c}{f}\right) = 300$ pounds.

Tension (10 days old) $t = 287$ pounds

or say $\left(\frac{t}{f}\right) = 30$ pounds.

The shearing stress of the mortar-joints is evidently greater than the tension, as samples tested tore across the tile and could not be sheared off.

The modulus of rupture (about 10 days old) was

$k = 91$ pounds

or, say, $\left(\frac{k}{f}\right) = 10$ pounds.

Of course older specimens would prove much stronger.

There is only one valid objection that the writer has heard so far against these arches, and this is, that in case of any uneven settlement they might prove dangerous; as, of course, the margin in which the curve of pressure can safely shift in case of changed surroundings is very small. The writer does not think the objection very great, however, as settlements are apt to ruin any arch and should be carefully provided against in every case. The arches have some very great advantages. The principal one, of course, is their lightness of construction and saving of weight on the floors, walls and foundations. Then, too, in most cases iron beams can be entirely dispensed with, the arches resting directly on the brick walls; of course, there must be weight enough on the wall to resist the horizontal thrust, or else iron tie-rods must be resorted to. The former is calculated as already explained for retaining walls. If tie-rods are used, they are calculated as explained above for the example of the 7″ flat floor arch. An example of these 3″-tile arches may be of interest.

Example IX.

A segmental 3″ tile arch, built as explained above, has 20′ clear span with a rise at the centre of 20″. The floor is loaded at the rate of 150 pounds per square foot. Is the arch safe?

Example of tile arch. We divide the arch into five parts or voussoirs, each voussoir carrying 2′ of floor = 300 pounds; we make (Fig. 118) $ab = 300$ pounds, $bc = 300$ pounds, etc., and find

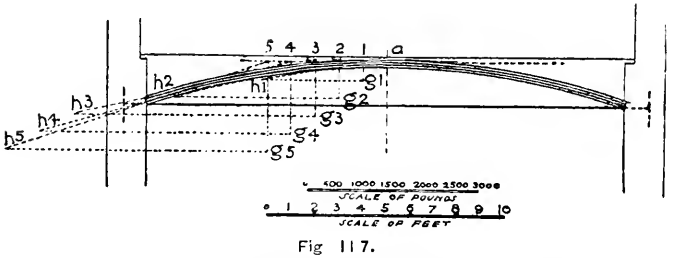


Fig 117.

the horizontal pressures (Fig. 117) $g_1 h_1$, $g_2 h_2$, etc. The last one $g_5 h_5$ is the largest and scales 4500 units or pounds. We now make (Fig.

118) $a\ o = g_5\ h_5 = 4500$ pounds and draw *ob*, *oc*, *od*, etc. We next construct the curve of pressure $a\ K$ and find that it coincides as closely as possible with the centre line of the arch. This means that

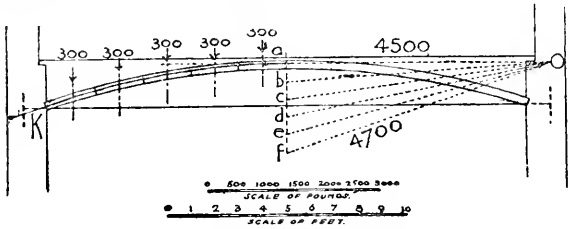


Fig. 118.

the pressure on each joint will be uniformly distributed. That on the lower joint is, of course, the largest and is $= o\ f$, which scales 4700 pounds. The area of the joint is, of course, $a = 3.\ 12 = 36$ square inches, therefore the greatest stress per square inch will be

$$\frac{4700}{36} = 130 \text{ pounds compression.}$$

As the tests gave us 300 pounds compression, per square inch, as safe stress of a sample only twelve days old, the arch is, of course, perfectly safe.

If, however, instead of a uniform load, we had to provide for a very heavy concentrated load, or heavy-moving loads, or vibrations, it would not be advisable to use these arches.

Danger of sliding. So far we have simply considered the danger of compression or tension at the joints of an arch; there is, however, another element of danger, though one that does not arise frequently, viz.: the danger of one voussoir sliding past the other. Where strong and quick-setting cements are used this danger is, of course, not very great. But in other cases, and particularly in large arches, it must be guarded against. The angle of friction of brick against brick (or stone against stone) being generally assumed at 30°, care must be taken that the angle formed by the curve of pressure at the joint, with a normal to the joint (at the point of intersection K) does not form an angle greater than 30°. If the angle be greater than 30° there is danger of sliding; if smaller, there is, of course, no danger. Thus, if in Fig. 114 we erect through K_4 a normal $K_4\ X$ to the joint, the angle $X\ K_4\ i_3$ should not exceed 30°.

Danger of shearing. In arches with heavy-concentrated loads at single points, there might, in rare cases, be danger of the

load shearing right through the arch. The resistance to shearing would, of course, be directly as the vertical area of cross-section of the arch, and in such cases this area must be made large enough to resist any tendency to shear.

Depth at crown. Arches are frequently built shallower at the crown and increasing gradually in depth towards the spring, the amount being regulated, of course, by the curve of pressure and Formulæ (44) and (45).

To establish the first experimental thickness at the crown of an arch, many engineers use the empyrical formula:

$$x = y.\sqrt{r} \qquad (67)$$

Where $x =$ the depth of arch, at crown, in inches.

Where $r =$ the radius at crown, in inches.

Where $y =$ a constant, as follows:

For cut stone, in blocks:	$y = 0{,}3$
For brickwork	$y = 0{,}4$
For rubblework	$y = 0{,}45$

When Portland cement is used, a somewhat lower value may be assumed for y. The depth thus established for crown is only experimental, of course, and must be varied by calculation of curve of pressure, etc.

Approximate rule. In cases where the architect does not feel the necessity for such a close calculation of the arch, it will be sufficient to find the curve of pressure. If this curve of pressure comes *within* the inner third of arch-ring, at every point, the arch is safe, provided the thrust on each joint, divided by the area of joint, does not exceed *one-half* of the safe compressive stress of the material, or:

$$\frac{p}{a} = \frac{1}{2}\cdot\left(\frac{c}{f}\right) \qquad (68)$$

Where $p =$ the thrust on joint, in pounds.

Where $a =$ the area of joint, in square inches.

Where $\left(\frac{c}{f}\right) =$ the safe resistance to compression, per square inch, of the material.

Vaulted and Groined Arches. Vaulted and groined arches are calculated on the same principles as ordinary arches and domes; though, in groined arches, as a rule, the ribs do all the work, the spandrels between the ribs being filled with stone slabs or other light material; in some European examples even flower-pots, plastered on the under side, have been used for this purpose.

CHAPTER VI.

FLOOR BEAMS AND GIRDERS.

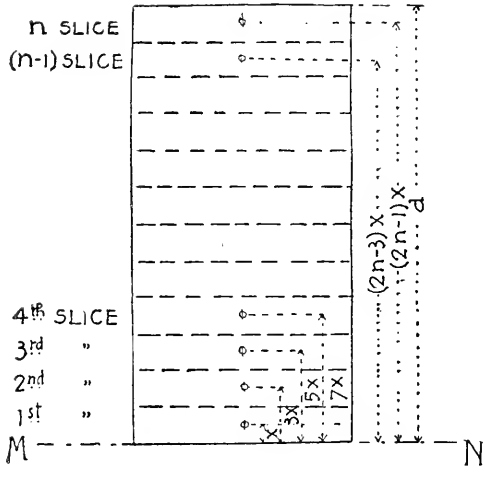


Fig. 119.

Moment of Inertia. THE writer has so often been asked for more information as to the meaning of the term *Moment of Inertia* that a few more words on this subject may not be out of place.

All matter, if once set in motion, will continue in motion unless stopped by gravity, resistance of the atmosphere, friction or some other force; similarly, matter, if once at rest, will so remain unless started into motion by some external force. Formerly it was believed, however, that all matter had a certain repugnance to being moved, which had to be first overcome, before a body could be moved. Probably in connection with some such theory the term arose.

In reality matter is perfectly indifferent whether it be in motion or in a state of rest, and this indifference is termed "Inertia." As used to-day, however, the term Moment of Inertia is simply a symbol or name for a certain part of the formula by which is calculated the force necessary to move a body around a certain axis with a given

velocity in a certain space of time; or, what amounts to the same thing, the resistance necessary to stop a body so moving.

In making the above calculation the "sum of the product of the weight of each particle of the body into the square of its distance from the axis" has to be taken into consideration, and is part of the formula; and, as this sum will, of course, vary as the size of the body varies, or as the location or direction of the axis varies, it would be difficult to express it so as to cover every case, and therefore it is called the "Moment of Inertia." Hence the general law or formula given covers every case, as it contains the Moment of Inertia, which varies, and has to be calculated for each case from the known size and weight of the body and the location and direction of the axis.

In plane figures, which, of course, have no thickness or weight, the *area* of each particle is taken in place of its weight; hence in all plane figures the Moment of Inertia is equal to the "sum of the products of the area of each particle of the figure multiplied by the square of its distance from the axis."

Calculation of Moment of Inertia.

Thus if we had a rectangular figure (119) b inches wide and d inches deep revolving around an axis M–N, we would divide it into many thin slices of equal height, say n slices each of a height $= 2.\ X.$

The distance of the centre of gravity of the first slice from the axis M–N will, of course be $= \frac{1}{2}.\ 2.\ X. = 1.\ X$

The distance of the centre of gravity of the second slice will be $= 3.\ X,$

that of the third slice will be $= 5.\ X,$

that of the fourth slice will be $= 7.\ X,$

that of the last slice but one will be $= (2\ n - 3).\ X.$

and that of the last slice will be $= (2\ n - 1).\ X$

The area of each slice will, of course, be $= 2.\ X.\ b$; therefore the Moment of Inertia of the whole section around the axis M–N will be (see p. 8),

$$i = 2.\ X.\ b.\ (1.\ X)^2 + 2.\ X.\ b.\ (3.\ X)^2 + 2.\ X.\ b.\ (5.\ X)^2 + 2.\ X.\ b.\ (7.\ X)^2 + \text{etc.} \ldots\ldots + 2.\ X.\ b.\ [(2n-3).X]^2 + 2.\ X.\ b.[(2n-1).X]^2 \text{ or,}$$

$$i = 2.X.^3 b.\ [1^2 + 3^2 + 5^2 + 7^2 + \text{etc.} \ldots\ldots + (2n-3)^2 + (2n-1)^2]$$

now the larger n is, that is the thinner we make our slices, the nearer will the above approximate:

$$i = 2.\ X^3.\ b.\left[\frac{4}{3}.\ n^3\right]$$

$$= 8.\ X^3.\ n^3.\frac{b}{3}$$

Therefore, as: 2. X. $n = d$ we have, by cubing,

8. $X^3.\ n^3 = d^3$; inserting this in above, we have:

$$i = \frac{d^3.\ b.}{3} \text{ or } \frac{b.\ d^3}{3}$$

The same value as given for i in Table I, section No. 20. Of course it would be very tedious to calculate the Moment of Inertia in every case; besides, unless the slices were assumed to be very thin, the result would be inaccurate; the writer has therefore given in Table I, the exact Moments of Inertia of every section likely to arise in practice.

Moment of Resistance. The Moment of Inertia applies to the whole section, the "Moment of Resistance," however, applies only to each individual fibre, and varies for each; it being equal to the Moment of Inertia of the whole section divided by the distance of the fibre from the axis.

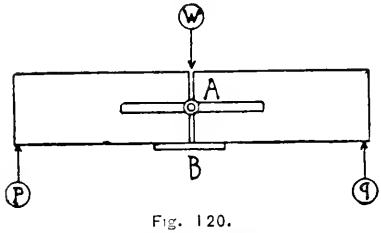


Fig. 120.

Now to show the connection of the Moments of Inertia and Resistance with transverse strains, let us consider the effect of a weight on a beam (supported at both ends).

If we consider the beam as cut in two and hinged at the point A (where the weight is applied), Fig. 120; further, if we consider a piece of rubber nailed to the bottom of each side of the beam, it is evident that the effect of the weight will be, as per Fig. 121.

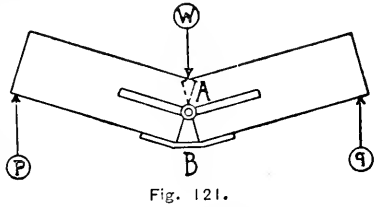


Fig. 121.

Effect of load on beam. Examining this closer we find that the corners of the beams above A (or their fibres) will crush each other, while those below A, are separated farther from each other, and the piece of rubber at B greatly stretched. It is evident, therefore, that the fibres nearest A experience the least change, and

that the amount of change of all the fibres is directly proportionate to their distance from A (as the length of all lines drawn parallel to the base of a triangle, are proportionate to their distance from the axis); further, that the fibres at A experience no change whatever. Now, if instead of considering the effect of a load on a hinged beam we

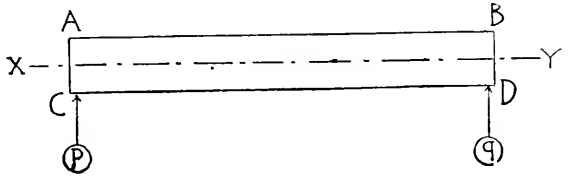


Fig. 122.

took an unbroken beam, the effect would be similar, but, instead of being concentrated at one point, it would be distributed along the entire beam; thus the beam A B D C (Fig. 122) which is not loaded, becomes when loaded, the slightly curved beam (A B D C) Fig. 123.

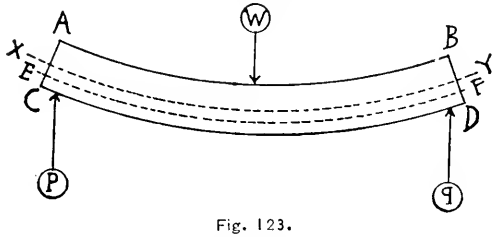


Fig. 123.

It is evident that the fibres along the upper edge are compressed or A B is shorter than before; on the other hand the fibres along C D are elongated, or in tension, and C D is longer than before; if we now take any other layer of fibres as E F, they—being below the neutral (and central)[1] axis X–Y—are evidently elongated; but not so much so, as C D: and a little thought will clearly show that their elongation is proportioned to the elongation of the fibres C D, directly as their respective distances from the neutral axis X–Y. It is further evident that the neutral axis X–Y is the same length as before, or its fibres are not strained; it is, therefore, at this point that the strain changes from one of tension to one of compression.

In Fig. 124 we have an isometrical view of a loaded beam.

[1] As a rule the neutral axis can be safely assumed to be central, but it is not necessarily so. In materials, such as cast-iron, stone, etc., where the resistance of the fibres to compression and tension varies greatly, the axis will be far from the centre, nearer the weaker fibres.

Rotation around neutral axis. Let us now consider an infinitesimally thin (cross) section of fibres A B C D in reference to their own neutral axis M–N. It is evident that if we were to double the load on the beam, so as to bend it still more, that the fibres along A B would be compressed towards or would *move* towards the centre of the beam; the fibres along D C on the contrary would be elongated or would *move* away from the centre of the beam.

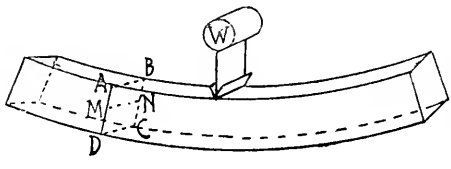


Fig. 124.

The fibres along M–N, being neither stretched nor compressed, would remain stationary.

The fibres between M–N and A B would all move towards the centre of the beam, the amount of motion being proportionate to their distance from M–N; the fibres between M–N and D C on the contrary would move away from the centre of the beam the amount of motion being proportionate to their distance from M–N; a little thought, therefore, shows clearly that the section A B C D turns or rotates on its neutral axis M–N, whenever additional weight is imposed on the beam.

This is why we consider in the calculations the moment of *Inertia* or the moment of resistance of a *cross-section* as *rotating* on its neutral axis.

Now let us take the additional weight off the beam and it will spring back to its former shape, and, of course, the fibres of the infinitesimally thin section A B C D will resume their normal shape; that is, those that were compressed will stretch themselves again, while those that were stretched will compress themselves back to their former shape and position, and those along the neutral axis will remain constant; or, in other words, this thin layer of fibres A B C D can be considered as a double wedge-shaped figure A B A_1 B_1 M N D C

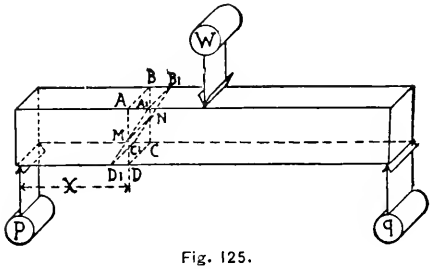


Fig. 125.

D_1 C_1 (Fig. 125) the base of the wedges becoming larger or smaller as the weight on the beam is varied.

Resistance of Wedge. Now to proceed to the calculation of the resistance of this wedge. It is evident that whatever may be the external strain on the beam at the section A B C D, the beam will owe whatever resistance it has at that point to the resistances of the fibres of the section or wedge to compression and tension.

Now considering the right-hand side of the beam as rigid, and the section A B C D as the point of fulcrum of the external forces, we have only one external force p, tending to turn the left-hand side of the beam upwards around the section A B C D, its total tendency, effect or moment m at A B C D, we know is $m = p.\ x$ (law of the lever).

Now to resist this we have the opposition of the fibres in the wedge A B A_1 B_1 M N to compression and the opposition to tension of the fibres in the wedge D C D_1 C_1 M N. For the sake of convenience, we will still consider these wedges, as wedges but so infinitesimally thin that we can safely put down the amount of their contents as equal to the area of their sides, so that — if A B $= b$ (the width of beam) and A D $= d$ (the depth of beam) — we can safely call *each* wedge as equal to $b.\ \frac{d}{2}$.

Now as the centre of gravity of a wedge is at $\frac{1}{3}$ of the height from its base, or $\frac{2}{3}$ of the height from its apex (and as the height of each wedge is $= \frac{d}{2}$) it would be $= \frac{2}{3} . \frac{d}{2} = \frac{d}{3}$ from axis M–N. The moment of a wedge at any axis M–N is equal to the contents of the wedge multiplied by the distance of its centre of gravity from the axis, the whole multiplied by the stress of the fibres, (that is their resistance to tension or compression). Now the contents of each wedge being $= b.\ \frac{d}{2}$, the distance of centre of gravity from M–N $= \frac{d}{3}$, and the stress being say $= s$, we have for the resistance of *each* wedge

$$= b.\ \frac{d}{2} . \frac{d}{3} . s$$

$$= \frac{b.\ d^2}{6} . s$$

Now if the stress on the fibres along the extreme upper or lower edges $= k$ (or the modulus of rupture), it is evident that the *average* stress on the fibres in either wedge will $= \frac{k}{2}$, or $s = \frac{k}{2}$ (for the

stress on each fibre being directly proportionate to the distance from the neutral axis the stress on the average will be equal to half that on the base). Now inserting $\frac{k}{2}$ for s in the above formula, and multiplying also by 2, (as there are two wedges resisting), we have the total resistance to rupture or bending of the section A B C D (A_1 B_1 C_1 D_1)

$$= \frac{b.\,d^2}{6} \cdot \frac{k}{2} \cdot 2$$

$$= \frac{b.\,d^2}{6} \cdot k$$

Now, by reference to Table I, section No. 2, we find that $\frac{b.d^2}{6}$ = Moment of Resistance for the section A B C D; therefore, we have proved the rule, that when the beam is at the point of rupture at any point of its length the bending moment at that point is equal to the moment of resistance of its cross-section at said point multiplied by the modulus of rupture.

Where girders or beams are of wood, it becomes of the highest importance that they should be sound and perfectly dry. The former that they may have sufficient strength, the latter that they may resist decay for the longest period possible.

Formation of Wood. Every architect, therefore, should study thoroughly the different kinds of timber in use in his locality, so as to be able to distinguish their different qualities. The strength of wood depends, as we know, on the resistance of its fibres to separation. It stands to reason that the young or newly formed parts of a tree will offer less resistance than the older or more thoroughly set parts. The formation of wood in trees is in circular layers, around the entire tree, just inside of the bark. As a rule one layer of wood is formed every year, and these layers are known, therefore, as the "annular rings," which can be distinctly seen when the trunk is sawed across. These rings are formed by the (returning) sap, which, in the spring, flows upwards between the bark and wood, supplies the leaves, and returning in the fall is arrested in its altered state, between the bark and last annular ring of wood. Here it hardens, forming the new annular ring. As subsequent rings form around it, their tendency in hardening is to shrink or compress and harden still more the inner rings, which hardening (by compression) is also assisted by the shrinkage of the bark. In a sound tree, therefore, the strongest wood is at the heart or centre of growth. The

heart, however, is rarely at the exact centre of the trunk, as the sap flows more freely on the side exposed to the effects of the sun and wind; and, of course, the rings on this side are thicker, thus leaving the heart constantly, relatively, nearer to the unexposed side.

Heart-Wood. From the above it will be readily seen that timber should be selected from the region of the heart, or it should be what is known as "heart-wood." The outer layers should be rejected, as they are not only softer and weaker, but, being full of sap, are liable to rapid decay. To tell whether or no the timber is "heart-wood" one need but look at the end, and see whether it contains the centre of the rings. No bark should be allowed on timber, for not only has it no strength itself, but the more recent annular rings near it, are about as valueless.

Medullary Rays. In some timbers, notably oak, distinct rays are noticed, crossing the annular rings and radiating from the centre. These are the "medullary rays," and are elements of weakness. Care should be taken that they do not cross the end of the timber horizontally, as shown at A in Fig. 126, but as near vertically as possible, see B in Fig. 127. The beautiful appearance of quartered oak and other woods is obtained by cutting the planks so that their surfaces will show slanting cuts through these medullary rays.

Seasoning cracks. All timber cracks more or less in seasoning, nor need these cracks cause much worry, unless they are very deep and long. They are, to a certain extent, signs of the amount of seasoning the timber has had. They should be avoided, as much as possible, near the centre of the timber, if regularly loaded, or near the point of greatest bending moment, where the

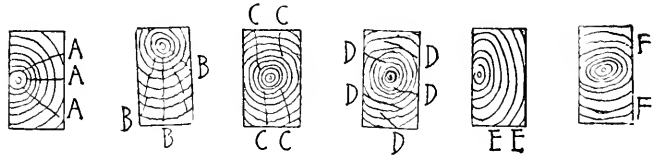


Fig. 126. Fig. 127. Fig. 128. Fig. 129. Fig. 130. Fig. 131.

loads are irregular. If timber without serious cracks cannot be obtained, allowance should be made for these, by increasing its size.

Vertical, or nearly vertical cracks (as C, Fig. 128) are not objectionable, and do not weaken the timber. But horizontal cracks (as D, Fig. 129), are decidedly so, and should not be allowed.

Knots. Knots in timber are another element of weakness. They are the hearts, where branches grow out of the trunk. If they are of nearly the same color as the wood, and their rings gradually die out into it, they need not be seriously feared. If, however, they are very dark or black, they are sure to shrink and fall out in time, leaving, of course, a hole and weakness at that place. Dead knots, — that is, loose knots, — in a piece of timber, mean, as a rule, that the heart is decaying. Knots should be avoided at the centre of a beam, regularly loaded, and at the point of greatest bending moment, where the loads are irregular. The farther the knots (and cracks) are from these points the better.

Wind-shakes. Timber with "wind-shakes" should be entirely avoided, as it has no strength. These are caused by the wind shaking tall trees, loosening the rings from each other, so that when the timber is sawed, the wood is full of small, almost separate pieces or splinters at these points.

A timber with wind-shakes should be condemned as unsound.

A timber with the rings at the end showing nearly vertical (E Fig. 130) will be much stronger than one showing them nearly horizontal. (F Fig. 131.)

Signs of sound Timber. To tell sound timber, Lord Bacon recommended to speak through it to a friend from end to end. If the voice is distinctly heard at the other end it is sound. If the voice comes abruptly or indistinctly it is knotty, imperfect at the heart, or decayed. More recent authorities recommend listening to the ticking of a watch at the other end, or the scratching of a pin on its surface. If, in sawing across a piece it makes a clean cut, it is neither too green nor decayed. The same if the section looks bright and smells sweet. If the section is soft or splinters up badly it is decayed. If it wets the saw it is full of sap and green. If a blow on timber rings out clearly it is sound; if it sounds soft, subdued, or dull, it is very green or else decayed. The color at freshly-sawed spots should be uniform throughout; timbers of darker cross-section are generally stronger than those of lighter color (of the same kind of wood.)

The annular rings should be perfectly regular. The closer they are, the stronger the wood. Their direction should be parallel to the axis throughout the length of the timber, or it will surely twist in time, and is, besides, much weaker. Where the rings at both ends are not in the same direction the timber has either twisted in

growing, or has a "wandering heart,"—that is, a crooked one. Such timber should be condemned. Besides looking at the rings at the end, a longitudinal cut near the heart will show whether it has grown regularly and straight, or whether it has twisted or wandered.

The weight of timber is important in judging its quality. If specimens of a wood are much heavier than the well-known weight of that wood, when seasoned, they may be condemned as green and full of sap. If they are much lighter than thoroughly seasoned specimens of the same wood, they are very probably decayed.

Methods of Seasoning. Tredgold claims that timber is "seasoned" when it has lost one-fifth of its original weight (when green); and "dry" when it has lost one-third. Some timbers, however, lose nearly one-half of their original weight in drying. Many methods are used to season or dry timber quickly.

The best method, however, is to stack the timber on dry ground (in as dry an atmosphere as possible) and in such a position that the air can circulate, as freely as possible around each piece. Sheds are built over the timber to protect it from the sun, rain, and also from severe winds as far as possible.

Timber dried slowly, in this manner, is the best. It will crack somewhat, but not so much so as hastily dried timber. Many processes are used to keep it from cracking, the most effective being to bore the timber from end to end, at the centre, where the loss of material does not weaken it much, while the hole greatly relieves the strain from shrinkage. Some authorities claim that two years' exposure is sufficient, though formerly timber was kept very much longer. But even two years is rarely granted with our modern conditions, and most of the seasoning is done after the timber is in the building. Hence its frequent decay. There are many artificial methods for drying timber, but they are expensive. The best known is to place it in a kiln and force a rapid current of heated air past it, this is known as "kiln-drying." It is very apt to badly "check" or crack the wood. To preserve timber, besides charring, the "creosoting" process is most effective. The timber is placed in an iron chamber, from which the air is exhausted; after which creosote is forced in under a high pressure, filling, of course, all the pores which have been forced open by the suction of the departing air. Creosoted wood, however, cannot be used in dwellings, as the least application of warmed air to it, causes a strong odor, and would render the building untenantable.

Manner of Shrinkage. In shrinking, the distance *between* rings remains constant, and it is for this reason that the finest floors are made from quartered stuff; for (besides their greater beauty), the rings being all on end, no horizontal shrinkage will take place; the width of boards remaining constant, and the shrinkage being only in their thickness; neither will timber shrink on end or in its length. Figures 132 and 133 show how timber will shrink. The first from a quartered log, the other from one with parallel cuts. The dotted part shows the shrinkage. The side-pieces G in Fig. 133 will curl, as shown, besides shrinking. By observing the directions of the annular rings, therefore, the future behavior of the timber can be readily predicted. Of course, the figures are greatly exaggerated to show the effect more clearly.

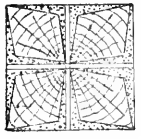


Fig. 132.

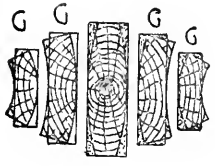


Fig. 133.

Decay of Timber. If the heart is not straight its entire length, the piece will twist lengthwise. Shrinkage is a serious danger, but the chief danger in the use of timber lies in its decay. All timber will decay in time, but if it is properly dried, before being built in, and all sap-wood discarded, and then so placed that no moisture can get to the timber, while fresh air has access to all parts of it, it will last for a very long time; some woods even for many centuries. In proportion as we neglect the above rules, will its life be short-lived. There are two kinds of decay, *wet* and *dry* rot. The wet rot is caused by alternating exposures to dampness and dryness; or by exposure to moisture and heat; the dry-rot, by confining the timber in an air-tight place. In wet rot there is "an excess of evaporation;" in dry rot there is an "imperfect evaporation." Beams with ends built solidly into walls are apt to rot; also beams surrounded solidly with fire-proof materials; beams in damp, close, and imperfectly ventilated cellars; sleepers bedded solidly in damp mortar or concrete, and covered with impervious papers or other materials; also timbers exposed only at intervals to water or dampness, or timbers in "solid" timbered floors.

Dry rot is like a contagious disease, and will gradually not only eat up the entire timber, but will attack all adjoining sound woodwork. Where rotted woodwork is removed, all adjoining woodwork,

masonry, etc., should be thoroughly scraped and washed with strong acids.

Ventilation necessary. Where wood has, of necessity, to be surrounded with fireproof materials, a system of pipes or other arrangements, should be made to force air to same through holes, either in the floors or ceilings, but in no case connecting two floors; the holes can then be made small enough not to allow the passage of fire. Where the air is forced in under pressure it would be advisable at times to force in disinfectants, such as steam containing evaporated carbolic acid, fumes of sulphur, etc.

Coating woodwork with paint or other preparations will only rot the wood, unless it has been first thoroughly dried and every particle of sap removed.

Cross-bridging. Timber must not be used too thin, or it will be apt to twist. For this reason floor-beams should not be used thinner than three inches. To avoid twisting and curling, cross-bridging is resorted to. That is, strips usually $2'' \times 3''$ are cut between the beams, from the bottom of one to the top of the next one, the ends being cut (in a mitre-box), so as to fit accurately against the sides of beams, and each end nailed with at least two strong nails. The strips are always placed in double courses, across the beams, the courses crossing each other like the letter x between each pair of beams.

This is known as "herring-bone" cross-bridging. Care should be taken that all the parallel pieces in each course are in the same line or plane. The lines of cross-bridging can be placed as frequently as desired, for the more there are, the stiffer will be the floor. About six feet between the lines is a good average. Sometimes solid blocks are used between the beams, in place of the herring-bone bridging. Cross-bridging is also of great help to a floor by relieving an individual beam from any great weight accidentally placed on it (such as one leg of a safe, or one end of a book-case), and distributing the weight to the adjoining beams. Unequal settlements of the individual beams are thus avoided. Where a floor shows signs of weakness, or lacks stiffness, or where it is desirable to force old beams, that

Stiffening weak floors. cannot be well removed, to do more work, two lines of slightly wedge-shaped blocks are driven tightly between the beams, in place of the cross-bridging. The beams are then bored, and an iron rod is run between the lines of wedges, from the outer beam at one end to the outer beam at the other, and, of course, at right angles to all. At one end the rod has a thread and

nut, and by screwing up the latter the beams are all forced upwards, "cambered," and the entire floor arched. It will be found much stronger and stiffer; but, of course, will need levelling for both floor and ceiling. Under the head and nut at ends of rod, there must be ample washers, or the sides of end beams will be crushed in, and the effect of the rod destroyed.

Girders, which cannot be stiffened sideways, should be, at least, half as thick as they are deep, to avoid lateral flexure.

Framing of beams. In using wooden beams and girders, much framing has to be resorted to. The used joints between timbers are numerous, but only a very few need special mention here. Beams should not rest on girders, if it can be avoided, on account of the additional dropping caused by the sum of the shrinkage of both, where one is over the other. If framing is too expensive, bolt a wide piece to the under side of the girder, sufficiently wider than the girder to allow the beams to rest on it, each side. If this is not practicable bolt pieces onto each side of the girder, at the bottom, and notch out the beams to rest against and over these pieces. The bearing of a beam should always be as near its bottom as possible. If a beam is notched so as to bear near its centre, it will split longitudinally. Where a notch of more than one-third the height of beam, from the bottom, is necessary, a wrought-iron strap or belt should be secured around the end of beam, to keep it from splitting lengthwise.

If framing can be used, the best method is the "tusk and tenon" joint, as shown in Figs. 134 and 135. In the one case the tenon goes through the girder and is secured by a wooden wedge on the other side; in the other it goes in only about a length equal to twice its depth, and is spiked from the top of girder. The latter is the most used. By both methods the girder is weakened but very little, the principal cut being near its neutral axis, while the beam gets bearing near its bottom, and its tenon is thoroughly strengthened to prevent its shearing off. The dimensions given in the figures are all in parts of the height of beams. Headers and trimmers at fire-places and other openings are frequently framed together, though it would be more advisable to use "stirrup-irons." The short tail-beams, however, can be safely tenoned into the header.

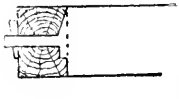


Fig. 134.

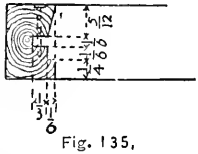


Fig. 135.

In calculating the strength of framed timber, the point where the mortise, etc., are cut, should be carefully calculated by itself, as the cutting frequently renders it dangerously weak, at this point, if not allowed for. For the same reason plumbers should not be allowed to cut timbers. As a rule, however, cuts near the wall are not dangerous, as the beam being of uniform size throughout, there is usually an excess of strength near the wall.

Stirrup-irons. Stirrup-irons are made of wrought-iron; they are secured to one timber in order to provide a resting-place for another timber, usually at right angles to and carried by the former. They should always lap over the farther side of the carrying timber, to prevent slipping, as shown in Fig. 136.

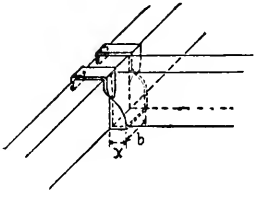


Fig. 136.

The iron should be sufficiently wide not to crush the beam, where resting on it; the section of iron must be sufficient not to shear off each side of beams. The twist must not be too sudden, or it will straighten out and let the carried timber down. To put the above in formulæ we should have: for the width of stirrup-iron (x)

Width of Stirrup-irons.
$$x = \frac{s}{b.\left(\frac{c}{f}\right)}. \qquad (69)$$

Where x = the width of stirrup-iron, in inches.

Where s = the shearing strain, in lbs., on end of beam, being carried.

Where b = the width of beam being carried, in inches.

Where $\left(\frac{c}{f}\right)$ = the safe resistance, in pounds, to compression, across the fibres, of the beam, being carried.

For the thickness of stirrup-iron we should have:

$$y = \frac{s}{2.x.\left(\frac{g}{f}\right)} \qquad (70)$$

Which for wrought-iron (Table IV.) becomes,

Thickness of Stirrup-iron.
$$y = \frac{s}{16000.x} \qquad (71)$$

Where y = the thickness of stirrup-iron, in inches.

Where s = the shearing strain on end of beam, in lbs.

Where x = is found by formula (69).

Providing, however, that y should never be *less* than one-quarter inch thick.

Example.

A girder carries the end of a beam, on which there is a uniform load of two thousand pounds. The beam is four inches thick, and of Georgia pine. What size must the stirrup-iron be?

Example stirrup-irons. The shearing strain at each end of the beam will, of course, be one thousand pounds, which will be the load on stirrup-irons. (See Table VII). From Table IV we find for Georgia pine, *across* the fibres, $\left(\frac{c}{f}\right) = 200$, we have, therefore, for the width of stirrup-iron from Formula (69)

$$x = \frac{1000}{4.200} = 1\tfrac{1}{4}''$$

Therefore the thickness of iron from Formula (71) should be

$$y = \frac{1000}{16000.1\frac{1}{4}} = \frac{1}{20}''$$

we must make the iron however at least $\frac{1}{4}''$ thick and therefore use a section of $1\frac{1}{4} \times \frac{1}{4}''$.

In calculating ordinary floor-beams the shearing strain can be overlooked, as a rule; for, in calculating transverse strength we allow only the safe stress on the fibres of the upper and lower edges, while the intermediate fibres are less and less strained, those at the neutral axis not at all. The reserve strength of these only partially used fibres will generally be found quite ample to take up the shearing strain.

Rectangular beams. The formulæ for transverse strength are quite complicated, but for rectangular sections (wooden beams) they can be very much simplified *provided we are calculating for strength only and not taking deflection into account.*

Remembering that the moment of resistance of a rectangular section is (Table I) $= \frac{b.d^2}{6}$ and inserting into Formula (18) the value for m according to the manner of loading and taken from (Table VII), we should have:

For uniform load on beam.

Transverse strength of rectangular beams.

$$u = \frac{b.d^2}{9.\mathrm{L}} \cdot \left(\frac{k}{f}\right) \qquad (72)$$

For centre load on beam.

$$w = \frac{b.d^2}{18.\mathrm{L}} \cdot \left(\frac{k}{f}\right) \qquad (73)$$

For load at any point of beam.

$$w_1 = \frac{b.d^2.L}{72.M.N} \cdot \left(\frac{k}{f}\right) \qquad (74)$$

For uniform load on cantilever.

$$u = \frac{b.d^2}{36.L} \cdot \left(\frac{k}{f}\right) \qquad (75)$$

For load concentrated at end of cantilever.

$$w = \frac{b.d^2.}{72.L} \cdot \left(\frac{k}{f}\right) \qquad (76)$$

For load at any point of cantilever.

$$w_1 = \frac{b.d^2}{72.Y} \cdot \left(\frac{k}{f}\right) \qquad (77)$$

Where u = safe uniform load, in pounds.

Where w = safe centre load on beam, in pounds; or safe load at end of cantilever, in pounds.

Where w_1 = safe concentrated load, in pounds, at any point.

Where Y = length, in feet, from wall to concentrated load (in cantilever).

Where M and N = the respective lengths, in feet, from concentrated load on beam to each support.

Where L = the length, in feet, of span of beam, or length of cantilever.

Where b = the breadth of beam, in inches.

Where d = the depth of beam, in inches.

Where $\left(\frac{k}{f}\right)$ = the safe modulus of rupture, per square inch, of the material of beam or cantilever (see Table IV).

The above formulæ are for rectangular wooden beams supported against lateral flexure (or yielding sideways). Where beams or girders are not supported sideways the thickness should be equal to at least half of the depth.

No allowance for deflection.

The above formulæ make no allowance for deflection, and except in cases, such as factories, etc., where strength only need be considered and not the danger of cracking plastering, or getting floors too uneven for machinery, are really of but little value. They are so easily understood that the simplest example will answer:

Example.

Take a 3″ × 10″ *hemlock timber and* 9 *feet long* (*clear span*), *loaded in different ways, what will it safely carry? taking no account of deflection.*

The safe modulus of rupture $\left(\frac{k}{f}\right)$ for hemlock from Table IV is

$$= 750 \text{ pounds.}$$

If both ends are supported and the load is uniformly distributed the beam will safely carry, (Formula 72):

$$u = \frac{3.10^2}{9.9} \cdot 750 = 2778 \text{ pounds.}$$

If both ends are supported and the load concentrated at the centre, the beam will safely carry, (Formula 73):

$$w = \frac{3.10^2}{18.9} \cdot 750 = 1389 \text{ pounds.}$$

If both ends are supported and the load is concentrated at a point I, distant four feet from one support (and five feet from the other) the beam will safely carry, (Formula 74):

$$w_1 = \frac{3.10^2.9}{72.4.5} \cdot 750 = 1406 \text{ pounds.}$$

If one end of the timber is built in and the other end free and the load uniformly distributed, the cantilever will safely carry, (Formula 75):

$$u = \frac{3.10^2}{36.9} \cdot 750 = 694 \text{ pounds.}$$

If one end is built in and the other end free, and the load concentrated at the free end, the cantilever will safely carry, (Formula 76):

$$w = \frac{3.10^2}{72.9} \cdot 750 = 347 \text{ pounds.}$$

If one end is built in and the other end free, and the load concentrated at a point I, which is 5 feet from the built-in end, the cantilever will safely carry, (Formula 77):

$$w_1 = \frac{3.10^2}{72.5} \cdot 750 = 625 \text{ pounds.}$$

Where, however, the span of the beam, in feet, greatly exceeds the depth in inches (see Table VIII), and regard must be had to deflection, the formulæ (28) and (29), also (37) to (42) should always be used, inserting for i its value from Table I, section No. 2, or: —

$$i = \frac{b.d^3}{12}$$

Where b = the thickness of timber, in inches.

Where d = the depth of timber, in inches.

Where i = the moment of inertia of the cross-section, in inches.

Table IX, however, gives a much easier method of calculating wooden beams, allowing for both rupture and deflection, and Formulæ (72) to (77) have only been given here, as they are often erro-

neously given in text-books, as the only calculations necessary for beams.

Basis of Tables XII and XIII. To still further simplify to the architect the labor of calculating wooden beams or girders, the writer has constructed Tables XII and XIII.

Table XII is calculated for floor beams of dwellings, offices, churches, etc., at 90 pounds per square foot, including weight of construction. The beams are supposed to be cross-bridged.

Table XIII is for isolated girders, or lintels, uniformly loaded, and supported sideways. When not supported sideways decrease the load, or else use timber at least half as thick as it is deep. In no case will beams or girders (with the loads given) deflect sufficiently to crack plastering. For convenience Table XII has been divided into two parts, the first part giving beams of from 5′ 0″ to 15′ 0″ span, the second part of from 15′ 0″ to 29′ 0″ span.

How to use Table XII. The use of the table is very simple and enables us to select the most economical beam in each case. For instance, we have say a span of 21′ 6″. We use the second part of Table XII. The vertical dotted line between 21′ 0″ and 22′ 0″ is, of course, our line for 21′ 6″. We pass our finger down this line till we strike the curve. To the left opposite the point at which we struck the curve, we read:

21.6 spruce, W. P. 56 — 4-14-14 or:

at 21′ 6′ span we can use spruce or white-pine floor beams, of 56 inches sectional area each, viz.: 4″ thick, 14″ deep and 14″ from centres. Of course we can use any other beam *below* this point, as they are all stronger and stiffer, but we must not use any other beam *above* this point. Now then, is a 4″ × 14″ beam of spruce or white pine, and 14″ from centres the most economical beam. We pass to the columns at the right of the curve and there read in the first column 48″. This means that while the sectional area of the beam is 56 square inches, it is equal to only 48 square inches *per square foot of floor*, as the beams are more than one foot from centres. In this column the areas are all reduced to the "area per square foot of floor," so that we can see at a glance if there is any cheaper beam *below* our point. We find below it, in fact, many cheaper beams, the smallest area (per square foot of floor) being, of course, the most economical. The smallest area we find is 36, 0 or 36 square inches of section per square foot of floor (this we find three times, in the sixteenth, twenty-ninth and thirty-first lines from the bottom). Passing to the left we find they represent, respectively, a Georgia pine

beam, 3″ thick, 16″ deep and 16″ from centres; or a Georgia pine beam 3″ thick, 14″ deep and 14″ from centres; or a white oak beam 3″ thick, 16″ deep and 16″ from centres. If therefore, we do not consider depth, or distance from centres, it would simply be a question, which is cheaper, 48 inches (or four feet "board-measure") of white pine or spruce, or 36 inches (or three feet "board-measure") of either white oak or Georgia pine. The four other columns on the right-hand side, are for the same purpose, only the figures for each kind of wood are in a column by themselves; so that, if we are limited to any kind of wood we can examine the figures for that wood by themselves. Take our last case and suppose we are limited to the use of hemlock; now from the point where our vertical line (21′ 6″) first struck the curve, we pass to the right-hand side of Table, to the second column, which is headed "Hemlock." From this point we seek the smallest figure *below* this level, but in the same column; we find, that the first figure we strike, viz: 41, 2 is the smallest, so we use this; passing along its level to the left we find it represents a hemlock beam of 48 square inches cross-section, or 3″ thick, 16″ deep and 14″ from centres.

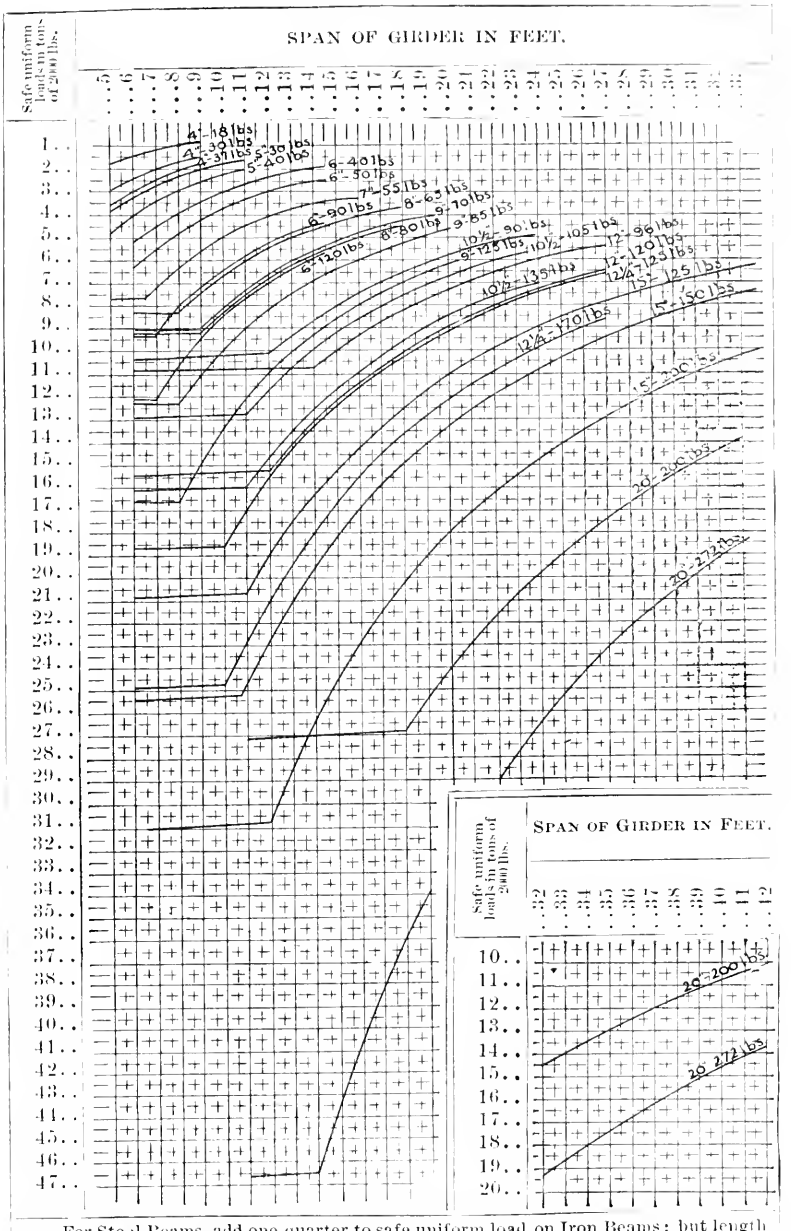
In case the size of the beam is known, its safe span can, of course, be found by reversing the above procedure, or if the depth of beam and span is settled, we can find the necessary thickness and distance between centres; in this way the Table, of course, covers every problem.

Table XIII is calculated for wooden girders of all sizes. Any thickness not given in the table can be obtained by taking the line for a girder of same depth, but one inch thick and multiplying by the thickness. For very short spans, look out for danger of horizontal shearing (see formula 13); where this danger exists, pass vertical bolts through ends of girder, or bolt thin iron plates, or straps, or even boards with vertical grain, to each side of girder, at ends.

How to use Table XIII. The use of this table is very simple. The vertical columns to the left give the safe uniform loads on girders (sufficiently stiff not to crack plastering) for different woods: these apply to the dotted parts of curves. The columns on the right-hand side give the same, but apply to the parts of curves drawn in full lines.

If we have a 6″ × 16″ Georgia pine beam of 20 feet span and want to know what it will carry, we select the curve marked at its upper end $6 \times 16 = 96$; we follow this curve till it intersects the vertical line 20′ 0″; as this is in the part of curve drawn full, we

IRON I-BEAM GIRDERS, BRACED SIDEWAYS.



For Steel Beams, add one-quarter to safe uniform load on Iron Beams; but length of span in feet must not exceed twice the depth of beam in inches, or deflection will be too great for plastering.

pass horizontally to the right and find under the column marked "Georgia Pine," 7980, which is the safe, uniform load in pounds. Supposing, however, we had simply settled the span, say 8 feet, and load, say 7000 pounds, and wished to select the most economical girder, being, we will say, limited to the use of white pine: the span not being great we will expect to strike the dotted part of curve, and therefore select the fourth (white pine) column *to the left*. We pass down to the nearest figure to 7000 and then pass horizontally to the right till we meet the vertical 8 feet line; this we find is, as we expected, at the dotted part, and therefore our selection of the left column was right. We follow the curve to its upper end and find it requires a girder $4'' \times 12'' = 48$ square inches. Now, can we use a cheaper girder? of course, all the lines *under* and *to the right* of our curve are stronger, so that if either has a smaller sectional area, we will use it. The next curve we find is a $6'' \times 10'' = 60''$; then comes a $4'' \times 14'' = 56''$; then an $8'' \times 10'' = 80''$; then a $6'' \times 12'' = 72''$ and so on; as none has a smaller area we will stick to our $4'' \times 12''$ girder, provided it is braced or supported sideways. If not, to avoid twisting or lateral flexure, we must select the next cheapest section, where the thickness is at least equal to half the depth;[1] the cheapest section beyond our curve that corresponds to this, we find is the $6'' \times 10''$ girder, which we should use if not braced sideways.

In the smaller sections of girders where the difference between the loads given from line to line is proportionally great, a safe load should be assumed between the two, according to the proximity to either line at which the curve cuts the vertical. The point where the curve cuts the *bottom* horizontal line of each part is the length of span for which the safe load opposite the line is calculated.

Heavier Floors. Where a different load than 90 pounds per square foot, must be provided for, we can either increase the thickness of beams as found in Table XII, or decrease their distance from centres, either in proportion to the additional amount of load. Or, if we wish to be more economical, we can calculate the safe uniform load on *each* floor beam, and consider it as a separate girder, supported sideways, using of course, Table XIII.

Basis of Tables XIV and XV. The Tables XIV and XV are very similar to the foregoing, but calculated for wrought-iron I-beams. Table XIV gives the size of beams and distance from centres required to carry different loads per square foot of floor, 150 pounds

[1] The rule for calculating the exact thickness will be found later, Formula (78).

per square foot of floor (including the weight of construction), however, being the usual load allowed for in churches, office-buildings, public halls, etc., where the space between beams is filled with arched brickwork, or straight hollow-brick arches, and then covered over with concrete. A careful estimate, however, should be made of the exact weight of construction per square foot, including the iron-work, and to this should be added 70 pounds per square foot, which is the greatest load likely ever to be produced if packed solidly with people. Furniture rarely weighs as much, though heavy safes should be provided for separately. The load on roofs should be 30 pounds additional to the weight of construction, to provide for the weight of snow or wind. Look out for tanks, etc., on roofs. Plastered ceilings hanging from roofs add about 10 pounds per square foot, and slate about the same. Where a different load than given in the Table must be provided for, the distance between centres of beams can be reduced, proportionally from the next greater load; or the weight on each beam can be figured and the beam treated as a girder, supported sideways, in that case using Table XV. Both tables are calculated for the beams not to deflect sufficiently to crack plastering.

How to use Table XIV.

The use of Table XIV is very simple. Supposing we have a span of 24 feet and a load of 150 pounds per square foot. We pass down the vertical line 24′ 0″ and strike first the 12″–96 pounds beam, which (for 150 pounds) is opposite (and three-quarter way between) 3′ 0″ and 3′ 4″ therefore 3′ 3″ from centres. The next beam is the 12″–120 pounds beam 4′ 0″ from centres; then the 12″–125 pounds beam 4′ 1″ from centres; then the 15″–125 pounds beam 5′ 0″ from centres and so on. It is simply a question, therefore, which "distance from centres" is most desirable and as a rule in fireproof buildings it is desirable to keep these as near alike as possible, so as not to have too many different spans of beam arches and centres. If economy is the only question, we divide the weight of beam by its distance from centres, and the curve giving the smallest result is, of course, the cheapest. Supposing, however, that we desire all distances from centres alike, say 5 feet. In that case we pass down the 150-pound column to and then along the horizontal line 5′ 0″ till we strike the vertical (span) line, in this case 24′ 0″, and then take the cheapest beam to the right of the point of intersection. Thus, in our case the nearest beam would be 15″–125 pounds; next comes $12\frac{1}{4}$″–170 pounds; then 15″–150 pounds, etc. As the nearest beam is the lightest in this case, we should select it. The weight of a beam

is always given per yard of length. The reason for this is that a square inch of wrought-iron, one yard long, weighs exactly 10 pounds. Therefore if we know the weight per yard in pounds we divide it by ten to obtain the exact area of cross-section in square inches; or if we know the area, we multiply by ten and obtain the exact weight per yard.

How to use Table XV. The use of Table XV, is very similar to that of Table XIII, but that the safe uniform load is given (in the first column) in tons of 2000 pounds each. The continuation of the two 20″ beams up to 42 feet span is given in the separate table, in the lower right-hand corner. To illustrate the Table: if we have a span of say 21 feet we pass down its vertical line; the first curve we strike is the 10½″–90 pounds beam, which is three-quarter space beyond the horizontal line 5 (tons); therefore a 10½″–90 pounds beam at 21 feet span will carry safely 5¾ tons uniform load, and will not deflect sufficiently to crack plaster. (Each full horizontal space represents one ton). The next beam at 21 feet span is 10½″–105 pounds, which will safely carry 6½ tons. Then comes the 12″–96 pounds beam, which will safely carry 7 tons, and so on down to the 20″–272 pounds beam, which will safely carry 33¾ tons.

If we know the span (say 17 feet) and uniform load (say 7½ tons) to be carried, we pass down the span line 17′ 0″ and then horizontally along the load line 7½ till they meet, which in our case is at the 9″–125 pounds beam; we can use this beam or any cheaper beam, whose curve is under it. We pass over the different curves *under* it, and find the cheapest to be the 12″–96 pounds beam, which we, of course, use.

Iron beams must be scraped clean of rust and be well painted. They should not be exposed to dampness, nor to salt air, or they will deteriorate and lose strength rapidly.

Steel beams. Steel beams are coming into use quite largely. They are cheaper to manufacture than iron beams, as they are made directly from the pig and practically in one process; while with iron beams the ore is first converted into cast iron, then puddled into the muck-bar, re-heated, and then rolled. Steel beams, however, are not apt to be of uniform quality. Some may be even very brittle; they are, however, very much stronger than iron (fully 25 per cent stronger), but as their deflection is only about 7, 3 per cent less than that of iron beams, there is but very little economy of material possible in their use. If steel beams are used they can be spaced one quarter distance (be-

tween centres) farther apart than given in Table XIV for iron beams; or they will safely carry one quarter more load than given in Table XV; but in no case, where full load is allowed, must the span in feet, (of steel beams), exceed twice the depth in inches. With full safe loads the deflection of steel beams will always be greater than that of iron beams (about $\frac{1}{3}$ larger). Where, therefore, it is desirable not to have a greater deflection than with iron beams, add only $7\frac{1}{2}$ per cent to the distances between centres or "safe loads" as given in Tables for iron beams, instead of 25 per cent.

Steel beams will undoubtedly supersede iron beams before many years have passed; but in the present state of their manufacture their use is hardly to be recommended. Their strength and consistency is very variable. It has been found in some cases that steel beams broke suddenly when jarred, (that is, were very brittle,) though test pieces off the ends of these same beams gave very satisfactory results. If steel is used, not only should samples of each piece be carefully tested, for tenacity, ductility, elasticity, elongation, etc., but the whole beam itself should be tested by actual loading. It will be readily seen that the expense of such tests would bar the use of steel, but no architect can afford to take any chances in such an important part of his building.

Many writers even claim, that, "within the elastic limit," the additional stiffness of steel over iron does not appear; and that it is only beyond this limit that steel is somewhat stiffer than iron.

Lateral Flexure in beams. In using iron and steel beams it is very important that they be supported sideways, so as not to yield to lateral flexure. Where the beams are isolated and unsupported sideways, the safe load must be diminished. Just how much to diminish this load is the question. The practice amongst iron workers is to consider the top flange as a column of the full length of the span, obliged to yield sideways, and with a load equal to the greatest strain on the flange. Modifying, therefore, Formula (3) to meet this view, we should have:

Beams not braced sideways.

$$w_t = \frac{w}{1 + \frac{y \cdot L^2}{b^2}} \qquad (78)$$

Where w_t = the safe load, in pounds, on a beam, girder, lintel or straight arch, etc., unsupported sideways.

Where w = the safe load, in pounds, on a beam, lintel or straight arch supported sideways.

Where L = the length of clear span, in feet, that beam, etc., is unsupported sideways.

Where b = the least breadth in inches of top flange, or least thickness of beam, lintel or arch.

Where y = a constant, as found in Table XVI.

(In place of w we can use r = the moment of resistance of beam supported sideways, and in place of w_1 we use r_1 = the moment of resistance of beam *not* supported sideways.)

The above practice, however, would seem to diminish the weight unnecessarily, particularly where the beam, girder, etc., is of uniform section throughout; for while the beam in that case, would, be equally strong at all points, it would be strained to the maximum compression only at the point of greatest bending-moment, the strain diminishing towards each support, where the compression would

TABLE XVI.

VALUE OF Y IN FORMULA (78).

Material of beam, girder, lintel, straight arch, etc.	Value of y for girders, beams, etc., of *variable* cross-sections.	Value of y for beams, girders, lintels, straight arches, etc., of *uniform* cross-section throughout.
Cast-Iron	0,5184	0,2304
Wrought-Iron	0,0432	0,0192
Steel	0,0346	0,0154
Wood	0,5702	0,2534
Stone	3,4560	1,5360
Brick	5,7024	2,5344

cease entirely. To consider, therefore, the whole as a long column carrying a weight equal to this maximum compressive strain, seems unreasonable. Box has shown, however, that the maximum tendency to deflect laterally is when we consider the top flange (or top half in rectangular beams, lintels and straight arches) as a column equal to two thirds of the span (unsupported sideways) loaded with a weight equal to one-third of the greatest compressive strain at any point. This greatest compressive strain is always at the point of greatest bending moment (usually the centre of span), and is equal to the area of top flange, multiplied by $\left(\frac{c}{f}\right)$[1]. In case of plate girders the angle-irons and part of web between angle-irons should be included in the area. Box's theory is given in Formula (5); if then we take

[1] This is not quite correct. The greatest compressive strain is really a little less, as will be explained in Vol. II.

one-third of this "maximum tendency to deflect" as safe, we should have the same Formula as (78) but with a smaller value for y. The writer would recommend using the larger value for y, where, as in plate girders, trusses, etc., the section of top flange or chord is diminished, varying according to the compressive strain at each point; and using the smaller value for y, where the section of beam, girder or top chord is uniform throughout.

Use of Table XVI.

Thus the $10\frac{1}{2}'' - 90$ pounds beam at 20 feet span will safely carry (if supported sideways) a uniform load of 5,9 tons or 11800 pounds (see Table XV.) The width of flange being $4\frac{1}{2}''$, and this width and its thickness, of course, being uniform throughout the entire length of beam, we use the smaller value for y (second column) and have for the actual safe uniform load, if the beam is not secured against lateral flexure:

$$w_1 = \frac{11800}{1 + 0{,}0192.\ \frac{20^2}{4\frac{1}{2}^2}} = \frac{11800}{1 + 0{,}379}$$

$$= \frac{11800}{1{,}379} = 8557 \text{ pounds, or } 4{,}28 \text{ tons.}$$

Had we used the larger value for $y = 0{,}0432$ we should have had

$$w_1 = \frac{11800}{1 + 0{,}854} = \frac{11800}{1{,}854} = 6365 \text{ pounds, or } 3{,}18 \text{ tons,}$$

which closely resembles the value (3,29) given in the Iron Companies' hand-books, but is an excessive reduction under the circumstances.

Doubled Beams.

Where two or more beams are used to carry the same load, as girders for instance, or as lintels in a wall, they should be firmly bolted together, with cast-iron separators between. In this case use for b in Formula (78) the total width, from outside to outside of all flanges, and including in b the spaces between. The separators are made to fit exactly between the inner sides of webs and top and bottom flanges. The separator is swelled out for the bolt to pass through. Sometimes there are two bolts to each separator, but it is better (weakening the beam less) to have but one at the centre of web. The size of separators and bolts vary, of course, to suit the different sizes of beams. They should be placed apart about as frequently as twenty times the width of flange of a single beam. Where beams are placed in a floor, the floor arches usually provide the side bracing. But in order to avoid unequal deflections, and possible cracks in the arches, (from unequal or moving loads or from vibrations) and also to take up the thrust on the end beams of each floor, it is necessary to place

Tie-rods.

lines of tie-rods across the entire line of beams. The size of these rods can be calculated as already explained in the Chapter on Arches (p. 166); they are usually made, however, from $\frac{5}{8}''$ to $\frac{7}{8}''$ diameter. Each rod extends from the outside web of one beam to the outside web of the next beam. The next rod is a little to one side of it, so that the rods do not really form one straight line, but every other rod falls in the same line. Care must be taken not to get the rods too long, or there will have to be several washers under the head and nut, making a very unsightly job, to say the least. Contractors will do this, however, for the sake of the convenience of ordering the rods all of one or two lengths. Where, therefore, the beams are not spaced evenly the contractor should be warned against this. One end of the rod has a "head" welded on, the other has a "screw-end," which need not be "up-set;" the nut is screwed along this end, thus forcing both nut and head to bear against the beams solidly. The distance between lines of tie-rods, would depend somewhat on our calculation, if made; the usual practice, however, is to place them apart a distance equal to about twenty times the width of flange of a single beam.

Flitch-plate Girder. Sometimes where wooden girders have heavier loads to carry than they are capable of doing, and yet iron girders cannot be afforded, a sheet of plate-iron is bolted between two wooden girders. In this case care must be taken to so proportion the iron, that in taking its share of the load, it will *deflect* equally with the wooden girders, otherwise the bolts would surely shear off, or else crush and tear the wood.

We consider the two wooden girders as one girder and calculate (or read from Table XIII) their safe load, taking care not to exceed 0,03 inches of deflection per foot of span. We then, from Table VII or Formulæ (37) to (41) obtain the exact amount of their deflection under this load. We now calculate the iron plate, for deflection only, inserting the above amount of deflection, and for the load the balance to be borne by the iron-work. An example will best illustrate this:

Example.

A Flitch-plate girder of 20-foot span consists of two Georgia pine beams each 6″ × 16″ with a sheet of plate-iron 16″ deep bolted between them. The girder carries a load of 13000 pounds at its centre; of what thickness should the plate be? The girder supports a plastered ceiling.

Strength of wooden part. From Table XIII we find that a Georgia pine beam 6″ × 16″ of 20-foot span will safely carry without cracking plaster 7980 pounds uniform load, or 3990 pounds at its centre (See Case (6) Table VII,) so that the two wooden beams together carry 7980 pounds of the load, leaving a balance of 5020 pounds for the iron plate to carry. The deflection of a 20-foot span Georgia pine beam 6″ × 16″ with 3990 pounds centre load will be, Formula (40)

$$\delta = \frac{1}{48} \cdot \frac{3990.\ 240^3}{e.\quad i.}$$

e for Georgia pine (Table IV) is $= 1200000$ and

$$i = \frac{b.d^3}{12} \text{ (Table I. section No. 2), or}$$

$$i = \frac{6.16^3}{12} = 2048, \text{ therefore}$$

$$\delta = \frac{1}{48} \cdot \frac{3990.\quad 240^3}{1200000.\ 2048} = 0{,}47''$$

Size of Iron Plate. We now have a wrought-iron plate which must carry 5020 pounds centre load, of a span of 20 feet, 16″ deep, and must deflect under this load only 0,47″.

Inserting these values in Formula (40) we have:

$$0{,}47 = \frac{1}{48} \cdot \frac{5020.240^3}{e.\quad i.}$$

From Table IV we have for wrought-iron

$$e = 27000000$$

While for i, we have (Table I. Section No. 2)

$$i = \frac{b.\,d^3}{12} = \frac{b.\,16^3}{12} = 341.b$$

Inserting these values and transposing we have:

$$b = \frac{5020.\quad 240^3}{48.\ 27000000.\ 341.\ 0{,}47} = 0{,}33$$

Or the plate would have to be $\frac{1}{3}''$ × 16″. Now to make sure that this deflection does not cause too great fibre strains in the iron, we can calculate these from Formulæ (18) and (22). The bending moment at the centre will be (22)

$$m = \frac{5020.\ 240}{4} = 301200$$

The moment of resistance will be (Table I. Section No. 2)

$$r = \frac{b.\,d^2}{6} = \frac{0{,}33.16^2}{6} = 14$$

And from (18) $\frac{m}{\left(\frac{k}{f}\right)} = r$, or transposing and inserting values,

$$\left(\frac{k}{f}\right) = \frac{301200}{14} = 21514 \text{ pounds.}$$

As the safe modulus of rupture of wrought-iron is only 12000 pounds (Table IV) we must increase the thickness of our plate. Let us call the plate $\frac{5}{8}'' \times 16''$, we should then have

$$r = \frac{5.\ 16^2}{8.\ 6} = 26{,}67 \text{ and}$$

$$\left(\frac{k}{f}\right) = \frac{301200}{26{,}67} = 11256.$$

So that the plate would be a trifle too strong. This would mean that both plate and beams would deflect less. The exact amount might be obtained by experimenting, allowing the beams to carry a little less and the plate a little more, until their deflections were the same, but such a calculation would have no practical value. We know that the deflection will be less than 0, 47″ and further, that plastering would not crack, unless the deflection exceeded $\frac{3}{8}$ of an inch (Formula 28) as

$$20.0{,}03 = 0,\ 6''$$

Size of Bolts. In regard to the bolts, the best position for them would, of course, be along the neutral axis, that is, at half the height of the beam. For here there would be no strain on them. But to place them with sufficient frequency along this line would tend to weaken it too much, encouraging the destruction of the beam from

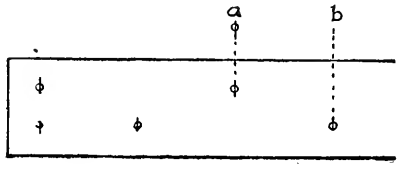


Fig. 137.

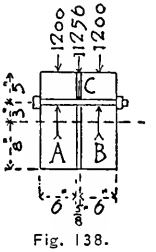


Fig. 138.

longitudinal shearing along this line. For this reason the bolts are placed, alternating, above and below the line, forming two lines of bolts, as shown in Fig. (137). The end bolts are doubled as shown; the horizontal distance, $a-b$, between two bolts should be about equal to the depth of the beam. If we place the bolts in our exam-

ple, say 3″ above and 3″ below the neutral axis, we can readily calculate the size required. Take a cross-section of the beam (Fig. 138) showing one of the upper bolts. Now the fibre strains along the upper edge of the girder, we know are $\left(\frac{k}{f}\right)$ or 1200 pounds per square inch, for the wood, and we just found the balance of the load coming on the iron would strain this on the extreme upper edge $=11256$ pounds per square inch. As the centre line of the bolt is only 3″ from the neutral axis or $\frac{3}{8}$ of the distance from neutral axis to the extreme upper fibres, the strains on the fibres along this line will be, of course, on the wood $\frac{3}{8}$ of 1200, or 450 pounds per square inch: and on the iron $\frac{3}{8}$ of $11256 = 4221$ pounds per square inch. Now, supposing the bolt to be 1″ in diameter. It then presses on each side against a surface of wood $= 1'' \times 6''$ or $=$ six square inches. The fibre strain being 450 pounds per square inch, the total pressure on the bolt from the wood, each side, is:

$$6.450 = 2700 \text{ pounds.}$$

On the iron we have a surface of $1'' \times \frac{5}{8}'' = \frac{5}{8}$ square inches. And as the fibre strain at the bolt is 4221, the total strain on the bolt from the iron is $= \frac{5}{8}$. $4221 = 2638$ pounds. Or, our bolt virtually becomes a beam of wrought-iron, circular and of 1″ diameter in cross-section, supported at the points A and B, which are $6\frac{5}{8}''$ apart, and loaded on its centre C with a weight of 2638 pounds.

Therefore we have, at centre, bending-moment (Formula 22)

$$m = \frac{2638.\ 6\frac{5}{8}}{4} = 4369.$$

From Table I, Section No. 7, we know that for a circular section, the moment of resistance is,

$$r = \frac{11}{14} \cdot v^3 = \frac{11}{14} \cdot \left(\frac{1}{2}\right)^3 = 0{,}098$$

Now for solid circular bolts, and which are acted on really along their whole length it is customary to take $\left(\frac{k}{f}\right)$ the safe modulus of rupture rather higher than for beams. Where the bolts or pins have heads and nuts at their ends firmly holding together the parts acting across them they are taken at 18000 pounds for steel and at 15000 pounds for iron. We have therefore transposing (Formula 18) for the required moment of resistance

$$r = \frac{4369}{15000} = 0{,}291.$$ Inserting this value for r in the

above we have for the radius of bolt,

$$\frac{11}{14} \cdot r^3 = 0{,}291 \text{ and}$$

$$r = \sqrt[3]{\frac{11}{14} \cdot 0{,}291} = \sqrt[3]{0{,}3701}$$

$$= 0{,}718''$$

Or the diameter of bolt should be 1,436″ or say 1 7-16″. But 1″ will be quite ample, as we must remember that the strains calculated will come only on the one bolt at the centre of span of beam; and that, as the beam remains of same cross section its whole length the extreme fibre strains decrease rapidly towards the supports, and therefore also the strains on the bolts. The end bolts are doubled however, to resist the starting there of a tendency to longitudinal shearing. We might further calculate the danger of the bolt crushing the iron plate at its bearing against it; or crushing the wood each side; or the danger of the iron bolt being sheared off by the iron plate between the wooden beams; or the danger of the iron bolt shearing off the wood in front of it, that is tearing its way out through the wood; but the strains are so small, that we can readily see that none of these dangers exist.

Keyed Girders. Another method of adding to the sum of the separate strengths of the girders is to place one under the other making a straight joint between the two parts and to drive in hard wood keys, as shown in Fig. 140.

The keys can either be made a tritle thicker than the holes and the beams then firmly bolted together so as to take hold of keys securely; or, keys can be shaped in two wedged-shaped pieces to each key, and driven into the hole from opposite sides, after the beams are firmly bolted. In the latter case, care must be taken that the joint between the opposite wedges is slanting or diagonal, and *not horizontal* or else, of course, the keys would be useless. Either method allows, for tightening up, after shrinkage has taken place. Iron bands are frequently used in place of bolts but they are more clumsy, less liable to *all* fit exactly, and besides do not allow for tightening up so easily as with bolts. Where beams are very wide, however, the

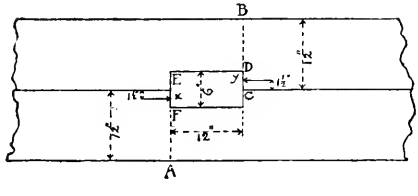


Fig. 139.

bands are very advantageous. Tredgold says the keys should be twice as wide, as high; and that the sum of all their heights should equal one and a third times the depth of girder. They can be easily calculated, however. As the main strain on them is a horizontal shearing strain, and the stress or resistance to shearing is greatest *across* the grain the keys should of course, be placed with their grain running as nearly as possible vertically. Of course, as the greatest horizontal shearing exists near the supports, the end wedges should be the strongest; it is customary, however, to make them all of the same size for convenience of execution. The amount of the horizontal shearing is found by Formula (13).

Horizontal shearing on Keys.

Besides the horizontal shearing strain there will also be a crushing strain on the sides of wedges, which will be greatest, where the greatest fibre strains exist. This of course, is at the point of the greatest bending moment on the beam. Let us consider the wedge at A – B Fig. (140) which has been drawn enlarged in Fig. (139.)

Compression on Keys.

The lower half of the girder, being in tension, in trying to stretch its fibres meets with the resistance of the wedge along E F, therefore tends to crush or compress this surface. The amount of this compression, per square inch, will be equal to the average fibre strain between E and F. Now the fibre strain at A can be readily found, by finding the "bending moment" at A and dividing this by the moment of resistance of the girder (see Table I) and Formula (18). This

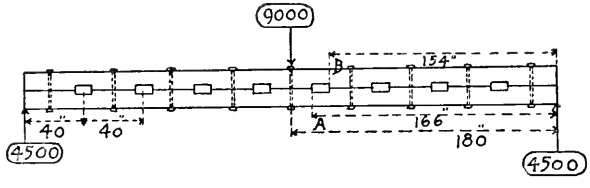


Fig. 140.

gives the fibre strains at A. The average fibre strain on E F will be to the strain at A as the distance of x from the neutral axis, is to the depth of half the beam; x being the centre of E F, or:

Extreme fibre strain at A : average fibre strain on wedge = E A : E x. The amount of the compression on E F will of course equal the area of wedge at E F (that is E F multiplied by the breadth of

girder), and this area multiplied by the average fibre strain on E F. The greatest compression on E F will of course be at F, and equal to twice the average fibre strain, as E F = 2. E x.

In the same way, we find that the upper half of the girder, being in compression, is forcing its fibres towards the centre causing compression on the surface D C. The amount of this compression is found similarly as for that on E F, the only difference being in the difference in bending moments at B and A. The key therefore becomes virtually a cantilever, the built-in part being between E F and C, and the load applied on the free end C D, the load being a uniform one and equal to the amount of the compression on C D.

Weakest Point of Girder. The weakest point of the girder itself will be either at the point of greatest bending moment, or at key nearest to it, where, of course the girder will not be of full section, being weakened in the part cut away for key. An example will more fully illustrate all of the foregoing.

Example.

A spruce girder (Fig. 140) of 30-foot clear span is built up of two girders 10″ × 12″ each, making the whole section 10″ × 24″. Georgia pine keys are used, each 6″ × 12″ (and, of course) 10″ across girder; they are placed with grain vertically 3′ 4″ between centres and same distance from centre of last key to support. The girder helps support a plastered ceiling. What is the safe centre load on girder?

Calculation of Keyed girder. The girder is (d =) 24″ deep and (L =) thirty feet long; now $1\frac{1}{6}$. L would be 35, therefore d is less than $1\frac{1}{6}$ L, and from rule contained in Table VIII for spruce we must calculate for deflection, not rupture, in order to be safe. Formula (40) gives the rule for deflection of a centre load on a girder or beam. It is:

$$\delta = \frac{1}{48} \cdot \frac{w. l^3}{e. i.} \text{ or transposing,}$$

$$w = \frac{\delta. 48. e. i}{l^3} \text{ where } w \text{ would be the safe centre load, in}$$

pounds. Now in order not to crack plastering, we have from Formula (28)

$$\delta = L. 0{,}03 \text{ or}$$
$$\delta = 30. 0{,}03$$
$$= 0{,}9.$$

From Table IV we have for spruce:

$e = 850000$

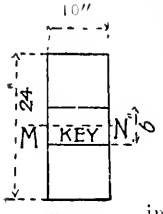


Fig. 141.

From Table I. Section No. 17, we have for the weakest section of the girder, which would be through a key, and as shown in Fig. 141,

$$i = \frac{b}{12} \cdot (d^3 - d_1^3)$$

$$= \frac{10}{12} \cdot (24^3 - 6^3) = 11340, \text{ therefore}$$

inserting these values in the transposed Formula(40)

$$w = \frac{0.9.48.850000.11340}{360^3}$$

$$= 8925$$

Or the safe centre load, not to crack plastering, would be, say 9000 pounds.

End Keys. Now let us try the keys. We first take the greatest horizontal shearing, which will be at the end keys.

The vertical shearing at these keys will be equal to the reaction (see Table VII, or Formula 11.)

As the load is central, each reaction will, of course, be one-half the load, or 4500 pounds, therefore the vertical shearing strain at end key, will be (a little less than)

$$x = 4500$$

Now from Formula (13) we know that the horizontal shearing strain at the same point is:

$$\frac{3}{2} \cdot \frac{x}{a}$$

For the area we take the full area of cross-section or $a = 10.24 = 240$, therefore horizontal shearing strain:

$\frac{3.4500}{2.240} = 28{,}125$ pounds per square inch. The amount of this strain that will act on each key is, of course, equal to the area at the neutral axis from centre to centre of key, or $40.10 = 400$ square inches multiplied by the strain per square inch, or

$$400.\ 28{,}125 = 11250 \text{ pounds.}$$

To resist this we have a key $12'' \times 10'' = 120$ square inches, being sheared across the grain. From Table IV we know that the safe shearing stress of Georgia pine across the grain or fibres, is:

$\left(\frac{g}{f}\right) = 570$ pounds so that the key could safely stand an amount of horizontal shearing

$$= 570.\ 120 = 68400 \text{ pounds}$$

or more than six times the actual strain. Had we, however, placed the grain of the key horizontal, the shearing would be with the grain or along the fibres; the safe shearing stress this way for Georgia pine (Table IV) is only 50 pounds per square inch, so that the key would only have resisted

$$= 50.120 = 6000 \text{ pounds,}$$

or it would have been in serious danger of splitting in two.

Central Keys. Now take the Key A B immediately to the right of the weight. The bending-moment at A will be (Table VII)

$$m_A = 4500.166 = 747000$$

and at B

$$m_B = 4500.154 = 693000$$

Now at A and B, the girder being uncut, the moment of resistance will be (Table I, Section No. 2)

$$r = \frac{10.24^2}{6} = 960$$

Dividing the bending moment by the moment of resistance (Formula 18 transposed) gives the extreme fibre strains,

$$\text{at A} = \frac{747000}{960} = 778 \text{ pounds.}$$

$$\text{and at B} = \frac{693000}{960} = 722 \text{ pounds.}$$

Now the centres (x and y, see Fig. 139) of each side of key will be $1\frac{1}{2}''$ from neutral axis, the extreme fibres being, of course, $12''$ distant from neutral axis, therefore average strain on side of key at A. (Or x, Fig. 139) $= \frac{1\frac{1}{2}}{12} . 778 = 97$ pounds,

and at B (or y, Fig. 139) $= \frac{1\frac{1}{2}}{12} . 722 = 90$ pounds.

The extreme compression, will, of course, be on the lower edge of key, at A and will be $= 2.97 = 194$ pounds per square inch. From Table IV we find that Georgia pine will safely stand a pressure of 200 pounds per square inch, *across* the fibres, so that we are just a little inside of the safety mark. We now have to consider our key as a cantilever with cross-section $10''$ wide and $12''$ deep, projecting $3''$ beyond the support and loaded uniformly with a weight equal to 90 pounds per square inch, or

$$u = 90.3.10 = 2700 \text{ pounds.}$$

Now the bending moment at support is, (Formula 25.)

$$m = \frac{2700.3}{2} = 4050 \text{ pounds.}$$

The moment of resistance (Table I, Section 2) is

$$r = \frac{10.12^2}{6} = 240$$

Therefore (Formula 18) the extreme fibre strains on key

$$= \frac{m}{r} = \frac{4050}{240} = 17 \text{ pounds.}$$

Or not enough to be even considered seriously.

Notched girders. Another method of combining and strengthening wooden girders, is to cut them with saw-shaped notches, as shown in (Fig. 142) and fit the teeth closely together, firmly bolting the two parts together, so as to force them to act

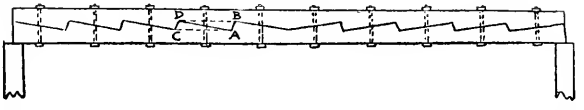


Fig. 142.

together as one girder. Sometimes the top surface slants towards each end, and iron bands are driven on towards the centre, till they are tight. But bolts are more reliable, and not likely to slip; where the girder is broad, they should be doubled, that is, placed in pairs across the width of girder. The distance between bolts should not exceed twice the depth of girder. Great care must be taken to get the right side up.[1] Many text-books even being careless in this matter. It must be remembered that the upper fibres are in compression, crowding towards the centre, while the lower ones, in tension, are pressing away.

The girder must therefore be placed, as shown, so that the two sets of fibres will meet at the short joints and oppose each other. The girder is easily calculated similarly to the former example. The crushing on C D or A B can be found, and also the stress on their extreme edges; this must not exceed the safe stress of the material for compression along the fibres. Then D B or C A must have area sufficient to resist the horizontal shearing strain.

Size of Bearings. In all these girders the most careful fitting of joints is necessary; then, too, the ends must have sufficient bearing not to crush under the load. Thus, take the former example, the reaction was 4500 pounds; the safe resistance of spruce to crushing across the fibres is (Table IV) = 75 pounds.

We need therefore an area $= \frac{4500}{75} = 60$ square inches, and as the girder is 10″ broad it should bear on each support $\frac{60}{10} = 6$

[1] The reasons for placing the notched girder in the position shown in Figure 142 have been fully stated by me in a letter published May 10, 1890, in the *American Architect*, Vol. XXVIII, No. 750.

inches. The end of girder should be deep enough to resist vertical shearing. In our case it is trifling, and we need not consider it. In all of these examples we have omitted the weight of the girder, to avoid complication. This should really be taken into account, in such a long girder, and treated as an additional but uniform load.

Continuous Girders. When girders run over three or more supports in one piece, that is, are not cut apart or jointed over the supports, the existing strains and reactions of ordinary girders, are very much altered. These are known as "continuous girders." If we have (Fig. 143) three supports, and run a continuous girder over them in one piece and load the girder on each side it will act as

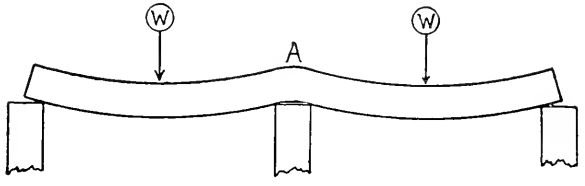


Fig. 143.

shown in Fig. 143; if the girder is cut it will act as shown in Fig. 144. Very little thought will show that the fibres at A not being able to separate in the first case, though they want to, must cause considerable tension in the upper fibres at A. This tension, of course, takes up or counterbalances part of the compression existing there, and the result is that the first or continuous girder (Fig. 143) is considerably stronger, that is, it is less strained and considerably stiffer, than the sectional or jointed girder (Fig. 144). Again we

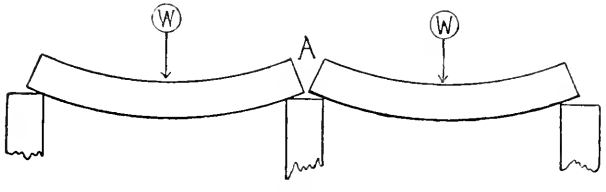


Fig. 144.

can readily see that the great tension and conflict of the opposite stresses at A would tend to cause more pressure on the central post in Fig. 143, than on the central post in Fig. 144, and this, in fact, is the case.

TABLE XVII. — CONTINUOUS GIRDERS.

Illustration.	Description.	Amount of Reactions.	Amount of Greatest Bending Moments.	Amount of Greatest Deflections.
	Two equal spans each carrying a central load but loads not equal. $w_1 \leq w_{11}$ $l = l_1$	*Left reaction.* $p = \frac{13 \cdot w_1 - 3 \cdot w_{11}}{32}$ *Centre reaction.* $r = \frac{11}{16} \cdot (w_1 + w_{11})$ *Right reaction.* $q = \frac{13 \cdot w_{11} - 3 \cdot w_1}{32}$	*Located at r* $m = \frac{3}{32} \cdot l \cdot (w_1 + w_{11})$	*Deflection in left span l* $\delta_1 = \frac{23 \cdot w_1 - 9 \cdot w_{11}}{1536 \cdot e \cdot i} \cdot l^3$ *Deflection in right span l_1* $\delta_{11} = \frac{23 \cdot w_{11} - 9 \cdot w_1}{1536 \cdot e \cdot i} \cdot l_1^3$
	Two equal spans carrying a central load = w, loads equal. $w = w_1$ $l = l_1$	*End reactions.* $p = q = \frac{5}{16} \cdot w$ or $= \frac{5}{32} \cdot (w + w_1)$ *Centre reaction.* $r = \frac{11}{8} \cdot w$ or $= \frac{11}{16} \cdot (w + w_1)$	*Located at r* $m = \frac{3}{16} \cdot l \cdot w$ or $= \frac{3}{32} \cdot l \cdot (w + w_1)$	*Deflection in either span* $\delta = \frac{w \cdot l^3}{110 \cdot e \cdot i}$ or $= \frac{l^3}{220 \cdot e \cdot i} \cdot (w + w_1)$
	Two equal spans each loaded with a	*End reactions.* $p = q = \frac{3}{8} \cdot u$ or $= \frac{3}{16} \cdot (u + u_1)$	*Located at r* $m = \frac{u \cdot l}{8}$ or	*Deflection in either span* $\delta = \frac{u \cdot l^3}{185 \cdot e \cdot i}$ or

Where $\delta, \delta_1, \delta_{11}$ = the amount of deflection in inches, if girder of uniform cross-section throughout.

" e = the modulus of elasticity of the material, in pounds-inch, (see Table IV).

" i = the moment of inertia of the cross-section, in inches, (see Table I)

[*Table XVII continued.*]

	uniform load $= u$ $u = u_1$ $l = l_1$	*Centre reaction.* $r = \frac{5}{4} \cdot u$ *or* $= \frac{5}{8} \cdot (u + u_1)$	$= \frac{l}{16} \cdot (u + u_1)$		$= \frac{l^3}{370 \cdot e \cdot i} \cdot (u + u_1)$
	Three equal spans each carrying a central load $= w$, all loads equal $w = w_1 = w_{11}$ $l = l_1 = l_{11}$	*End reactions.* $p = q = \frac{7}{20} \cdot w$ *or* $= \frac{7}{60} \cdot (w + w_1 + w_{11})$ *Central reactions.* $r = s = \frac{23}{20} \cdot w$ *or* $= \frac{23}{60} \cdot (w + w_1 + w_{11})$	*Located at r or s* $m = \frac{3 \cdot w \cdot l}{20}$ *or* $= \frac{l}{20} \cdot (w + w_1 + w_{11})$		*Deflection in central span* $\delta = \frac{w \cdot l^3}{480 \cdot e \cdot i}$ *or* $= \frac{l^3}{1440 \cdot e \cdot i} \cdot (w + w_1 + w_{11})$ *Deflection in either end span* $\delta = \frac{w \cdot l^3}{87 \cdot e \cdot i}$ *or* $= \frac{l^3}{261 \cdot e \cdot i} \cdot (w + w_1 + w_{11})$
	Three equal spans each loaded with a uniform load $= u$ $u = u_1 = u_{11}$ $l = l_1 = l_{11}$	*End reactions.* $p = q = \frac{2}{5} \cdot u$ *or* $= \frac{2}{15} \cdot (u + u_1 + u_{11})$ *Central reactions.* $r = s = \frac{11}{10} \cdot u$ *or* $= \frac{11}{30} \cdot (u + u_1 + u_{11})$	*Located at r or s* $m = \frac{u \cdot l}{10}$ *or* $= \frac{l}{30} \cdot (u + u_1 + u_{11})$		*Deflection in central span* $\delta = \frac{u \cdot l^3}{1920 \cdot e \cdot i}$ *or* $= \frac{l^3}{5760 \cdot e \cdot i} \cdot (u + u_1 + u_{11})$ *Deflection in either end span* $\delta = \frac{u \cdot l^3}{145 \cdot e \cdot i}$ *or* $= \frac{l^3}{435 \cdot e \cdot i} \cdot (u + u_1 + u_{11})$

Where w, w_1, w_{11} = central concentrated loads in pounds, on either span, being equal, when so stated.

" l, l_1, l_{11} = the length of respective spans, in inches, all being equal.

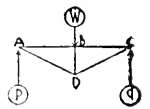
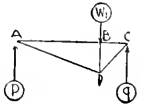
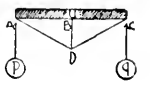
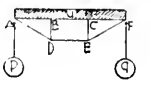
" u, u_1, u_{11} = uniform loads on each span, in pounds, all being equal.

" $p, r, s, q,$ = the amount of respective reactions, in pounds.

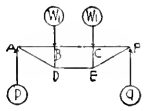
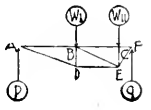
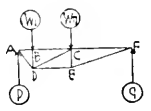
" m = the bending moment, in pounds-inch.

TABLE XVIII.

TRUSSED BEAMS.

Illustrations.	Description.	Compression in Struts.	Compression in Beam.	Tension in Rods.	Amount of Reactions.
	*Trussed Beam* with one centre load $= w$ $A\,B = B\,C$	*Compression in B D* $= + w$	*Compression in A B* $= + \frac{w}{2} \cdot \frac{A\,B}{B\,D}$ *Compression in B C* same as in *A B*	*Tension in A D* $= - \frac{w}{2} \cdot \frac{A\,D}{B\,D}$ *Tension in C D* same as in *A D*	$p = \frac{w}{2}$ $q = \frac{w}{2}$
	*Trussed Beam* with one load $= w_1$ (not central,) at any point $A\,B \lesssim B\,C$	*Compression in B D* $= + w_1$	*Compression in A B* $= + p \cdot \frac{A\,B}{B\,D}$ *Compression in B C* same as in *A B*, or $= + q \cdot \frac{B\,C}{B\,D}$	*Tension in A D* $= - p \cdot \frac{A\,D}{B\,D}$ *Tension in C D* $= - q \cdot \frac{C\,D}{B\,D}$	$p = w_1 \cdot \frac{B\,C}{A\,C}$ $q = w_1 \cdot \frac{A\,B}{A\,C}$
	*Trussed Beam* with uniform load $= u$ and one central strut. $A\,B = B\,C$	*Compression in B D* $= + \frac{5}{8} \cdot u$	*Compression in A B* $= + \frac{5}{16} \cdot u \cdot \frac{A\,B}{B\,D}$ *Compression in B C* same as in *A B*	*Tension in A D* $= - \frac{5}{16} \cdot u \cdot \frac{A\,D}{B\,D}$ *Tension in C D* same as in *A D*	$p = \frac{u}{2}$ $q = \frac{u}{2}$
	*Trussed Beam* with uniform load $= u$ and two struts, dividing beam into three equal parts. $A\,B = B\,C = C\,F$	*Compression in B D* $= + \frac{11}{30} \cdot u$ *Compression in C E* same as in *B D*	*Compression in A B* $= + \frac{11}{30} \cdot u \cdot \frac{A\,B}{B\,D}$ *Compression in B C and Compression in C F* same as in *A B*	*Tension in A D* $= - \frac{11}{30} \cdot u \cdot \frac{A\,D}{B\,D}$ *Tension in D E* $=$ — compression in *A B* *Tension in F E* same as in *A D*	$p = \frac{u}{2}$ $q = \frac{u}{2}$

[*Table XVIII continued.*]

	*Trussed Beam* with two equal loads each $= w_{\text{I}}$ and two struts at equal distances from ends. $A\,B = C\,F$	*Compression in B D* $= + w_{\text{I}}$ *Compression in C E* same as in *B D*	*Compression in A B* $= + w_{\text{I}} \cdot \frac{A\,B}{B\,D}$ *Compression in B C* and *Compression in C F* same as in *A B*	*Tension in A D* $= - w_{\text{I}} \cdot \frac{A\,D}{B\,D}$ *Tension in D E* $=$ — compression in *A B* *Tension in F E* same as in *A D*	$p = w_{\text{I}}$ $q = w_{\text{I}}$
	*Trussed Beam* with two unequal loads w_{I} and w_{II} at any points. Providing p smaller than w_{I} and q larger than w_{II}	*Compression in B D* $= + p$ *Compression in B E* $= + (q - w_{\text{II}}) \cdot \frac{B\,E}{B\,D}$ *Compression in C E* $= + w_{\text{II}}$	*Compression in A B* $= + p \cdot \frac{A\,B}{B\,D}$ *Compression in B C* same as in *C F* *Compression in C F* $= + q \cdot \frac{C\,F}{B\,D}$	*Tension in A D* $= - p \cdot \frac{A\,D}{B\,D}$ *Tension in D E* $=$ — compression in *A B* *Tension in F E* $= - q \cdot \frac{F\,E}{B\,D}$	$p = \frac{w_{\text{I}} \cdot B\,F + w_{\text{II}} \cdot C\,F}{A\,F}$ $q = \frac{w_{\text{I}} \cdot A\,B + w_{\text{II}} \cdot A\,C}{A\,F}$
	*Trussed Beam* with two unequal loads w_{I} and w_{II} at any points. Providing p larger than w_{I} and q smaller than w_{II}	*Compression in B D* $= + w_{\text{I}}$ *Compression in C D* $= + (p - w_{\text{I}}) \cdot \frac{C\,D}{B\,D}$ *Compression in C E* $= + q$	*Compression in A B* $= + p \cdot \frac{A\,B}{B\,D}$ *Compression in B C* same as in *A B* *Compression in C F* $= + q \cdot \frac{C\,F}{B\,D}$	*Tension in A D* $= - p \cdot \frac{A\,D}{B\,D}$ *Tension in D E* $=$ — compression in *C F* *Tension in F E* $= - q \cdot \frac{F\,E}{B\,D}$	$p = \frac{w_{\text{I}} \cdot B\,F + w_{\text{II}} \cdot C\,F}{A\,F}$ $q = \frac{w_{\text{I}} \cdot A\,B + w_{\text{II}} \cdot A\,C}{A\,F}$

Where $p =$ the amount of the left reaction, in pounds.
" $q =$ the amount of right reaction, in pounds.
" w, w_{I}, $w_{\text{II}} =$ concentrated loads, in pounds.
" $u =$ uniform load, in pounds, over whole beam.
" *A B*, *B C*, *C F*, *B D*, *B E*, *C D*, *C E*, *A D*, *D E*, *F E* = the length of *longitudinal central* axes of these pieces, and must *all* be expressed *uniformly*, that is, *all* expressed either in feet or inches.

The amounts of compression in either struts or beam parts will be the total compression in each, expressed in pounds; to obtain the compression *per square inch*, divide the amount by the area of cross-section of the strut or part.

The amounts of tension in rods will be the total tension in each part, expressed in pounds; to obtain the tension *per square inch*, divide the amount by the area of cross-section of rod.

In Table XVII, pages 218 and 219, are given the various formulæ for reactions, greatest bending moments and deflections, for the most usual cases of continuous girders. The architect can, if he wishes, neglect to allow for the additional strength and stiffness of continuous girders, as both are on the safe side. But he must never overlook the fact *that the central reactions are much greater*, or in other words, that the end supports carry *less*, and the central supports carry *more*, than when the girders are cut.

Variation of Reactions.

Bending moments can be figured, at any desired point along a continuous girder, as usual, subtracting from the sum of the reactions on one side multiplied by their respective distances from the point, the sum of all the weights on the same side, multiplied by their respective distances from the point. Sometimes the result will be negative, which means a reversal of the usual stresses and strains. Otherwise the rules and formulæ hold good, the same as for other girders or beams. Table XVII gives all necessary information at a glance.

Strength is frequently added to a girder or beam by trussing it, as shown in Table XVIII, pages 220 and 221. One or two struts are placed against the lower edge of a beam and a rod passed over them and secured to each end of the beam; by stretching this rod the beam becomes the compression chord of a truss and also a continuous girder running over one or two supports. There must therefore be enough material in the beam to stand the compression, and *in addition* to this enough to stand the transverse strains on the continuous girder. If the loads are concentrated immediately over the braces, there will be no transverse strain whatever, but the braces will be compressed the full amount of the respective loads on each. In the case of uniform loads, transverse strains cannot be avoided, of course, but where loads are concentrated the struts should always be placed immediately under them. Even where loads are placed very unevenly, it is better to have the panels of the truss irregular, thus avoiding cross or transverse strains. This same rule holds good in designing trusses of any kind.

Trussed Beams.

Struts placed under load.

Table XVIII shows very clearly the amount and kind of strains in each part of trussed beams. Where there are two struts and they are of any length care must be taken by diagonal braces or otherwise, to keep the lower ends of braces from tipping towards each other. Theoretically they cannot tip, but

Necessary Conditions.

practically, sometimes, they do. Care must be taken that the beam is braced sideways, or else it must be figured for its safety against lateral flexure (Formula 5.) Then it must have material enough not to shear off at supports, nor to crush its under side where lying on support. The ends of rods must have sufficient bearing not to crush the wood. Iron shoes are sometimes used, but if very large are apt to rot the wood. In that case it is well to have a few small holes in the shoes, to allow ventilation to end of timber. If iron straps and bolts are used at the end, care must be taken that the strap does not tear apart at bolt holes; that it does not crush itself against bolts; that it does not shear off the bolts, and that it does not crush in the end of timber. Care must also be taken to have enough bolts, so that they do not crush the wood before them, and to keep the bolts from shearing out, that is tearing out the wood before them. In all trusses and trussed works the joints must be carefully designed to cover all these points. Many architects give tremendous sizes for timbers and rods in trusses, thus adding unnecessary weight, but when it comes to the joint, they overlook it, and then are surprised when the truss gives out. The next time they add more timber and more iron, till they learn the lesson. It must be remembered that the strength of a truss is only equal to the strength of its weakest part, be that part a member or only a part of a joint. This subject will be fully dealt with in the chapter on Trusses.

Importance of Joints.

Depth Desirable.

The deeper the truss is made, that is, the further we separate the top and bottom chords, the stronger will it be; besides additional depth adds very much to the stiffness of a truss.

Deflection of Girders and Beams.

All trussed beams, and all trusses should be "cambered up," that is, built up above their natural lines sufficiently to allow for settling back into their correct lines, when loaded. The amount of the camber should equal the calculated deflection. For all beams, girders, etc., of uniform cross-section throughout, the deflection can be calculated from Formulæ (37) to (42) according to the manner of loading. For wrought-iron beams and plate-girders of uniform cross-section throughout, the deflection can be calculated from the same formulæ; where, however, the load is uniform and it is desired to simplify the calculation, the deflection can be quite closely calculated from the following Formula:

Uniform Cross-section and Load.

$$\delta = \frac{L^2}{75.d} \qquad (79)$$

Where δ = the greatest deflection at centre, in inches, of a wrought-iron beam or plate girder of uniform cross-section throughout, and carrying its total safe uniform load, calculated for rupture only.

Where L = the length of span, in feet.

Where d = the total depth of beam or girder in inches.

If beam or plate girder is of steel, use $64\frac{1}{2}$ instead of 75.

If the load is not uniform, change the result, as provided in cases (1) to (8), Table VII.

For a centre load we should use $93\frac{3}{4}$ in place of 75 or

Uniform Cross-section, Centre Load.

$$\delta = \frac{L^2}{93\frac{3}{4}.\,d} \qquad (80)$$

Where values are the same, as for Formula (79) except that beam or girder carries its total safe centre load, calculated for rupture only.

If beam or girder is of steel use $80\frac{2}{3}$ instead of $93\frac{3}{4}$.

Therefore not to crack plastering and yet to carry their full safe loads, wrought-iron beams or plate girders should never exceed in length (measured in feet) twice and a quarter times the depth (measured in inches), if the load is uniform, or

Safe length, uniform Cross-section and Load.

$$2\tfrac{1}{4}.\,d = L \qquad (81)$$

Where L = the ultimate length of span (not to crack plastering), in feet, of a wrought-iron beam or plate girder, of uniform cross-section throughout and uniformly loaded with its total safe load.

Where d = the total depth of beam or girder in inches.

If beam or girder is of steel, use 2 instead of $2\frac{1}{4}$.

If the load is central the length in feet should not exceed $2\frac{4}{5}$ times the depth in inches, or

Safe length, uniform Cross-section, Centre Load.

$$2\tfrac{4}{5}.\,d = L. \qquad (82)$$

Where L = the ultimate length of span in feet (not to crack plastering), of a wrought-iron beam or plate girder, of uniform cross-section throughout, and loaded at its centre with its total safe load.

Where d = the total depth of beam or girder in inches.

If beam or girder is of steel use $2\frac{2}{5}$ instead of $2\frac{4}{5}$.

Deepest beam most economical. One thing should always be remembered, when using iron beams, and that is, that the deepest beam is *always* not only the stiffest, but the most economical. For instance, if we find it necessary to use a $10\frac{1}{2}''$ beam — 105 pounds per yard,

it will be cheaper to use instead the 12″ beam — 96 pounds per yard. The latter beam not only weighs 9 pounds per yard less, but it will carry more, and deflect less, owing to its extra two inches of depth. This same rule holds good for nearly all sections.

Deflection of Trusses. To obtain the deflections of trussed beams or girders by the rules already given would be very complicated. For these cases, however, Box gives an approximate rule, which answers every purpose. He calculates the amount of extension in the tension (usually the lower) chord, and the amount of contraction in the compression (usually the upper) chord, due to the strains in each, and from these, obtains the deflections. Of course the *average* strain in each chord must be taken and not the greatest strain at any one point in either. In a truss, where each part is proportioned in size to resist exactly the compressive or tensional strain on the part, every part will, of course, be strained alike; the strain in the compressive member being $= \left(\frac{c}{f}\right)$ per square inch, throughout the whole length, and in the tension member $= \left(\frac{t}{f}\right)$ per square inch, throughout the whole length.

The same holds good practically for plate girders, where the top and bottom flanges are diminished towards the ends, in proportion to the bending moment. But where, as in wrought-iron beams (and in many trusses), the flanges are made, for the sake of convenience, of uniform cross-section throughout their entire length, the "average" strain will, of course, be much less, and consequently the beam or girder stiffer.

Average Strain in Chords. If we construct the graphical representation of the bending moments at each point of beam (as will be explained in the next Chapter) and divide the area of this figure in inch-pounds by the length of span in inches, we will obtain the average strain in either flange, provided the flange is of uniform cross-section throughout, or

Uniform Cross-section.
$$v = \frac{a}{l} \qquad (83)$$

Where $v =$ the *average* strain, in pounds, on top or bottom flange or chord, where beam or girder is of uniform cross-section throughout.

Where $l =$ the length of span, in inches.

Where $a =$ the area in pounds-inch of the graphical figure giving the bending moment at all points of beam.

To obtain the dimensions of this figure measure its base line (or horizontal measurement) in inches, and its height (or vertical measurement) in pounds, assuming the greatest vertical measurement as $= \left(\frac{c}{f}\right)$ or $= \left(\frac{t}{f}\right)$, in pounds, according to which flange we are examining.

Thus, in the case of a uniform load, this figure would be a parabola, with a base of length equal to the span measured in inches, and a height equal to the greatest fibre strains in pounds; the average strain therefore in the compression member of a beam, girder or truss, of uniform cross-section throughout would be, — (remembering that the area of a parabola is equal to two-thirds of the product of its height into its base),

Uniform load and Cross-section.

$$v = \frac{\frac{2}{3} \cdot l \cdot \left(\frac{c}{f}\right)}{l} \text{ or}$$

$$v = \frac{2}{3} \cdot \left(\frac{c}{f}\right) \qquad (84)$$

Where $v =$ the *average* strain, in pounds, in compression flange or chord of a beam, girder or truss of uniform cross-section throughout and carrying its total safe uniform load.

Where $\left(\frac{c}{f}\right) =$ the safe resistance to compression per square inch of the material.

It is supposed, of course, that at the point of greatest bending moment — or where the greatest compression strain exists — that the part is designed to resist or exert a stress $= \left(\frac{c}{f}\right)$ per square inch. If the greatest compression stress is less, insert its value in place of $\left(\frac{c}{f}\right)$. Of course, it must never be greater than $\left(\frac{c}{f}\right)$.

Similarly we should have

Uniform Load and Cross-section.

$$v = \frac{2}{3} \cdot \left(\frac{t}{f}\right) \qquad (85)$$

Where $v =$ the *average* strain, in pounds, in tension flange or chord of a beam, girder or truss of uniform cross-section throughout, and carrying its total safe uniform load.

Where $\left(\frac{t}{f}\right) =$ the safe resistance to tension, per square inch, of the material.

It being understood that at the point of greatest bending moment — or where the greatest tension strain exists — that the part is de-

signed to resist or exert a stress $= \left(\frac{f}{f'}\right)$ per square inch. If this greatest tensional stress is less than $\left(\frac{f}{f'}\right)$ insert its value in its place in Formula (85). Of course, it must never be greater than $\left(\frac{f}{f'}\right)$.

For a beam, girder or truss with a load concentrated at the centre, but with flanges or chords of uniform cross-section throughout, the average strain would be just one-half that at the centre; for, the bending-moment graphical-figure will be a triangle, and inserting the values in Formula (83) would give for the compression member:

Centre Load Uniform Cross-Section.

$$r = \tfrac{1}{2} \cdot \left(\frac{c}{f'}\right) \qquad (86)$$

and for the tension member:

$$r = \tfrac{1}{2} \cdot \left(\frac{f}{f'}\right) \qquad (87)$$

The meaning of letters being the same as in Formulæ (84) and (85), but the total safe load being concentrated at the centre instead of uniformly distributed.

To obtain the amount of contraction or expansion due to this average strain, use the following Formula:

Expansion or Contraction from Strain.

$$x = \frac{r \cdot l}{e} \qquad (88)$$

Where $r =$ the *average* strain, in pounds per square inch, in either chord or flange.

Where $l =$ the length of span, in inches.

Where $e =$ the modulus of elasticity of the material, in pounds-inch.

Where $x =$ the total amount of extension or contraction, in inches, of the chord or flange.

Now let us apply the above rules to beams, plate girders, and trussed beams. Taking the case of a beam or plate girder or truss with parallel flanges or chords.

Figure 145 shows the same, after the deflection has taken place. We can now assume approximately, that $C\,A$ is equal to one-half the difference between the contraction of $G\,C$ and the elongation of $H\,B$, or, what amounts to the same thing, that $C\,A$

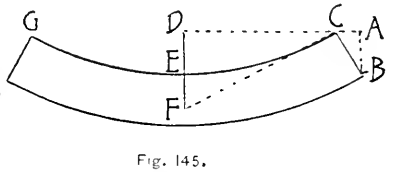


Fig. 145.

is equal to one-half the sum of the contraction of the one and the elongation of the other.

Further, we can assume that approximately, $A\ B = d$ or the depth of beam, and $C\ D = \frac{l}{2}$ or one half the span.

The curve $C\ E\ C'$ will approximate a parabola, so that if we draw a tangent $C\ F$ to the same at C, we know that $D\ E = E\ F = \frac{D\ F}{2}$ or $D\ F = 2.\ D\ E$. But as $D\ E$ represents the deflection (δ) of the beam, we have

$$D\ F = 2.\ \delta$$

Now as $C\ F$ is normal to $C\ B$, and $C\ D$ normal to $A\ B$, we know that angles $D\ C\ F = A\ B\ C$; further, as both triangles are right angle triangles, we know that they are similar, therefore:

$$D\ F : C\ A :: D\ C : A\ B, \text{ or}$$

$$2.\ \delta : C\ A :: \frac{l}{2} : d \text{ or}$$

$$\delta = \frac{C\ A.\frac{l}{2}}{2.\ d} = \frac{C\ A.\ l}{4.\ d}.$$

If now we assume the sum of the extension and contraction of the two flanges or chords to be $= x$.

We have $C\ A = \frac{x}{2}$ or

Deflection of Parallel Flanges or Chords, any Cross-section.

$$\delta = \frac{x.\ l}{8.\ d} \qquad (89)$$

Where $\delta =$ the deflection, in inches, of a beam, plate girder or truss, with parallel flanges or chords.

Where $x =$ the sum of the amount of extension in tension chord, plus the amount of contraction in compression chord.

Where $l =$ the length of span, in inches.

Where $d =$ the total depth of beam, girder or truss in inches.

Take the case of a wrought-iron plate girder or beam of uniform cross-section throughout carrying its full uniform load, we should have the strain at the centre on the extreme fibres $= 12000$ pounds per square inch. Now the average strain on both upper and lower flanges would be, Formulæ (84) and (85).

$$r = \tfrac{2}{3}.\quad 12000 = 8000 \text{ pounds}$$

per square inch. Therefore amount of contraction in upper flange

Formula (88), (and remembering that, from Table IV, $\epsilon = 27000000$)

$$x = \frac{8000.\,l}{27000000} = \frac{l}{3375}$$

The elongation of the bottom flange would be an equal amount, therefore the sum of the two

$$x_1 = 2.\,x = \frac{2.\,l}{3375}$$

$$= \frac{l}{1687,5}$$

Inserting these values in Formula (89) we have the deflection

$$\delta = \frac{l^2}{8.1687,5.\,d} = \frac{l^2}{13500.\,d}$$

and inserting for $l^2 = 144.\,L^2$, we have

$$\delta = \frac{144.\,L^2}{13500.\,d.}$$

$$= \frac{L^2}{93\frac{3}{4}.\,d.}$$

Had we assumed that the area of flanges or chords diminished towards the supports in proportion to the bending moment or actual stresses required, the average strain would, of course, be 12000 pounds per square inch throughout the entire length, no matter how the load might be applied.

Inserting this value in Formula (88) we should have had, for the amount of contraction of top flange

$$x = \frac{12000.\,l}{27000000} = \frac{l}{2250}$$

The same for the extension of bottom chord, or

$$x_1 = 2.\,\frac{l}{2250} = \frac{l}{1125}$$

Inserting this in Formula (89) we have for the deflection:

$$\delta = \frac{l^2}{8.1125.\,d} = \frac{l^2}{9000.\,d}$$

Inserting $144\,L^2 = l^2$ we have

$$\delta = \frac{144.\,L^2}{9000.\,d} \text{ or}$$

Parallel flanges or Chords, Diminished Cross-section, any loads.

$$\delta = \frac{L^2}{62\frac{1}{2}.\,d} \qquad (90)$$

Where δ = the greatest deflection, in inches, of a wrought-iron plate girder, or wrought-iron truss, with parallel flanges or chords, and where the areas of flanges or chords are gradually diminished

towards supports, and no matter how the load is applied; in no part however must the stresses, per square inch exceed respectively either $\left(\frac{c}{f}\right)$ or $\left(\frac{t}{f}\right)$.

Where $L =$ the length of span, in feet.

Where $d =$ the total depth (height) in inches, from top of top flange or chord to bottom of bottom flange or chord.

If girder or truss is of steel, use $53\frac{2}{3}$ instead of $62\frac{1}{2}$.

From Formula (90) and Formula (28) we get the rule that (no matter how the load is applied) if we want to carry the full safe load and not have deflection enough to crack plastering the length in feet must not exceed $1\frac{7}{8}$ times the total depth in inches.

For:

$$L.\ 0{,}03 = \frac{L^2}{62\frac{1}{2}.\ d} \text{ or}$$

$$L = 62\tfrac{1}{2}.\ 0{,}03.\ d$$

$$= 1{,}875.\ d \text{ or say}$$

Safe Length, Diminished Cross-section, any Load, Parallel Flanges or Chords.

$$L = 1\tfrac{7}{8}.\ d \qquad (91)$$

Where $L =$ the length, in feet, of a wrought-iron plate girder or wrought-iron truss, with parallel flanges or chords and with area of flanges or chords diminishing gradually towards supports and no matter how the load is applied; in no part however must the stresses, per square inch, exceed respectively either $\left(\frac{c}{f}\right)$ or $\left(\frac{t}{f}\right)$.

Where $d =$ the total depth (height), in inches, from top of top flange or chord to bottom of bottom flange or chord.

If girder or truss is of steel, use $1\frac{2}{3}$ instead of $1\frac{7}{8}$.

We see therefore that a beam of diminishing cross-section throughout is only about $\frac{2}{3}$ as stiff, as one with uniform cross-section, as its amount of deflection will be one-half more than that of the latter. Both deflections are approximate only, however, as we see by comparing the amount for the uniform cross-section to that obtained from Formula (79). The deflection for varying cross-sections how-

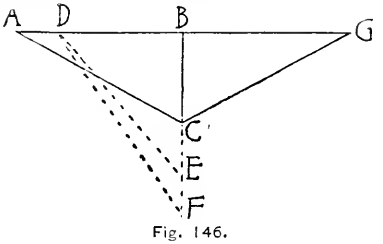


Fig. 146.

ever can be assumed as nearly enough correct, as these are never diminished so much practically as we have assumed in theory. Now taking the case of a trussed beam.

Deflection Trussed Beam. In Figure 146, let $A\ B$ be one half of a trussed beam, let $B\ C$ be the strut and $A\ C$ the tie. We will consider the load concentrated at B. Now the first effect is to shorten $A\ B$ by compression, let us say to $D\ B$.

Then, of course, $A\ D$ will represent one half of the contraction in the whole beam $A\ G$. Now the end of rod A moving to D will, of course, let the point C down to E, if we make $D\ E = A\ C$.

But there will be an elongation in $D\ E$ besides, due to the tension in it, which will let it down still further, say to F, if $D\ F = A\ C +$ elongation in $A\ C$, of course the point B will move down too, but we can overlook this to avoid complication. We now have $C\ F$ representing the amount of the deflection. To this should be added the amount of contraction of $B\ C$ due to the compression in it. We can readily find $C\ F$.

We know that

$$B\ F = \sqrt[2]{D\ F^2 - D\ B^2}$$

Now $D\ F$ we know is $= A\ C$ plus the elongation of $A\ C$ due to the tension in it, which we can find from Formula (88). From same formula we find the amount of contraction in $A\ G$ of which $A\ D$ is one-half, subtracting this from $A\ B$ or $\frac{l}{2}$ leaves, of course, $D\ B$.

Now having found $B\ F$ we substract from it $B\ C$, the length of which is known, and the balance is of course the deflection $C\ F$; to this we add the contraction of $B\ C$ and obtain the total deflection of the whole trussed beam.

If the load had been a uniform load, instead of a concentrated one over the strut, there would be a deflection in that part of $A\ G$ which would be acting as a continuous girder. But this deflection would take place between B and G and between B and A and would not affect the deflection of the whole trussed beam.

An example will make much of the foregoing more clear.

Example.

Trussed Beam. *A trussed Georgia Pine beam is* 16″ *deep and of* 24 *feet clear span; it bears* 16″ *on each support and is trussed as*

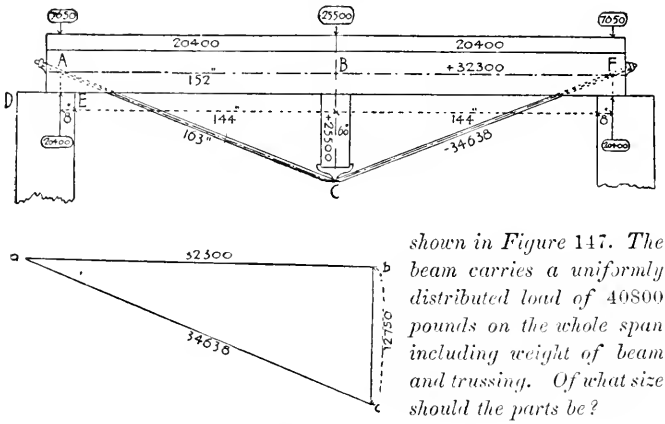


Fig. 147.

shown in Figure 147. *The beam carries a uniformly distributed load of* 40800 *pounds on the whole span including weight of beam and trussing. Of what size should the parts be?*

We draw the longitudinal neutral axes of each part, namely $A\ B$, $B\ C$ and $A\ C$. The latter is so drawn that the neutral axis of the reaction, which is of course half way between end of girder and E (or 8″ from E) will also pass through A.

In designing trusses this should always be borne in mind, that so far as possible all the neutral axes at each joint should go through the same point.

Cross-strains in Beams. The beam $A\ F$ virtually becomes a continuous girder, of two equal spans of 12 feet or 144″ each, uniformly loaded with 20400 pounds each, and supported at three points A, B and F. From table XVII we know that the greatest bending moment is at B and

$$= \frac{u.l}{8} = \frac{20400.144}{8} = 367200 \text{ pounds-inch.}$$

The modulus of rupture for Georgia pine (Table IV) is

$\left(\frac{k}{f}\right) = 1200$, therefore moment of resistance (r) from Formula (18) and Table I, section No. 2,

$$r = \frac{b.d^2}{6} = \frac{367200}{1200} \text{ or}$$

$$b.d^2 = 1836$$

Now we know that $d = 16$, or $d^2 = 256$, therefore

$$b = \frac{1836}{256} = 7{,}2 \text{ or say we need a beam } 7\tfrac{1}{4}'' \text{ x } 16'' \text{ for the}$$

transverse strain. We must add to this however for the additional compression due to the trussing.

Compression in Strut. The amount of the load carried by strut $C\ B$, see Table XVII, is

$= \frac{5}{8}$. a from each side, or

$= 25500$ on the strut $B\ C$, of which

$= 12750$ from each side.

If now we make at any scale a vertical line $b\ c = half$ the load carried at point B or $= 12750$ in our case, and draw $b\ a$ horizontally and $a\ c$ parallel to $A\ C$, we find the strain in $B\ A$ by measuring $b\ a = (32300$ pounds) or in $A\ C$ by measuring $a\ c = (34638$ pounds) both measured at same scale as $b\ c$. We find, further, in passing around the triangle $c\ b\ a\ c$ — ($c\ b$ being the direction of the reaction at A), that $b\ a$ is pushing towards A, therefore compression; and that $a\ c$ is pulling away from A, therefore tension. Using the usual signs of $+$ for compression, and $-$ for tension, we have then:

Compression in Beam.

$A\ B = +\ 32300$ pounds.

$A\ C = -\ 34638$ pounds.

$B\ C = +\ 25500$ pounds.

Had we used Table XVIII we should have had the same result for:

Compression in $A\ B = \frac{25500}{2} \cdot \frac{A\ B}{B\ C} = +\ 32300$ pounds and

Tension in $A\ C = \frac{25500}{2} \cdot \frac{A\ C}{B\ C} = -\ 34638$ pounds.

Now the safe resistance of Georgia pine to compression along fibres (Table IV) is

$$\left(\frac{c}{f}\right) = 750 \text{ pounds.}$$

If $A\ B$ were very long, or the beam very shallow or very thin, we should still further reduce $\left(\frac{c}{f}\right)$ by using Formulae (3), or (5). But we can readily see that the beam will not bend much by vertical flexure due to compression, nor will it deflect laterally very much, so we can safely allow the maximum safe stress per square inch, or 750 pounds, that is, consider $A\ B$ a short column.

The necessary area to resist the compression, Formula (2) is:

$$32300 = a.\ 750 \text{ or}$$

$$a = \frac{32300}{750} = 43 \text{ square inches.}$$

As the beam is 16″ deep, this would mean an additional thickness $= \frac{4}{16}\frac{3}{} = 2\frac{11}{16}$

Adding this to the $7\frac{1}{4}$″ already found to be necessary, we have

$$7\tfrac{1}{4} + 2\tfrac{11}{16} = 9\tfrac{15}{16}$$

or the beam would need to be, say 10″ x 16″.

Size of Strut. Now the size of $B\ C$ must be made sufficient not to crush in the soft underside of the beam at B. The bearing here would be across the fibres of the beam, and we find (Table IV) that the safe compressive stress of Georgia pine across the fibres is $\left(\frac{c}{f}\right) = 200$ pounds. We need therefore an area

$$a = \frac{25500}{200} = 128 \text{ inches.}$$

As the beam is only 10″ wide the strut $B\ C$ will have to measure, $\frac{128}{10} = 12\frac{4}{5}$ inches the other way, or we will say it could be 10″ x 12″. This strut itself might be made of softer wood than Georgia pine, say of spruce; the average compression on it is

$$\frac{25500}{10.12} = 212 \text{ pounds per square inch.}$$

Now spruce will stand a compression on end (Table IV) of $\left(\frac{c}{f}\right) = 650$ or, even if spruce is used, the actual strain would be less than one-third of the safe stress. At the foot of the strut $B\ C$ we put an iron plate, to prevent the rod from crushing in the wood. The rod itself must bear on the plate at least

Iron Shoe to Strut. $\frac{25500}{12000} = 2{,}1$ square inches, or it would crush the iron — (12000 pounds being the safe resistance of wrought-iron to crushing).

Size of Tie-rod. The safe tensional stress of wrought-iron being 12000 pounds per square inch (Table IV), we have the necessary area for tie-rod $A\ C$ from Formula (6)

$$34638 = a \,.\, 12000 \text{ or}$$

$$a = \frac{34638}{12000} = 2{,}886 \text{ square inches.}$$

From a table of areas we find that we should require a rod of $1\frac{15}{16}$″ diameter, or say a 2″ rod.

The area of a 2″ rod being $= 3{,}14$ square inches the actual tensional stress, per square inch on the rod, will be only

$$\frac{34638}{3{,}14} = 11312 \text{ pounds per square inch.}$$

Size of Washer. We must now proportion the bearing of the washer at "*A*" end of tie-rod. The amount of the crushing coming on washer will be whichever of the two strains at *A*, (viz. *B A* and *A C*) is the lesser, or *B A* in our case, which is 32300 pounds. We must therefore have area enough to the washer not to crush the end of beam (or along its fibres), the safe resistance of which we already found to be: $\left(\frac{c}{f}\right) = 750$ pounds per square inch; we need therefore

$$\frac{32300}{750} = 43 \text{ square inches.}$$

The washer therefore should be about

$6\frac{1}{2}''$ by $6\frac{1}{2}''$

Upset Screw-end. The end of the rod *must* have an "upset" screw-end; that is, the threads are raised above the end of rod all around, so that the area at the bottom of sinkage, between two adjoining threads, is still equal to the full area of rod. If the end is not "upset" the whole rod will have to be made enough larger to allow for the cutting of the screw at the end, which would be a wilful extravagance.

It is unnecessary to calculate the size of nuts, heads, threads, etc., as, if these are made the regulation sizes, they are more than amply strong. **Central Swivel.** It should be remarked here that in all trussed beams, if there is not a central swivel, for tightening the rod, that there should be a nut at *each* end of the rod; and not a head at one end and a nut at the other. Otherwise in tightening the rod from one side only it is apt to tip the strut or crush it into the beam on side being tightened. We must still however calculate the vertical shearing across the beam at the supports, which we know equals the reaction, or 20400 pounds at each end. To resist this we have $10'' \times 16'' = 160$ square inches, less $3'' \times 16''$, cut out to allow rod end to pass, or say 112 square inches net, of Georgia pine, across the grain; and as $\left(\frac{g}{f}\right) = 570$ pounds per square inch (see Table IV); the safe vertical shearing stress at each support would be (Formula 7)

$112.570 = 63840$ pounds or more than three times the actual strain. **Bearing of Beam.** Then, too, we should see that the bearing of beam is not crushed. It bears on each reaction 16 inches, or has a bearing area $= 16.10 = 160$ square inches.

$\left(\frac{c}{f}\right)$ for Georgia pine, across the fibres, Table IV, is

$\left(\frac{c}{f}\right) = 200$, therefore the beam will bear safely at each end

$160.200 = 32000$ pounds or about one-half more than the reaction. There will be no horizontal shearing, of course, except in that part of beam under transverse strain, and this certainly cannot amount to much. The beam is therefore amply safe.

Deflection of Beam. Now let us calculate the deflection. The modulus of elasticity for Georgia pine, Table IV is: $e = 1200000$ pounds-inch. The average compression strain in $A\ F$ was 750 pounds per square inch, therefore the amount of contraction (Formula 88)[1]

$$x = \frac{750.304}{1200000} = 0{,}19 \text{ inches.}$$

Now $A\ D$ (in Fig. 146) will be one-half of this, or 0,095 inches.

The amount of elongation in $A\ C$ will be, remembering that we found the average stress to be only 11312 pounds per square inch, and that for wrought-iron $e = 27000000$ (Formula 88)

$$x = \frac{11312.163}{27000000} = 0{,}0682$$

The exact length of $A\ C$ (Fig. 147 should be 163,41 not 163″). Therefore $D\ F$ (Fig. 146) will be

$$D\,F = 163{,}41 + 0{,}0682 = 163{,}4782''$$
$$D\,B = 152 - 0{,}19$$
$$= 151'', 81$$

Therefore (Fig. 146)

$$B\,F = \sqrt[2]{163{,}4782^2 - 151{,}81^2}$$
$$= 60'', 655$$

Now $B\ C$ (Fig. 147) would be $= 60''$, deducting this from the above we should have a deflection $= 0'', 655$.

To this we must add the contraction of $B\ C$. The strut will be less than 60″ long, say about 50″. The average compressive stress per square inch we found $= 212$ pounds. The modulus of elasticity

[1] In reality the contraction of $A\ F$ would be much less, as the part figured for transverse strain only would very materially help to resist the compression, one-half of it being in tension.

for spruce, Table IV, is $e = 850000$, therefore contraction in strut (Formula 88)

$$x = \frac{212.50}{850000} = 0{,}0125$$

Adding this to the above we should have the total deflection

$$\delta = 0{,}655 + 0{,}0125$$
$$= 0{,}6675$$

This would be the amount we should have to "camber" up the beam, or say $\frac{3}{4}''$.

The safe deflection not to crack plastering, would be (Formula 28)

$$\delta = L.\ 0{,}03$$
$$= 24.0{,}03$$
$$= 0{,}72$$

So that our trussed beam is amply stiff.

Explanation of Tables XIX to XXV. Table XIX gives all the necessary data in regard to the use of Tables XX, XXI, XXII, XXIII, XXIV and XXV. These tables give all the necessary information in regard to all architectural sections which are rolled. Where, after the name of the company rolling the section there are several letters, it means that practically the same section is rolled by several companies. It should be remarked that except in the case of the simplest kind of beam work, it is cheaper to frame up plate girders, or trusses, of angles, tees, etc., as there is a strong pool in the rolling of I-beams and channel sections, which keeps the price of these two sections unreasonably high, in proportion to other rolled sections. Steel beams and sections are sold as cheap as iron, (they are really cheaper to manufacture), and where their uniformity can be relied on, should be used in preference, as they are much stronger and also a trifle stiffer. As a rule, however, the uniformity of steel in beams and other rolled sections cannot be relied on.

I-sections not economical.

One example of an iron beam will make the application of the Tables to transverse strains clear, and help to review the subject, before taking up the graphical method of calculating transverse strains.

Example.

Use of Tables XIX to XXV. *A wrought-iron I-beam of 25-foot clear span, carries a uniform load of 500 pounds per foot including weight of beam; also a concentrated load of 1000 pounds 10 feet from*

the right hand support. The beam is not supported sideways. What size beam should be used?

The total uniform load $u = 500.25 = 12500$ pounds of which one-half or 6250 pounds will go to each reaction; of the 1000 pounds load $\frac{180}{300}$ or $\frac{3}{5}$ will go to the nearer support q (Formula 15), therefore

$$q = 6250 + \tfrac{3}{5}.1000 = 6850$$

Similarly we should have (Formula 14)

$$p = 6250 + \tfrac{2}{5}.1000 = 6650$$

As a check the sum of the two loads should $= 13500$, and we have, in effect:

$$6850 + 6650 = 13500$$

To find the point of greatest bending moment begin at q pass to load 1000, and we will have passed over ten feet of uniform load or 5000 pounds, add to this the 1000 pounds making 6000 pounds, and we still are 850 pounds short of the reaction, we pass on therefore towards p one foot, which leaves 350 pounds more, and pass on another $\frac{7}{10}$ of

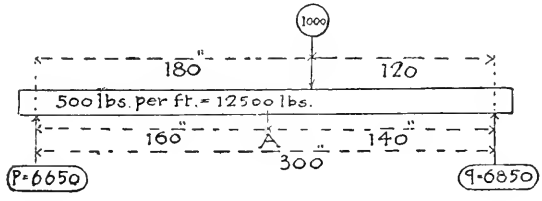


Fig. 148.

a foot (to A) which very closely makes the amount. The point of greatest bending moment therefore is at A, say 1′ 8″ to the left of the weight, or 140″ from q: As a check begin at p and we must pass along 160″ or 13′ 4″ of uniform load before reaching the point A, at 500 pounds a foot this would make $13\frac{1}{3}.500. = 6666$ or close enough to amount of reaction p for all practical purposes.

The uniform load per inch will be $\frac{500}{12} = 41\frac{2}{3}$ pounds.

Now the bending moment at A will be, taking the right-hand side (Formula 24)

$$m_A = 6850.140 - 41\tfrac{2}{3}.140.70 - 1000.20$$
$$= 530\,667 \text{ pounds-inch.}$$

As a check take the left-hand side (Formula 23)

$$m_A = 6650.160 - 41\tfrac{2}{3}.160.80$$

$= 530614$ pounds-inch, or near enough alike for all practical purposes.

Now the safe modulus of rupture for wrought-iron (Table IV) is $\left(\frac{k}{f}\right) = 12000$ pounds, therefore the required moment of resistance r from Formula (18)

$$r = \frac{530667}{12000} = 44,2$$

Looking at the Table XX we find the nearest moment of resistance to be 46,8 or we should use the $12'' - 120$ pounds per yard I-beam. But the beam is unsupported sideways. The width of top flange is $b = 5\frac{1}{2}''$. We now use Formula (78) to find out how much extra strength we require.

Reduction for Lateral Flexure. In inserting value for y, we use the second column of Table XVI, as the beam is, of course, of uniform cross-section throughout, and have

$$y = 0,0192.$$

In place of w we can insert the actual value r of the beam, and see what proportion of it is left to resist the transverse strength, after the lateral flexure is attended to,

$$\text{or } r_1 = \frac{r}{1 + \frac{0,0192.25^2}{5\frac{1}{2}^2}} = \frac{r}{1 + 0,3966} \text{ or}$$

$$r_1 = \frac{46,8}{1,3966} = 33,6 \text{ or the beam would not be strong}$$

enough. The next size would be the $12\frac{1}{4}'' - 125$ pounds per yard beam, but as the $15'' - 125$ pounds per yard beam would cost no more and be much stronger we will try that. Its width of flange is $b = 5''$ and moment of resistance $r = 57,93$. Inserting these values in (Formula 78) and using r in place of w we have

$$r_1 = \frac{57,93}{1 + \frac{0,0192.25^2}{5^2}} = \frac{57,93}{1,48}$$

$$= 39,14$$

The required moment of resistance was

$r = 44,2$ so that this is still short of the mark, and we should have to use the next section or the $15'' - 150$ pounds per yard beam. The moment of resistance of this beam is $r = 69,8$ its width of flange the same as before, therefore:

$$r_1 = \frac{69,8}{1,48} = 47,1$$

Or this beam would be a trifle too strong even if unsupported sideways. We need not bother with deflection, for the length of beam is only $1\frac{2}{3}$ times the span, and besides not even $\frac{2}{3}$ of the actual transverse strength of the beam is required to resist the vertical strains, and, of course, the deflection would be diminished accordingly.

Safe Uniform Load. The column in Table XX headed "Transverse Value," gives the safe uniform load, in pounds, if divided by the span in feet, for beams supported sideways. Of course the result should correspond with Table XV, except that the uniform load will be expressed in pounds here, while it is expressed in tons of 2000 pounds each in that table. For Tables XXI, XXII, XXIII, XXIV and XXV the use of the "Transverse Value" is similar, and as more fully explained in Table XIX.

CHAPTER VII.

Graphical Analysis of Transverse Strains.

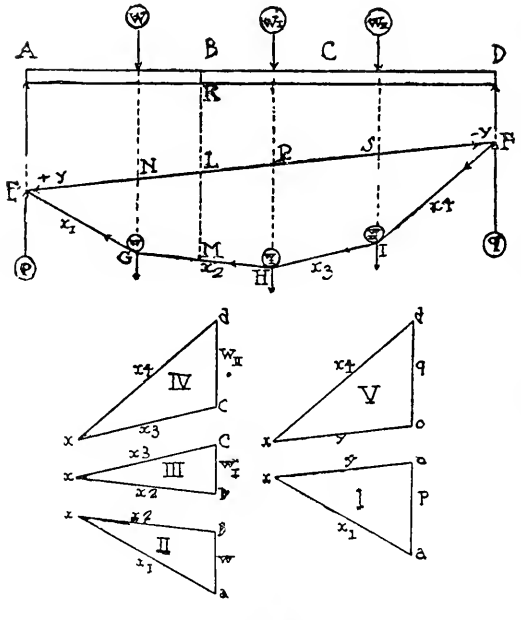


Fig. 149.

ALL the different calculations to ascertain the amounts of bending-moments, the required moments of resistance and inertia, the amounts of reactions, vertical shearing on beam, deflections, etc., can be done graphically, as well as arithmetically. In cases of complicated loads, or where it is desired to economize by reducing size of flanges, the graphical method is to be preferred, but in cases of uniform loads, or where there are but one or two concentrated loads,

the arithmetical method will probably save time. As a check, however, in important calculations, both methods might be used to advantage.

Basis of Graphical Method. If we have three concentrated loads w, w_1, and w_{11} on a beam $A\ D$ (Fig. 149), as represented by the arrows, we can also represent the reactions p and q by arrows in opposite directions, and we know that the loads and reactions all counterbalance each other. The equilibrium of these forces will not be disturbed if we add at E a force $= +y$, providing that at F we add an equal force, in the same line, but in opposite direction or $= -y$.

We have now at E two forces, $+y$ and p. If we draw at any scale a triangle $a\,o\,x$ (or I) where $a\,o$ parallel and $= p$, and where $o\,x$ parallel and $= +y$, we get a force $x\,a$, which would just counterbalance them, or $a\,x$, which would be their *resultant*. That is, a force $G\,E$ thrusting against E with an amount $a\,x$ (or x_1) and parallel $a\,x$ would have the same effect on E as the two forces $+y$ and p. Continuing x_1 till it intersects the vertical neutral axis through load w at G, we obtain the resultant x_2 of the two forces acting at G, namely x_1 and w (see triangle $b\,a\,x$ or II). Similarly we get resultant x_3 at H, of load w_1 and x_2, (see triangle $c\,b\,x$ or III); also resultant x_4 at I of load w_{11} and x_3 (see triangle $d\,c\,x$ or IV); and finally resultant $+y$, at F of reaction of q and x_4 (see triangle $o\,d\,x$ or V). As this resultant is $+y$ it must, of course, be resisted by a force $-y$ that the whole may remain in equilibrium. By comparing the triangles I, II, III, IV and V, we see that they might all have been drawn in one figure (Fig. 150) for $q + p = w_{11} + w_1 + w$, therefore:

$d\,o + o\,a = d\,c + c\,b + b\,a$,

further both V and IV contain $d\,x = x_4$

" " V " I " $o\,x = y$

" " II " I " $a\,x = x_1$

" " II " III " $b\,x = x_2$

" " III " IV " $c\,x = x_3$

We know further that the respective lines are parallel with each other.

In Fig. 150 then, we have $d\,c = w_{11}$

$c\,b = w_1$

$b\,a = w$

$a\,o = p$ and

$o\,d = q$

The distance $x\,y$ of *pole* x from *load line* $d\,a$ being arbitrary, and the position of pole x the same. The figure $E\,G\,H\,I\,F\,E$ (Fig 149) has many valuable qualities. If at any point K of beam we draw a vertical line $K\,L\,M$, then $L\,M$ will represent (as compared with the other vertical lines) the proportionate amount of bending moment at K. If we measure $L\,M$ in parts of the length of $A\,D$ and measure $x\,y$ (the distance of pole, Fig. 150) in units of the load line $d\,a$, then will the product of $L\,M$ and $x\,y$ represent the actual bending moment at K. That is, if we measure $L\,M$ in inches and — (having laid out $d\,c$, $c\,b$, etc., in pounds) — measure $x\,y$ in pounds, the bending moment at K will be $= x\,y.\ L\,M$ (in pounds-inch.) Similarly at w the bending moment would be

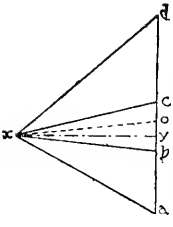


Fig. 150.

$= x\,y.\ N\,G$ (in pounds-inch.)

and at w_1 it would be $= x\,y.\ R\,H$ " " "

and at w_{11} it would be $= x\,y.\ S\,I$ " " "

measuring, in all cases, $x\,y$ in pounds and $N\,G$, $R\,H$ and $S\,I$ in inches.

Average Strain on Extreme Fibres. The area of $E\,G\,H\,I\,F\,E$, divided by the length of span in inches will give the average strain for the entire length on extreme top or bottom fibres of beam, providing the beam is of uniform cross-section throughout. The area should be figured by measuring all horizontal dimensions in inches, and all vertical dimensions in parts of the longest vertical ($R\,H$ in our case), this longest vertical being considered $= \left(\frac{c}{f}\right)$ for top, or $\left(\frac{t}{f}\right)$ for bottom fibres, or where these are practically equal $= \left(\frac{k}{f}\right)$.

The greatest bending moment on the beam will occur at the point where the longest vertical can be drawn through the figure. From this figure can also be found the shearing strains and deflection of beam, as we shall see later.

Distance of Pole. If now instead of selecting arbitrarily the distance $x\,y$ of the pole from load line $d\,a$ (Fig. 150) we had made this distance equal the safe modulus of rupture of the material, or $x\,y = \left(\frac{k}{f}\right)$ — measuring $x\,y$ in pounds at same scale as the load line $d\,a$ — it stands to reason that any vertical through the Figure

E G H I F E (Fig. 149) measured in inches, will represent the required moment of resistance, for if $L\,M.\,x\,y = m$, we know from

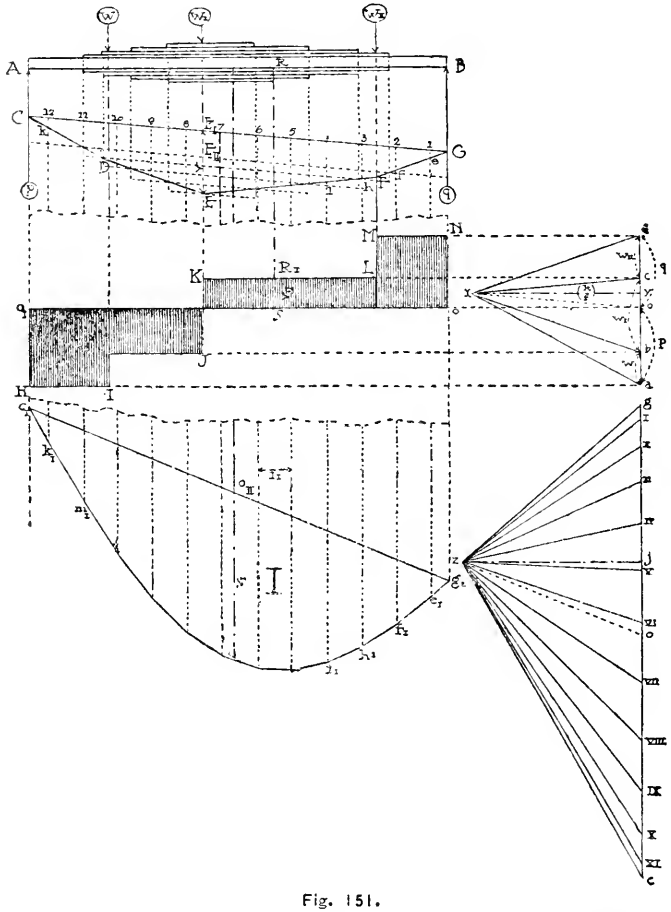


Fig. 151.

Formula (18), that $m = r.\left(\frac{k}{f}\right)$ and as we made $x\,y = \left(\frac{k}{f}\right)$, we have, inserting values in above:

$$L\,M.\left(\frac{k}{f}\right) = r.\left(\frac{k}{f}\right) \text{ or}$$

$$L\,M = r$$

Having thus shown the basis of the graphical method of analyzing transverse strains, we will now give the actual method without wasting further space on proofs.

Several Concentrated Loads. If there are three loads w, w_1 and w_{11} on a beam $A\ B$ (Fig. 151) we proceed as follows: at any convenient scale — to be known as the pounds-scale — lay off in pounds, $d\ c = w_{11}$; also $c\ b = w_1$ and $b\ a = w$. Let $A\ B = l$ measured in inches — this scale being called the inch-scale. Now select pole x at random, **Strain Diagram.** but at a distance (measured with pounds-scale) $x\ y = \left(\frac{k}{f}\right)$ = the safe modulus of rupture of the material. Draw $x\ d$, $x\ c$, $x\ b$ and $x\ a$. Now begin at any point G of reaction q, draw $G\ F$ parallel $d\ x$, till it intersects vertical w_{11} at F: then from F draw $F\ E$ parallel $c\ x$ to vertical w_1: then draw $E\ D$ parallel $b\ x$ to vertical w; and then $D\ C$ parallel $a\ x$ to reaction p. From C draw $C\ G$, and through x draw $x\ o$ parallel $C\ G$.

Reactions. We now have the following results:

$o\ d$ = reaction q (measured with pounds-scale.)

$a\ o$ = " p " " " "

any *vertical* through figure $C\ D\ E\ F\ G\ C$, (measured with inch-scale) gives the amount of r = required moment of resistance in inches, at point of beam where vertical is measured. The longest vertical passes through the point of greatest bending-moment in beam. Multiply any vertical (in inches) with $x\ y$ (in pounds) to obtain amount of bending-moment at point of beam through which vertical passes.

Moment of Resistance. or we should have: $r = v$ (92)

Where r = the required moment of resistance, in inches, at any point of beam, provided pole distance $x\ y = \left(\frac{k}{f}\right)$.

Where v = the length (measured with inch-scale) of the vertical through upper figure $C\ D\ E\ F\ G\ C$ at point of beam for which r is sought.

And further:

Bending-moment. $m = v.\ x\ y$ (93)

Where m = the bending moment at any point of beam in pounds-inch.

Where v = the same value as in Formula (92)

Where $x\ y$ = the length, (measured with pound-scale) of distance of pole x from load line, in upper strain diagram $x\ a\ d$.

If now we draw horizontal lines through d, c, b and a; and through o the horizontal line for horizontal axis; and continue these lines until they intersect their respective load verticals w_{11}, w_1 and w, the shaded figure $O_1\ H\ I\ J\ K\ L\ M\ N\ O\ O_1$ will give the vertical shearing strain along beam. Any vertical (as $R_1\ S$) drawn through this figure to horizontal axis and measured with pounds-scale, gives the amount of vertical shearing at the point of beam (R) through which vertical is drawn. Or,

Vertical Cross-shearing.

$$s = v_{11} \qquad (94)$$

Where s = the amount of vertical shearing strain in pounds, at any point of beam.

Where v_{11} = the length (measured with pounds-scale) of vertical through figure $O_1\ H\ I\ J\ K\ L\ M\ N\ O\ O_1$ dropped from point of beam for which strain s is sought.

We now divide $G\ C$ into any number of *equal* parts — say twelve in our case — and begin with a half part, or

G to 1 = 12 to $C = \frac{1}{24}$. $G\ C$; also

1 to 2 = 2 to 3 = 3 to 4 = 4 to 5, etc. $= \frac{1}{12}$. $G\ C$

and make the new lower load line $g\ c$ with inch-scale so that

Deflection Diagram.

g to I = length of vertical 1 e

further I to II = length of vertical 2 f

" II " III = " " " 3 h

" III " IV = " " " 4 i, etc. unti.

" XI " c = " " " 12 k

Now select arbitrarily a pole z at any distance $z\ j$ from load line $g\ c$.

Now draw below the beam where convenient (say I. Fig. 151) beginning at g_1, the line $g_1\ e_1$ parallel $g\ z$ till it intersects the prolongation of 1 e (from above) at e_1; then draw $e_1 f_1$ parallel I z till it intersects vertical 2 f at f_1; and similarly draw $f_1\ h_1$ parallel II z; also $h_1\ i_1$ parallel III z, etc., to $m_1\ k_1$ parallel XI z and finally $k_1\ c_1$ parallel $c\ z$. The more parts (l_1) we divide the beam into, the nearer will this line $g_1\ e_1 f_1\ m_1\ k_1\ c_1$ approach a curve. The real line to measure deflections would be a curve with the above lines as tangents to it; we need not, however, bother to draw this curve for practical work. Now draw $c_1\ g_1$ and parallel thereto $z\ o$. Divide $g_1\ c_1$ at o_{11} so that[1]: $g_1\ o_{11} : c_1\ o_{11} = c\ o : g\ o$, then will o_{11} be the point of greatest deflection along beam. This will be further proven by

[1] Note that the division of the line $g_1\ o_{11}\ c_1$ is the reverse of the division of the line $g\ o\ c$.

the fact that the greatest vertical (in lower figure I) will pass through

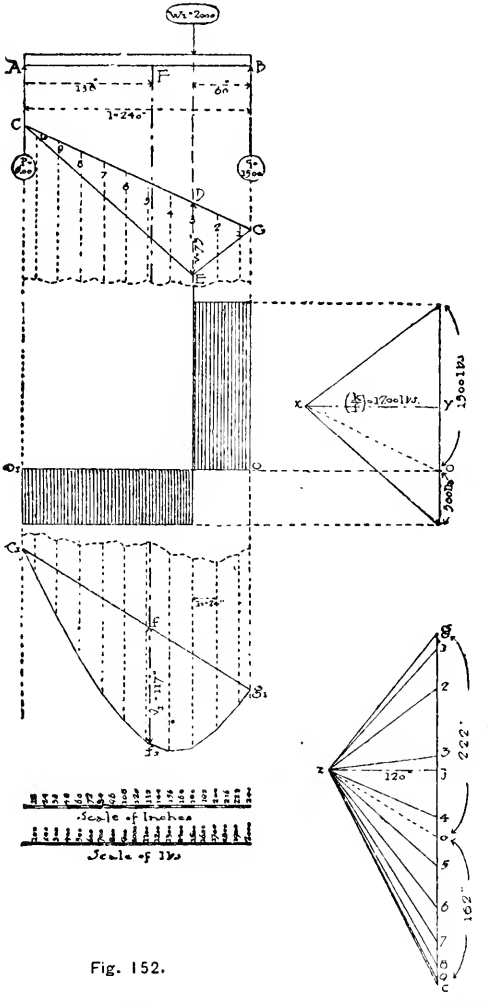


Fig. 152.

o_n, if the real curve were drawn. The figure $g, e, f, h, i, m, k, c, g,$ will measure the amount of deflection of beam at all points of

beam. The deflection at any point of beam being proportionate to length of its vertical through lower figure I. The *amount* of this deflection will be

Amount of Deflection, Definite Pole Distance.

$$\delta = \frac{v_1 . l_1 . zj . \left(\frac{k}{f}\right)}{e . i .} \qquad (95)$$

Where δ = the deflection, in inches, at any point of beam, if pole distance of upper strain diagram $(x\ y) = \left(\frac{k}{f}\right)$.

Where v_1 = the length of vertical, in inches, dropped from said point through lower figure I (see Fig. 151)

Where l_1 = the length, in inches, of each equal part 1 to 2, 2 to 3, 3 to 4, etc., into which beam was divided, [in our case $l_1 = \frac{1}{12} l$.]

Where i = the moment of inertia, of cross-section at said point, in inches.

Where zj = the distance (measured with inch-scale) of pole z from load line in lower strain diagram.

Where $\left(\frac{k}{f}\right)$ = the safe modulus of rupture, per square-inch, of the material.

Where e = the modulus of elasticity, in pounds-inch, of the material.

If we were to so proportion the beam that the moment of resistance at each point would exactly equal the required moment of resistance as found above, we should have:[1]

Deflection varying Cross-section.

$$\delta = \frac{v_1 . l_1 . zj . \left(\frac{k}{f}\right)}{v . \frac{d}{2} . e} \qquad (96)$$

Where $\delta, v_1, zj, \left(\frac{k}{f}\right)$, e and l same value as in Formula (95).

Where v = length of corresponding vertical in upper figure $C\ D\ G\ E\ C$, (to vertical v_1 of lower Fig. I) to be measured in inches.

Where $\frac{d}{2}$ = one-half the total depth of beam, in inches. Had we not made $x\ y = \left(\frac{k}{f}\right)$, we should have

Deflection Pole Distance arbitrary.

$$\delta = \frac{v_1 . l_1 . zj . x\ y .}{e . i .} \qquad (97)$$

[1] This would be the greatest possible deflection. If the beam were not so proportioned, but of uniform cross-section throughout, the deflection would be less.

Where δ, v_1, l_1, e, z j, and i same value as in Formula (95).

Where $x\ y =$ the length of pole distance from load line in upper strain diagram, measured in pounds.

The same formulæ and methods could be applied to cantilevers, but for these the arithmetical calculations are so very simple that it would be taking unnecessary trouble.

A few practical examples will make all of the foregoing more clear.

Example I.

Single concentrated Load. *A Georgia pine girder A B of 20-foot span carries a load w_1 of 2000 pounds 5′ 0″ from right reaction B. What size should the girder be?*

We draw (Figure 152) $A\ B = 240''$ at inch-scale, and locate w_1 at 60″ to the left of B. Now draw a vertical line $b\ a = 2000$ pounds at pounds-scale. Select point x anywhere, but distant $x\ y = 1200$ pounds. (1200 pounds being $= \left(\frac{k}{f}\right)$ or the safe modulus of rupture, per square-inch, of Georgia pine). Draw $x\ b$ and $x\ a$. Draw verticals through A, w_1 and B. On vertical A begin at any point C, draw $C\ E$ parallel $x\ a$, till it intersects verticals w_1 at E; then draw $E\ G$ till it intersects vertical B at G. Draw $G\ C$ and $o\ x$ parallel to $G\ C$. We scale $o\ b$, it scales 1500 pounds, so this, is the reaction at B. We scale $a\ o$, it scales 500 pounds and this is the reaction at A. The longest vertical through $C\ E\ G$ is vertical w_1, therefore greatest bending-moment is at w_1 which we know is the case. We scale $E\ D$ at inch-scale, it scales 75 inches, therefore the (greatest) *required* moment of resistance will be at w_1 and will be Formula (92).

$$r = 75.$$

From Table I, section No. 2, we know for rectangular beams,

$$r = \frac{b.\ d^2}{6}, \text{ therefore:}$$

$$\frac{b.\ d^2}{6} = 75, \text{ or}$$

$$b.\ d^2 = 450.$$

We will suppose the girder is not braced sideways, and needs to be pretty broad; let us try $b = 5''$, we have then:

$$5.\ d^2 = 450 \text{ or}$$

$$d^2 = \frac{450}{5} = 90 \text{ and}$$

$$d = 9,\ 5'' \text{ or the girder}$$

would have to be 5″ x 9½″ or say 5″ x 10″. The bending-moment at

w_1 is, of course, Formula (93) $= E\ D.\ x\ y = 75.1200 = 90000$ (pounds-inch).

Had we calculated arithmetically, we should have had, Formulæ (14) and (15):

reaction $A = \frac{60}{240}.\ 2000 = 500$ pounds.

" $B = \frac{180}{240}.\ 2000 = 1500$ pounds.

Bending moment at w_1 would be (right side) Formulæ (23) and (24). $m_{w1} = 1500.60 - 0.2000 = 90000$ (pounds-inch) or check (left) side $m_{w1} = 500.180 - 0.2000 = 90000$ (pounds-inch.) Therefore required moment of resistance, Formula (18)

$$r = \frac{90000}{1200} = 75.$$

or same result as graphically.

By drawing the horizontals from b between verticals B and w_1; from a between verticals A and w_1; and from o between verticals A and B we get the etched figure for measuring vertical shearing strains. We see at a glance that the shearing to the right of load is equal to the right reaction, and is constant at all points of right side of beam; while on the left side of load it is equal to the left reaction, and is constant at all points of the left side of beam. And this we know is the case. We need not bother with shearing, however, for we can readily see there is no danger. For even immediately to the right of the load, the weakest point in our case, we know that one-half of the fibres of cross-section are not strained at all, or we should have one-half of area or $\frac{5.10}{2} = 25$ square-inches to resist 1500 pounds of shearing, or $\frac{1500}{25} = 60$ pounds per square-inch, while the safe resistance, per square-inch, of Georgia pine to shearing across the grain is (Table IV) $\left(\frac{g}{f}\right) = 570$ pounds.

There is, however, some danger of excessive deflection; we draw, therefore, the figure $c_1 f_1 g_1$ by dividing the beam into ten equal parts, beginning and ending with half parts at the reaction, (each whole part being 24″ long, or $l_1 = \frac{240}{10} = 24''$)

We draw the verticals through these parts and get their lengths through figure $C\ E\ G$. These lengths we carry down in their proper succession on the load line $g\ j\ c$ of the lower strain diagram, begin-

ning at the top with the right vertical 1, putting immediately under this the length of second vertical 2, then 3 and so on till $g\,c =$ sum of lengths of all ten verticals through $C\,E\,G$. We now select z at random (in our case 120 inches from load line or $z\,j = 120''$). We now draw lines from z to g I, II, III, etc., to c. Construct figure $g_1 f_1 c_1$ by beginning at g_1 drawing line parallel to $z\,g$ until it intersects

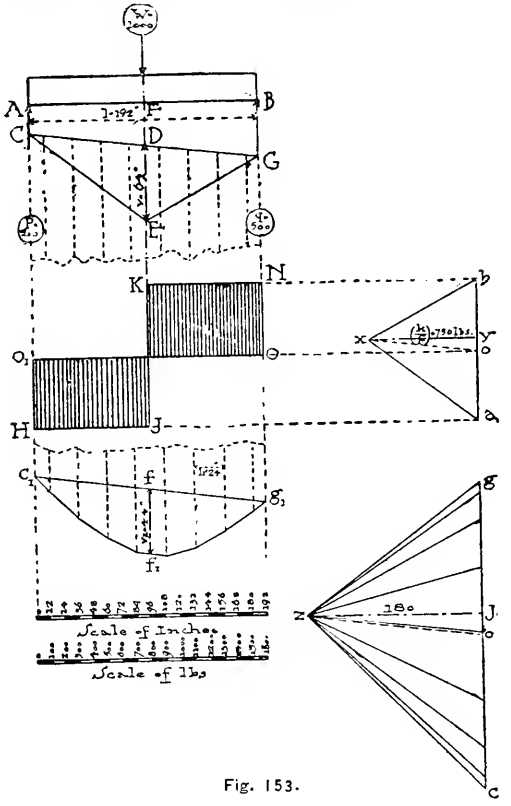


Fig. 153.

prolongation of first vertical 1; then line parallel to z I till it intersects prolongation of second vertical 2, etc. We now draw $z\,o$ parallel $c_1 g_1$. We scale $g\,o$ and find it scales 222″, also $c\,o$ which scales 162″; we divide $c_1 g_1$ at f, so that

$$c_1 f : f g_1 = 222 : 162.$$

Carrying vertical ff_1 through figure we find it scales $(v_1) = 117''$ continuing ff_1 up to beam it gives us point F as the point of greatest deflection, we find $A\,F$ scales $138''$. Had we used Formula (43) we should have located F at a distance from A or $A\ F = \sqrt{\frac{240^2 - 60^2}{3}}$ $= 134,\ 17''$. So that we have a sufficiently accurate result.

For the amount of deflection at F we use Formula (95); we know that (Table I, Section No. 2) $i = \frac{b.d^3}{12} = \frac{5.10^3}{12} = 417$, further for Georgia pine $\left(\frac{k}{f}\right) = 1200$ pounds.

$e = 1200000$ (inch-pounds.)
$l_1 = 24''$
$v_1 = ff_1 = 117''$
$zj = 120''$, therefore:

$$\delta = \frac{117.24.120.1200}{1200000.\quad 417}$$
$$= 0{,}808''$$

Had we calculated the deflection by Formula (41) we should have had:
remembering that $m = 180''$ and $n = 60''$ and $l + n = 240 + 60 = 300''$

$$\delta = \frac{2000.180.60.300}{9.240.1200000.417} \cdot \sqrt{\frac{180.300}{3}}$$
$$= 0{,}803''$$

Which proves the accuracy of the graphical method.

For a beam of 20 feet span the deflection not to crack plastering should not exceed, Formula (28).

$$\delta = 20.0{,}03 = 0{,}6''$$

Therefore, if our beam supports a plastered ceiling, it must be redesigned to be *stiffer*. Either made deeper, in which case it can be thinner, if braced sideways, or it can be thickened sufficiently to reduce the deflection, see Formula (31).

Example II.

Single centre load. *A hemlock girder A B (Fig. 153) of 16-foot span, carries a centre load w of 1000 pounds. What size should the girder be?*

We make $A\ B = 192''$ at inch scale; locate w at its centre F; make $b\,a$ at any scale — (pounds-scale) — equal 1000 pounds. Select pole x distant, $x\ y = 750$ pounds, from load line $b\ a$, (as 750

pounds $= \left(\frac{k}{f}\right)$ the safe modulus of rupture per square inch of hemlock). Draw $x\,b$ and $x\,a$. Begin at G, draw $G\,E$ parallel $b\,x$ to vertical through load, and then draw $E\,C$ parallel $a\,x$. Draw $C\,G$ and then $x\,o$ parallel $C\,G$, we find that o bisects $b\,a$ or $a\,o = o\,b = 500$ pounds. Each reaction is therefore one-half of the load; this we know is the case. Greatest line through $C\,G\,E$ we find is at $D\,E$, so that greatest bending-moment is at load; this we know is the case. $D\,E$ scales 64″ at inch-scale, therefore the required moment of resistance for the beam is, Formula (92):

$$r = 64.$$

and the greatest bending-moment at load, Formula (93):

$$m_w = 64.\ x\,y = 64.750$$
$$= 48000$$

Had we calculated arithmetically we should have obtained the same results, for Formula (22)

$$m_w = \frac{1000.192}{4} = 48000$$

and Formula (18):

$$r = \frac{48000}{750} = 64$$

Now from Table I, Section No. 2, we know that for rectangular sections:

$$r = \frac{b.d^2}{6} \text{ or}$$
$$b.d^2 = 64.6 = 384.$$

If we assume the beam as 4″ thick, we have then:

$$4.d^2 = 384.$$
$$d^2 = \frac{384}{4} = 96 \text{ or}$$
$$d = \sqrt{96} = 9{,}8'' \text{ or we will make the beam } 4'' \times 10''.$$

We draw the figure $O,\ H\,J\,K\,N\,O$ for shearing and find it is constant throughout the whole length of beam and equal to length $O,\ H$ or $N\ O$ measured at pounds scale, or 500 pounds. This is so small we need not bother with it.

To obtain the deflection diagram we divide $G\ C$ into eight equal parts, each part $l_1 = \frac{192}{8} = 24''$ and begin at each end with half parts, drawing the eight verticals through $C\ E\ G$.

We lay off their exact lengths in proper succession on the lower load line $g\ c$, beginning at the top with the right vertical. Select

pole z at random, in this case distant from load line $z\,j = 180''$. We now draw the figure $c_1\,g_1\,f_1$ and find greatest deflection is at its centre $f\,f_1$; for $z\,o$ parallel $c_1\,g_1$ bisects $g\,c$. We scale $f\,f_1$ at inch scale

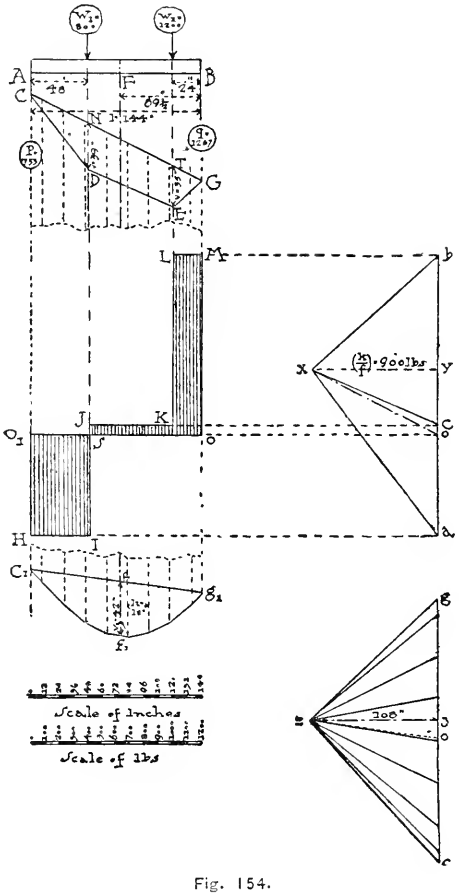


Fig. 154.

$= 44''$, therefore greatest deflection of beam at centre, Formula (95), remembering that (Table I, Section No. 2) $i = \frac{4.10^3}{12} = 333$ and

(Table IV) $e = 800000$

$$\delta = \frac{44.\ 24.\ 180.\ 750}{800000.\ 333} = 0{,}535''$$

Had we calculated the deflection arithmetically from Formula (40) we should have had:

$$\delta = \frac{1}{48} \cdot \frac{1000.\ 192^3}{800000.\ 333} = 0{,}548''$$

or practically the same result.

If the beam supported plastered work the deflection should not exceed, Formula (28)

$$\delta = 16.\ 0{,}03 = 0{,}48''$$

Still, unless we were very particular, the beam could be passed as practically stiff enough.

Example III.

Two concentrated loads. *A white pine beam A B Fig. 154, of 12-foot span carries two loads, one $w_{\text{I}} = 800$ pounds, four feet from left support, the other $w_{\text{II}} = 1200$ pounds, two feet from right support. What size should the beam be?*

Make $A\ B$ at inch scale $= 144$ inches, locate w_{I} so that $A\ w_{\text{I}} = 48''$, and w_{II} so that $B\ w_{\text{II}} = 24''$. At any (pounds scale) make $b\ c = 1200$ pounds and $c\ a = 800$ pounds. Select pole x distant from $b\ a$; $x\ y = 900$ pounds, the safe modulus of rupture per square inch of white pine; draw $x\ b$, $x\ c$ and $x\ a$. Construct $C\ D\ E\ G$ parallel to these lines. Draw $C\ G$, and parallel to same $x\ o$, then will $a\ o = 733$ pounds be reaction at A, and $o\ b = 1267$ pounds be reaction at B. We scale vertical $D\ N$ at $w_{\text{I}} = 39''$ and $T\ E$ at $w_{\text{II}} = 35''$, therefore greatest bending-moment is at w_{I} and Formula (93)

$$m_{w_{\text{I}}} = 39.\ 900 = 35100$$

Further, the required moment of resistance at w_{I} Formula (92) will be:

$$r = D\ N = 39.$$

Now from Table I, Section No. 2,

$$r = \frac{b.d^2}{6}, \text{ or}$$

$$b.d^2 = 6.39 = 234$$

Now if $b = 3''$ we should have

$$d^2 = \frac{234}{3} = 78 \text{ and}$$

$$d = \sqrt{78} = \text{say } 9'', \text{ or the beam would need to be } 3'' \times 9''.$$

We should have obtained practically the same results arithmetically, for: Formulæ (16) and (17):

Reaction at $A = \frac{800.96}{144} + \frac{1200.24}{144} = 733$

Reaction at $B = \frac{800.48}{144} + \frac{1200.120}{144} = 1267$

check: $A + B = w_{\text{I}} + w_{\text{II}} = 800 + 1200 = 2000$ pounds and $733 + 1267 = 2000$ pounds.

Beginning at B we have to pass over load w_{II} (1200 pounds) and on to w_{I}, before passing amount of reaction B (1267 pounds) therefore greatest bending-moment at w_{I}. We know from Formula (24) it would be:

$$m_{w_{\text{I}}} = 1267.96 - 72.1200 = 35232$$

and check from Formula (23)

$$m_{w_{\text{I}}} = 733.48 - 0.800 = 35184$$

being near enough for practical purposes. From Formula (18) we should have had:

$$r = \frac{35184}{900} = 39{,}09$$

We now draw the shearing diagram $O_{\text{I}}\ H\ I\ J\ K\ L\ M\ O$, as shown in Figure 154, and find the amount of shearing

from A to $w_{\text{I}} = O_{\text{I}}\ H =$ 733 pounds,
from w_{I} to $w_{\text{II}} = J\ S =$ 67 pounds,
from w_{II} to $B = M\ O = 1267$ pounds.

We can overlook it, for even at the weakest point of beam for resisting cross-shearing we have half the area, or $\frac{3.9}{2} = 13\frac{1}{2}$ square inches.

White pine will safely resist 250 pounds per square inch in cross-shearing (Table IV) or the beam would resist.

$13\frac{1}{2} . 250 = 3375$ pounds at its weakest point for cross-shearing, (viz.: at w_{I}) and twice as much at the reactions.

To find the deflection we divide $G\ C$ into eight equal parts, beginning with half parts (or $l_{\text{I}} = \frac{144}{8} = 18''$) and draw the verticals through $C\ D\ E\ G$. We now make the lower load line $g\ c$ equal the sum of these verticals, beginning at the top with the right vertical.

Select z distant from $g\ c$ (the load line) $z\ j = 108''$. Draw $z\ g$, $z\ c$, etc., and construct $g_{\text{I}}\ c_{\text{I}}\ f_{\text{I}}$ as before.

We draw $z\ o$ parallel $c_{\text{I}}\ g_{\text{I}}$. Now $g\ o$ measures 116 inches and $o\ c$

108 inches, therefore divide $c_1 g_1$ at f so that:

$$c_1 f : f g_1 = 116 : 108$$

Carrying the vertical $f f_1$ up to point F of beam, we find the point of greatest deflection F, where

$$B F = 69\tfrac{1}{2}'' \text{ and } A F = 74\tfrac{1}{2}''$$

We find $f f_1$ scales 42″, remembering that (Table I, Section No. 2) $i = \frac{3.9^3}{12} = 182$, and that for white pine Table IV $e = 850000$ pounds we have Formula (95):

$$\delta = \frac{42.18.108.900}{850000 .182} = 0{,}475''$$

Had we attempted to get this result arithmetically by inserting the values in Formula (41) (and remembering that n is always the nearer support, or in our case respectively 48″ and 24″, while m respectively 96″ and 120″) we should realize the advantage of the graphical method, for:

$$\delta = \frac{800.96.48.(144+48).\sqrt{\frac{96.(144+48)}{3}} + 1200.96.48.(144+24).\sqrt{\frac{120.(144+24)}{3}}}{9.144.850000.182}$$

If we figure out the above tedious formula we should have

$$\delta = 0{,}422''$$

or practically the same result as we obtained graphically.

The safe deflection, were the beam to carry plastering, should not exceed Formula (28)

$$\delta = 12.0,03 = 0{,}36''$$

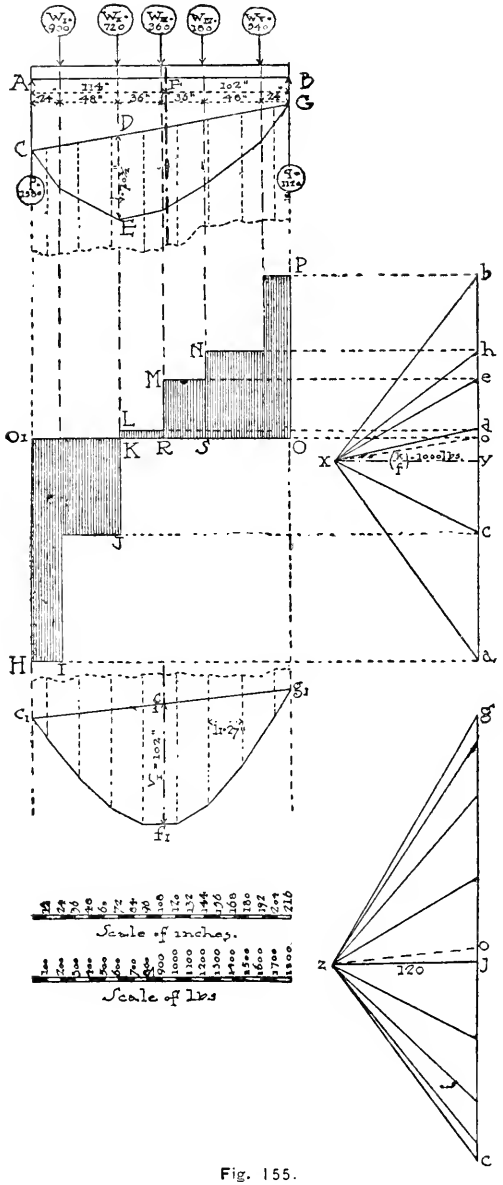
Our beam is therefore not nearly stiff enough, and we must make it thicker; or else if we wish to save material, we will make it thinner, but deeper; and then brace it sideways, see Formula (31).

Example IV.

Five Concentrated Loads. *A spruce girder A B of 18-foot span carries five loads, as shown in Figure 155. What size should the girder be?*

We draw $A B = 216''$ (inch scale); further

$b a = 2700$ pounds (sum of loads at pounds scale); make
$b h = w_{\text{v}} = 540$ pounds,
$h e = w_{\text{iv}} = 180$ pounds,
$e d = w_{\text{iii}} = 360$ pounds,
$d c = w_{\text{ii}} = 720$ pounds, and
$c a = w_{\text{i}} = 900$ pounds.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Scale of inches.
Scale of lbs

Fig. 155.

Select x distant $x\ y = 1000$ pounds from $b\ a$, (as $1000 = \left(\frac{k}{f}\right)$ for spruce, see Table IV). Draw $x\ b$, $x\ h$, $x\ c$, etc., and figure $C\ D\ G$. Draw $x\ o$ parallel $C\ G$; it divides load line as follows:

$a\ o = 1580$ pounds or reaction at A.

$o\ b = 1120$ pounds or reaction at B.

We find longest vertical through $C\ D\ G$, is at load w_{II}, therefore greatest bending-moment on beam at w_{II}; now $D\ E$ scales $70\frac{1}{2}''$, therefore Formula (93):

$$m_{w_{II}} = 70\tfrac{1}{2}.\ 1000 = 70500$$

and Formula (92)

$$r = 70,\ 5$$

From Table I, Section No. 2

$$r = \frac{b.d^2}{6} = 70,\ 5 \text{ and if } b = 5, \text{ we have}$$

$$5.\ d^2 = 6.\ 70,\ 5 \text{ or}$$

$$d^2 = 84,\ 6 \text{ and}$$

$$d = \sqrt{84,\ 6} = 9{,}2'' \text{ or say } 10'' \text{ which is the nearest size}$$

larger than 9,2", and of course wooden beams are never ordered to fractions of inches.

Had we worked arithmetically we should have had practically the same results.

From Formulæ (16) and (17) we should have had:

reaction at $A = 1580$ pounds,

reaction at $B = 1120$ pounds.

From rule for finding greatest bending-moment we should have located it at w_{II} and then had Formula (23)

$$m_{w_{II}} = 1580.\ 72 - 48.900 = 70560$$

and from Formula (18)

$$r = \frac{70560}{1000} = 70,\ 56$$

We now draw the shearing diagram $O_I\ II\ I\ J\ K\ L\ M\ N\ P\ O$ and find as follows:

Cross-shearing A to $w_I = II\ O = 1580$ pounds.
Cross-shearing w_I to $w_{II} = J\ K = 680$ pounds.
Cross-shearing w_{II} to $w_{III} = K\ L = 40$ pounds.
Cross-shearing w_{III} to $w_{IV} = M\ R = 400$ pounds.
Cross-shearing w_{IV} to $w_V = N\ S = 580$ pounds.
Cross-shearing w_V to $B = P\ O = 1120$ pounds.

We need not bother with it, therefore. For deflection we now

divide $C\,G$ again into eight equal parts, (or $l_1 = \frac{216}{8} = 27''$) beginning with half parts at C and G. We now make lower load line $g\,c =$ the sum of the eight verticals, putting the right vertical at the top from g down. We select pole z at a distance $z\,j = 120''$ from $g\,c$ and draw $z\,g$, $z\,c$, etc. We construct figure $g_1 f_1 c_1$ and draw $z\,o$ parallel to $c_1 g_1$. We now divi le $c_1 g_1$ at f, so that

$g_1 f : f c_1 = c\,o : o\,g$, carrying $f f_1$ up to beam, we have the point F, distant $102''$ from B, and $114''$ from A, which is the point of greatest deflection. We find that $f f_1$ scales $102''$, remembering that $e = 850000$ for spruce (Table IV), and that $i = \frac{5.10^3}{12} = 417$ (See Table I, Section No. 2) we have, Formula (95).

$$\delta = \frac{102.27.120.1000}{850000.417} = 0{,}93''$$

This would be too much for plastering, for if the girder supported plastering, the deflection should not exceed Formula (28)

$$\delta = 18.\ 0,\ 03 = 0{,}54''$$

We must therefore deepen the beam very materially.

We use Formula (31),

$$x = \frac{1}{b.d^3}$$

In our case it would be

$$x = \frac{1}{5.10^3} = \frac{1}{5000} = 0{,}0002$$

Supposing we were to make the beam $4'' \times 12''$, then we should have

$$x = \frac{1}{4.12^3} = 0,\ 000\,144$$

The deflection of the latter, then, would be

$\delta :\ 0,\ 93 = 0,\ 000144 :\ 0,\ 0002$ or

$$\delta = \frac{0,\ 93.\ 0.000144}{0,0002} = 0{,}67'' \text{ still too much deflection.}$$

Were we to make the beam $3'' \times 14''$, we should have:

$$x = \frac{1}{3.14^3} = 0{,}0001215$$

The corresponding deflection for this beam would be:

$\delta :\ 0,\ 93 = 0{,}0001215 :\ 0{,}0002$ or

$$\delta = \frac{0,\ 93.\ 0{,}0001215}{0{,}0002} = 0{,}565''$$

or just about what would be required in the way of stiffness.

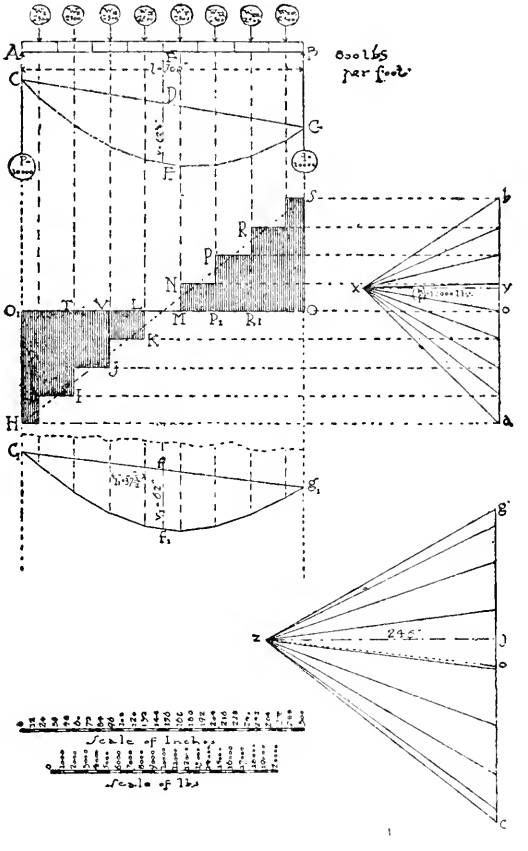


Fig. 156.

Had we used Formula (95) we should have had, remembering that now

$$i = \frac{3.14^3}{12} = 686$$

$$\delta = \frac{102.\,27.\,120.\,1000}{850000.\,686} = 0{,}568''$$

showing that we have made no mistake in applying Formula (31).

If we have any doubts as to whether a 3″ x 14″ stick is as strong as a 5″ x 10″ we use Formula (30) and have for the former

$$x = 3.14^2 = 588$$

while for the latter

$x = 5.10^2 = 500$, so that the 3″ x 14″ stick is actually much stronger, as well as much stiffer than the 5″ x 10″. It is, however, a very thin beam, and would be apt to warp or twist, unless braced sideways about every five feet of its length.

To attempt to get the deflection of the girder arithmetically would be a very tedious operation. It could be done, however, by inserting in Formula (41) the different values for n and m, remembering every time to make n the distance from each weight to the *nearer* support to respective weight, and m the distance from same weight to the further support.

Example V.

Uniform Load. *A wrought-iron beam of 25-foot span (Figure 156) carries a uniform load of 800 pounds per running foot of beam, including weight of beam. The beam is thoroughly braced sideways. What beam should be used?*

We draw $A\,B = 300''$ at inch scale, and then divide our uniform load into a number of equal sections, say eight, each

$$l_1 = \frac{300}{8} = 37\tfrac{1}{2}'' \text{ long.}$$

The total load on beam is

$$u = 25.800 = 20000 \text{ pounds.}$$

Each section therefore carries:

$$\frac{u}{8} = \frac{20000}{8} = 2500 \text{ pounds.}$$

We place our arrows w_{I}, w_{II}, etc., at the centre of each section, which will bring the end ones at $\frac{l_1}{2}$ distant from each support, so that these same verticals will answer when obtaining deflection figure.

We now make $b\,a = 20000$ pounds at pound scale, and divide it into eight equal parts, each equal $w_{\text{I}} = w_{\text{II}} = w_{\text{III}}$ etc., $= 2500$ pounds. We make $x\,y = 12000$ pounds, which is the $\left(\frac{k}{f}\right)$ for wrought-iron, see Table IV. We draw $x\,b$, $x\,a$, etc., and construct figure $C\,E\,G$, which will approach a parabola in outline. The more parts we take the nearer will it be to a parabola.

We draw $x\,o$ parallel $C\,G$ and find it bisects $b\,a$, or each reaction

is one-half the load or = 10000 pounds. This we know is the case. The longest vertical will, of course, be at the centre D of $C\ G$, or greatest bending-moment will be at the centre, this we know is the case. $D\ E$ scales (inch scale) $62\frac{1}{2}''$ which will be the *required* r or moment of resistance (Formula 92). The bending-moment at the centre will be, Formula (93).

$$m = 62\tfrac{1}{2}.\ 12000 = 750000$$

Had we used Formula (21) we should have had

$$m = \frac{20000.\ 300}{8} = 750000$$ or same result, and from Formula (18) for

$$r = \frac{750000}{12000} = 62\tfrac{1}{2}$$ also the same as before. From Table XX we find the nearest r to our required r (62,5) is 69,8 which calls for a $15'' - 150$ pounds beam; as the beam is braced sideways this will do, if sufficiently stiff.

In regard to shearing, we draw the figure $O_{\text{I}}\ II\ I\ J\ K\ L\ M\ N\ P\ R\ S\ O$ and find shearing on both sides of beam similar, increasing gradually from the centre to ends.[1]

It would be:

Cross shearing from A to w_{I} $= O_{\text{I}}\ II =$ 10000 pounds.
Cross shearing from w_{I} to w_{II} $= T\ I =$ 7500 pounds.
Cross shearing from w_{II} to w_{III} $= V\ J =$ 5000 pounds.
Cross shearing from w_{III} to w_{IV} $= L\ K =$ 2500 pounds.
Cross shearing from w_{IV} to w_{V} $= O =$ 0 pounds.
Cross shearing from w_{V} to w_{VI} $= M\ N =$ 2500 pounds.
Cross shearing from w_{VI} to w_{VII} $= P\ P_{\text{I}} =$ 5000 pounds.
Cross shearing from w_{VII} to w_{VIII} $= R\ R_{\text{I}} =$ 7500 pounds.
Cross shearing from w_{VIII} to B $= S\ O =$ 10000 pounds.

The area of web of a $15'' - 150$ pounds beam (Table XX) is 7,59 square inches; the safe resistance of wrought-iron to cross-shearing per square inch being $\left(\frac{q}{f}\right) = 8000$ pounds, we need not worry any further on that score.

To find the deflection we now make the lower load line $g\ c$ equal to the sum of the lengths of verticals w_{VIII}, w_{VII}, w_{VI}, etc., through parabola $C\ E\ G$, beginning at top g with length of right vertical w_{VIII}. We select z at random, scale $z\ j = 246''$ (inch scale), draw $z\ g$, $z\ c$, etc., and figure

[1] Had we taken more parts, the steps in shearing figure would become smaller and smaller till they would finally assume the straight line $II\ S$, which is the real outline of shearing figure.

$c_1 f_1 g_1$. We now draw $z\,o$ parallel $c_1\,g_1$ and find it bisects $g\,c$, or greatest deflection will be at centre of beam, which we know is the case. We scale $f\,f_1 = 62''$ (inch scale); find from Table XX for our $15'' - 150$ pounds beam $i = 523,5$ and from Table IV for wrought-iron $e = 27000000$, therefore, Formula (95):

$$\delta = \frac{62.37,5.246.12000}{27000000.523,5} = 0,486''$$

Had we figured arithmetically, Formula (39), we should have had

$$\delta = \frac{5.20000.300^3}{384.27000000.523,5} = 0,497''$$

or practically the same result.

The safe deflection for plastering should not exceed (28)

$$\delta = 25.0,03 = 0,75''$$

so that we are perfectly safe, providing our beam is well braced sideways.

Example VI.

Uniform and Concentrated Load. *A wrought-iron beam, braced sideways, of 30-foot span, Figure 157, carries a uniform load of 200 pounds per foot, including weight of beam. It carries also a concentrated load $w_1 = 10000$ pounds ten feet from the right-hand support. What beam should be used?*

We draw beam $A\,B = 360''$ at inch scale, we divide uniform load into, say, six equal parts, each 5 feet long, or $l_1 = 60''$. The total uniform load will be $u = 30.200 = 6000$ pounds, therefore each part $\frac{u}{6} = \frac{6000}{6} = 1000$ pounds. We draw arrows at the centre of each uniform part, so that the end arrows will be one-half part from supports. These arrows will therefore answer for our verticals, when drawing deflection figure.

At $120''$ from right hand support we locate the load $w_1 = 10000$ pounds.

We now make load line $b\,a = 16000$ pounds the total load and divide it, so that

$b\,l = w_{vii} =$ 1000 pounds.
$l\,h = w_{vi} =$ 1000 pounds.
$h\,f = w_{i} =$ 10000 pounds.
$f\,e = w_{v} =$ 1000 pounds.
$e\,d = w_{iv} =$ 1000 pounds.
$d\,c = w_{iii} =$ 1000 pounds.
$c\,a = w_{ii} =$ 1000 pounds.

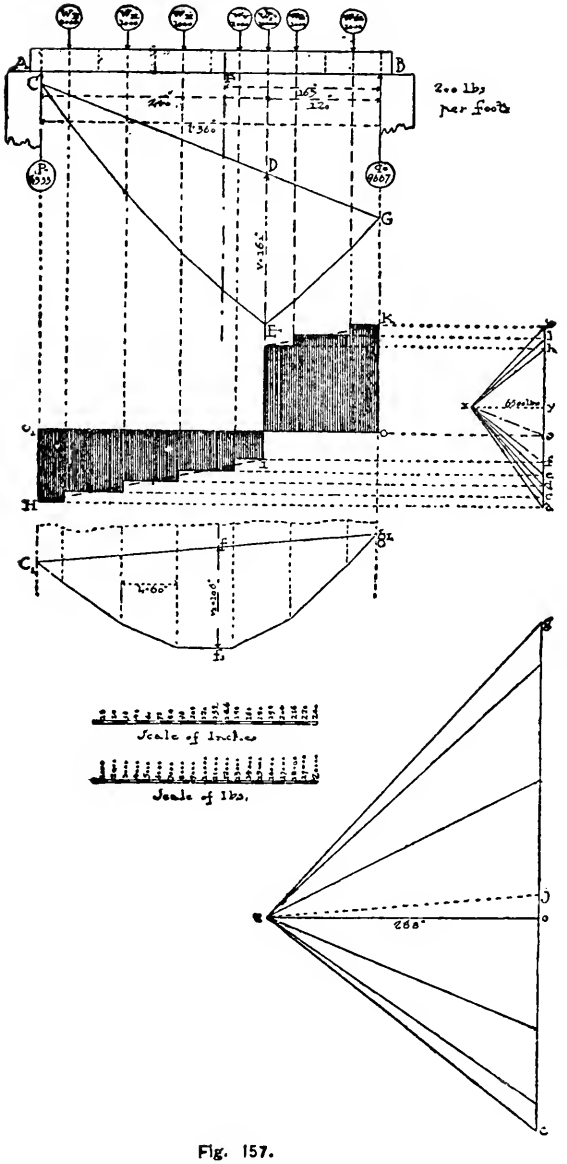


Fig. 157.

Select pole x distant from load line at random (for the sake of illustration, though it would be better to make $x\,y = \left(\frac{k}{f}\right) = 12000$ pounds.) We find $x\,y$ scales 6500 pounds. We now draw $x\,b, x\,l, x\,h, x\,f$, etc. And construct figure $C\,E\,G$. Draw $x\,o$ parallel $C\,G$ and we find $a\,o$ (or reaction A) scales $=$ 6333 pounds, and $o\,b$ (or reaction B) scales $=$ 9667 pounds.

The longest vertical is $D\,E = 161''$ (inch scale) therefore greatest bending-moment is at w_{I} and from Formula (93)

$$m_{w\text{I}} = 161.6500 = 1046500$$

For the *required* moment of resistance we have from Formula (18)

$$r = \frac{1046500}{12000} = 87{,}2$$

The *cheapest* or most economical nearest section we find — to this required r (87,2) is the $20'' - 200$ pounds beam of which the moment of resistance is $r = 123{,}8$.

Had we combined the formulæ for uniform and concentrated loads and worked out the problem arithmetically it would have been tedious, but we should have had similar results.

We can safely overlook shearing, but note that the real shearing figure would not be the shaded figure, but dotted figure $O_{\text{I}}\,H\,I\,J\,K\,O$

For finding the deflection we now draw lower load line $g\,c =$ the sum of the verticals through $C\,E\,G$, beginning at top with length of w_{VII}, then w_{VI}, w_{V}, w_{IV}, w_{III}, and w_{II} in their order. We take no notice of vertical w_{I} as it does not fall in one of the even divisions of $C\,G$ or $A\,B$ into lengths l_{I}. We select pole z distant $z\,j = 288''$ from load line, draw $z\,g$, $z\,c$, etc., and then figure $c_{\text{I}}\,f_{\text{I}}\,g_{\text{I}}$. We now draw $z\,o$ parallel $c_{\text{I}}\,g_{\text{I}}$ it divides $g\,c$, so that $g\,o = 295''$ and $o\,c = 245''$, we divide $c_{\text{I}}\,g_{\text{I}}$ in same proportion at f, and carry this up to F at beam, which is the point of greatest deflection of beam, and is distant $163''$ from B, and $197''$ from A. We scale $f\,f_{\text{I}} = 106''$ (inch scale) and have from Formula (97)

$$\delta = \frac{106.60.288.6500}{27000000.1238} = 0{,}357''$$

1238 being $= i$, the moment of inertia of beam as found in Table XX. The beam is therefore amply stiff even to carry plastering.

Irregular Cross-sections. The graphical method lends itself very readily to finding centres of gravity and neutral axes, as explained in the chapter on arches, and also for finding the moments of inertia of difficult cross-sections.

If we have an irregular figure *A B C D E* (Figure 158) we divide it into simple parts I, II, III and IV. We find the centres of gravity g_{I}, g_{II}, g_{III} and g_{IV} of each part and draw their respective horizontal neutral axes through these. Anywhere's make a line $a\,e$ = area of whole figure and divide it, so that:

To find Neutral Axis.

$a\,b$ = area of I
$b\,c$ = area of II
$c\,d$ = area of III and
$d\,e$ = area of IV.

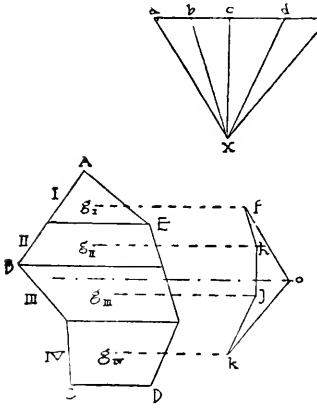


Fig. 158.

Select pole x at random, draw $x\,a$, $x\,b$, $x\,c$, $x\,d$, and $x\,e$.

From any point of horizontal g_{I} draw $f\,h$ parallel $b\,x$ till it insects horizontal g_{II}; then draw $h\,j$ parallel $c\,x$ to horizontal g_{III}; then $j\,k$ parallel $d\,x$ to last horizontal, and finally $k\,o$ parallel $x\,e$; and $f\,o$ parallel $a\,x$ till they intersect at o. A horizontal through o is the main neutral axis of the whole.[1] If we multiply the area of the figure $f\,o\,k\,j\,h$ by the area of the figure *A B C D E* (both in square inches) we have the value of moment of inertia i of *A B C D E* in inches, around its horizontal neutral axis o.

To find Moment of Inertia.

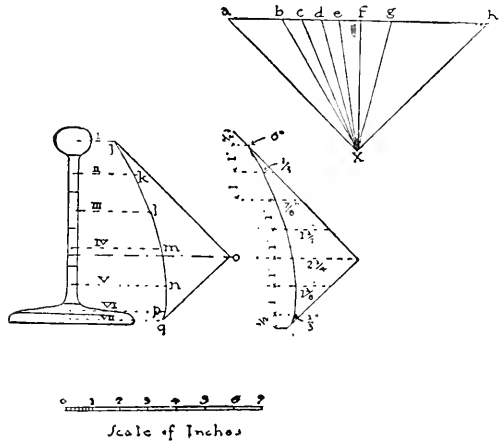


Fig. 159.

[1] The point of intersection of this line with a main neutral axis, found similarly, in any other direction, would be the centre of gravity of the whole figure.

To find area. A simple way of obtaining the area of the figure $f\,o\,k$ would be to draw horizontal lines through it at *equal* distances beginning with half distances at *top* and *bottom*, and to multiply the sum of these horizontals in length by the distance apart of any two horizontals, all measurements in inches. This will approximate quite closely both the area and moment of inertia. Of course the more parts we take in all of the processes, the closer will be our result.

A practical example will more fully illustrate the above.

Example VII.

Rolled Deck-beam. *Find horizontal neutral axis and the corresponding moment of inertia of a 7″ — 55 pounds per yard deck beam, resting on its flat flange* (*Figure* 159).

We will take the roll as one part, divide the web into four equal parts, the flange into two parts, one the base which will be practically rectangular, and its upper part which will be practically triangular. The whole area we know is for wrought-iron:

$$a = \frac{55}{10} = 5,5 \text{ square inches.}$$

The bottom rectangular part of flange will be

$$a_{\text{VII}} = 4\tfrac{1}{2} \cdot \tfrac{3}{8} = 1,7 \text{ square inches}$$

next triangular part

$$a_{\text{VI}} = \frac{4\frac{1}{2} \cdot \frac{3}{8}}{2} = 0,9$$

The web parts

$$a_{\text{II}} = a_{\text{III}} = a_{\text{IV}} = a_{\text{V}} = \frac{\frac{5}{16} \cdot 5\frac{1}{2}}{4} = 0,4 \text{ square inches each.}$$

Leaving for the roll at top $a_{\text{I}} = 1,3$

We now make the horizontal line $a\,h = 5,5''$ and divide it, so that

$a\,b = 1,3$ inches

$b\,c = c\,d = d\,e = e\,f = 0,4$ inches

$f\,g = 0,9$ inches and

$g\,h = 1,7$ inches.

Select x at random and draw $x\,a$, $x\,b$, $x\,c$, etc.

Draw the horizontal neutral axes I, II, III, etc., through their respective parts. Begin anywhere on I and draw $j\,k$ parallel $b\,x$ to line II; then $k\,l$ parallel $c\,x$ to III; then $l\,m$ parallel $d\,x$ to IV; then $m\,n$ parallel $e\,x$ to V; then $n\,p$ parallel $f\,x$ to VI; then $p\,g$ parallel

$g\,x$ to VII; Now draw from q the line $q\,o$ parallel $x\,h$, and from j the line $j\,o$ parallel $a\,x$ till they intersect at o. A horizontal through o is the neutral axis of whole beam. We will now make a new drawing of figure $j\,o\,q$ for the sake of clearness. Draw horizontals through it every inch in height beginning at both top and bottom with one-half inch. The top one scales nothing, the next $\frac{1}{3}''$, then $\frac{7}{8}''$, then $1\frac{1}{2}''$, then $2\frac{1}{4}''$, then $1\frac{1}{8}''$, and the bottom one $\frac{1}{3}''$, the sum of all being $6\frac{5}{12}''$ or $6{,}416''$. This multiplied by the height of the parts, which is one inch, would give us, of course, 6,416 square inches area. Multiplying this area by the area of the cross-section of deck beam 5,5 square inches, we should have

Horizontal Neutral Axis.

Area of Diagram.

$$i = 5{,}5.\ 6{,}416 = 35{,}288.$$

In Table XX it is given as 35, 1 so that we are not very far out.

Moment of Inertia of Beam.

If we had taken more parts, of course the result would have been more exact.

When constructing plate girders of large size, much material can be saved by making the flanges heaviest at the point of greatest bending-moment, and gradually reducing the flanges towards the supports.

Reducing Flanges, Plate Girders.

This is accomplished by making each flange at the point of greatest bending-moment of several thicknesses or layers of iron, the outer layer being the shortest, the next a little longer, etc. Of course the angles, which form part of the flange are kept of uniform size the whole length, as it would be awkward to attempt to use different sized angles. Generally (though not necessarily) the inner or first layer of the flange plates, is also run the entire length. Of course, where the flanges are gradually reduced in this way, it becomes necessary to figure the bending-moment and moment of resistance at many points along the plate girder to find where the plates can be reduced. This would be a wearisome job. By using the graphical method, however, it can be easily accomplished. Referring back to Figure 151, we take the point of greatest bending-moment (at w_1) of the beam $A\ B$. The required moment of resistance at this point, it will be remembered was the length (inch scale) of vertical E through $C\ D\ E\ F\ G$. We now decide what size angles we propose using and settle the necessary thickness of the flanges by Formula (36), inserting for the value of r, the length (inch scale) of v or vertical at E. Further a_1 will, of course, be the sum of the area of two angles, d the

total depth of girder in inches and b the breadth of flange, in inches, less rivet holes. The above is on the assumption that the distance $x\,y$ of pole x from load line $d\,a$ was equal to the safe modulus of rupture $\left(\frac{k}{f}\right)$ of steel or wrought-iron according to whichever material we were using, or we should have:

Thickness of Flanges.

$$x = \frac{\frac{v}{d} - a_1}{b} \qquad (98)$$

Where x = the thickness, in inches, of each flange of a plate girder at any point of its length.

Where v = the length of vertical, inch scale, through upper or resistance figure, providing we have assumed the distance of pole from load line (pound scale) $= \left(\frac{k}{f}\right)$ of the material.

Where d = the total depth, in inches, of the plate girder.

Where b = the width, less rivet holes, in inches, of the flange.

Where a_1 = the sum of the areas of cross-section, in square inches, of *two* of the angles used.

We now calculate as above, the thickness x of flange at point of greatest bending moment and then decide into how many layers or thicknesses we will divide the flanges. Say, in our case we decide to make the flange of four layers of plates, each $\frac{x}{4}$ or one quarter x in thickness. Then make

$$E_1\,E_{11} = a_1 .\, d \qquad (99)$$

Where $E_1\,E_{11}$ = the amount to be substracted (inch scale) from moment of resistance or vertical v and representing the work of two angles.

Where a_1 = the sum of the area of cross-section, in square inches, of the two angles.

Where d = the total depth, in inches, of the girder.

Where to drop off Plates. Now draw through E_{11} a parallel to base of figure $C\,G$, divide $E_{11}\,E$ into as many parts as we decide to use thicknesses of plates (four in our case) and draw parallel lines to base $C\,G$ through these parts. Vertically over the points where these lines intersect the curve or outline of figure $C\,D\,E\,F\,G$ will be the points at which to break off plates, as illustrated in drawing. This method, of course, is approximate, but it will be found sufficiently accurate for all practical purposes. It is not necessary that x

or $E\ E_n$ be divided into equal parts. Had we decided to use plates of varying thicknesses we should simply divide $E\ E_n$ in proportions to correspond to thicknesses of plates *in their proper order*, beginning at E_n with plate immediately next to angles and ending at E with extreme central outside plate. An example, more fully illustrating the above, will be given in the chapter on plate girders.

LE XII.

LOOR-BEAMS.

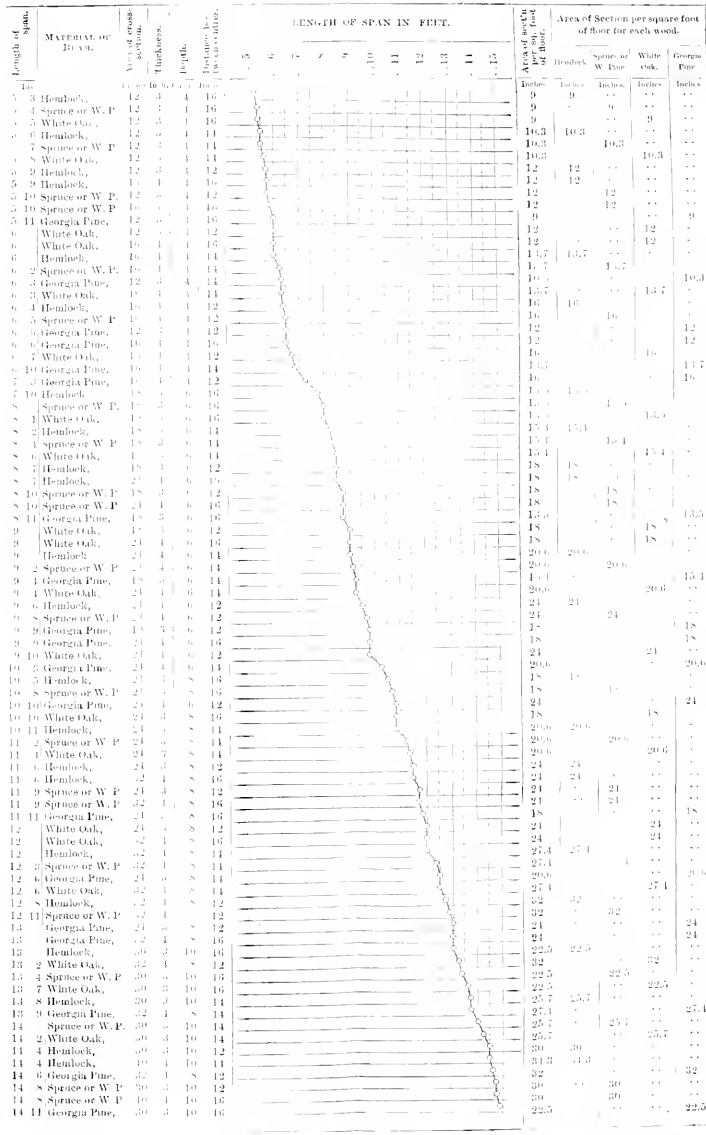
S PER SQUARE FOOT OF FLOOR.]

F SPAN IN FEET.							Area of sect'n per sq. foot of floor.	Area of Section per square foot of floor for each wood.			
								Hemlock.	Spruce or W. Pine.	White Oak.	Georgia Pine.
							Inches.	Inches.	Inches.	Inches.	Inches.
							9	9	..	..	..
							9	..	9	..	..
							9	..	..	9	..
							10.3	10.3	..	..	..
							10.3	..	10.3	..	..
							10.3	..	..	10.3	..
							12	12	..	..	..
							12	12	..	..	..
							12	..	12	..	..
							12	..	12	..	..
							9	..	..	..	9
							12	..	..	12	..
							12	..	..	12	..
							13.7	13.7	..	..	..
							13.7	..	13.7	..	..
							10.3	..	..	..	10.3
							13.7	..	..	13.7	..
							16	16	..	..	..
							16	..	16	..	..
							12	..	..	..	12
							12	..	..	..	12
							16	..	..	16	..
							13.7	..	..	..	13.7
							16	..	..	..	16
							13.5	13.5	..	..	..

Table XII.

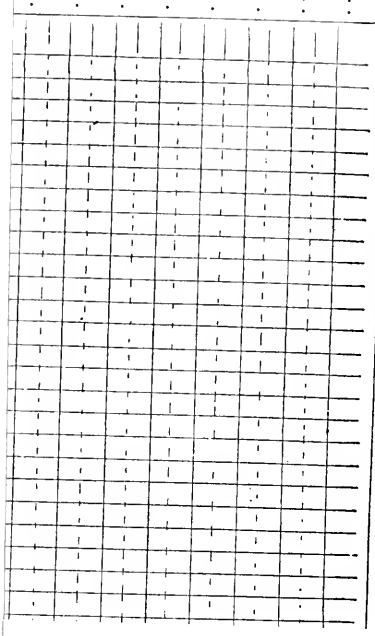
WOODEN FLOOR-BEAMS.

[Calculated for 90 Pounds per Square Foot of Floor.]



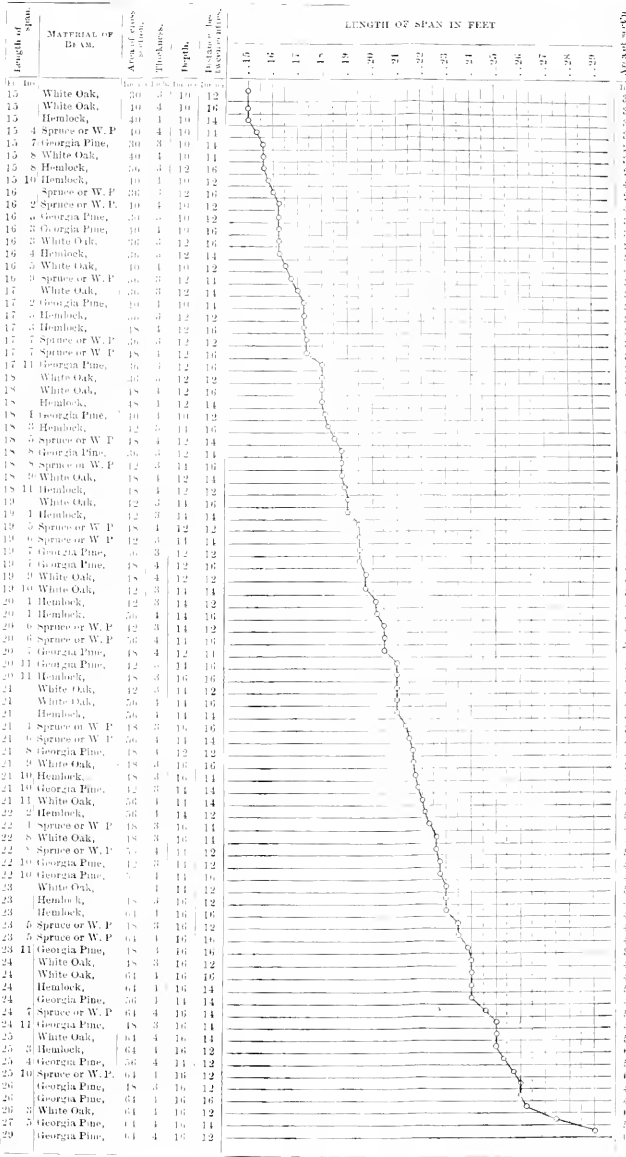
Length of span.	Material or Beam.	Area of cross section.	Thickness.	Depth.	Distance be-tween centres.	Area of section per sq. foot of floor.	Area of Section per square foot of floor for each wood.			
							Hemlock.	Spruce or W. Pine	White oak.	Georgia Pine
Ins.		In. sq.	In.	In.	Ins.	Inches	Inches	Inches	Inches	Inches
7	3 Hemlock,	12	3	4	16	9	9	..	..	..
7	4 Spruce or W. P	12	3	4	16	9	..	9	..	..
7	5 White Oak,	12	3	4	16	9	..	..	9	..
7	6 Hemlock,	12	3	4	14	10.3	10.3	..	..	..
7	7 Spruce or W. P	12	3	4	14	10.3	..	10.3	..	..
7	8 White Oak,	12	3	4	14	10.3	..	..	10.3	..
8	9 Hemlock,	12	3	4	12	12	12	..	..	..
5	9 Hemlock,	15	4	4	16	12	12	..	..	..
5	10 Spruce or W. P.	12	3	4	12	12	..	12	..	..
5	10 Spruce or W. P	16	4	4	16	12	..	12	..	..
5	11 Georgia Pine,	12	3	4	16	9	..	..	..	9
6	White Oak,	12	3	4	12	12	..	..	12	..
6	White Oak,	16	4	4	16	12	..	..	12	..
6	Hemlock,	16	4	4	14	13.7	13.7	..	..	..
6	2 Spruce or W. P.	16	4	4	14	13.7	..	13.7	..	..
6	3 Georgia Pine,	12	3	4	14	10.3	..	..	..	10.3
6	3 White Oak,	16	4	4	14	13.7	..	..	13.7	..
6	4 Hemlock,	16	4	4	12	16	16	..	..	..
6	5 Spruce or W. P	16	4	4	12	16	..	16	..	..
6	6 Georgia Pine,	12	3	4	12	12	..	..	..	12
6	6 Georgia Pine,	16	4	4	16	12	..	..	..	12
6	7 White Oak,	16	4	4	12	16	..	..	16	..
6	10 Georgia Pine,	16	4	4	14	13.7	..	..	..	13.7
7	8 Georgia Pine,	16	4	4	12	16	..	..	..	16
7	10 Hemlock,	18	4	6	16	14	14	..	..	..
8	Spruce or W. P.	18	4	6	16	14	..	14	..	..
8	1 White Oak,	18	4	6	16	18	..	..	18	..
8	2 Hemlock,	18	4	6	14	15.4	15.4	..	..	..
8	3 Spruce or W. P	18	4	6	14	15.4	..	15.4	..	..
8	6 White Oak,	18	4	6	14	15.4	..	..	15.4	..
8	7 Hemlock,	18	4	6	12	18	18	..	..	..
8	7 Hemlock,	24	4	6	16	18	18	..	..	..
8	10 Spruce or W. P	18	4	6	12	18	..	18	..	..
8	10 Spruce or W. P	24	4	6	16	18	..	18	..	..
8	11 Georgia Pine,	18	4	6	16	18	..	..	..	18
9	White Oak,	18	4	6	12	18	..	..	18	..
9	White Oak,	24	4	6	16	18	..	..	18	..
9	Hemlock	24	4	6	14	20.6	20.6	..	..	..
9	2 Spruce or W. P	24	4	6	14	20.6	..	20.6	..	..
9	3 Georgia Pine,	18	4	6	14	15.4	..	..	..	15.4
9	4 White Oak,	24	4	6	14	20.6	..	..	20.6	..
9	6 Hemlock,	24	4	6	12	24	24	..	..	..
9	8 Spruce or W. P	24	4	6	12	24	..	24	..	..
9	9 Georgia Pine,	18	4	6	12	18	..	..	..	18
9	9 Georgia Pine,	24	4	6	16	18	..	..	..	18
9	10 White Oak,	24	4	6	12	24	..	..	24	..
10	5 Georgia Pine,	24	4	6	14	20.6	..	..	..	20.6
10	5 Hemlock,	24	4	8	16	18	18	..	..	..
10	8 Spruce or W. P	24	..	8	16	18	..	18	..	..
10	10 Georgia Pine,	24	4	6	12	24	..	..	..	24
10	10 White Oak,	24	4	8	16	18	..	..	18	..
10	11 Hemlock,	24	..	8	14	20.6	20.6	..	..	..
11	2 Spruce or W. P	24	..	8	14	20.6	..	20.6	..	..
11	1 White Oak,	24	4	8	14	20.6	..	..	20.6	..
11	6 Hemlock,	24	4	8	16	24	24	..	..	..
11	6 Hemlock,	32	4	8	16	24	24	..	..	..
11	9 Spruce or W. P	24	4	8	12	24	..	24	..	..
11	9 Spruce or W. P	32	4	8	16	24	..	24	..	..
11	11 Georgia Pine,	24	..	8	16	18	..	..	..	18
12	White Oak,	24	..	8	12	24	..	..	24	..
12	White Oak,	32	4	8	16	24	..	..	24	..
12	Hemlock,	32	4	8	14	27.4	27.4	..	..	..
12	2 Spruce or W. P	32	4	8	14	27.4	..	27.4	..	..
12	6 Georgia Pine,	24	4	8	14	20.6	..	..	..	20.6
12	6 White Oak,	32	4	8	14	27.4	..	..	27.4	..
12	8 Hemlock,	32	4	8	12	32	32	..	..	..
12	11 Spruce or W. P	32	4	8	12	32	..	32	..	..
13	Georgia Pine,	24	4	8	12	24	..	..	..	24
13	Georgia Pine,	32	4	8	16	24	..	..	..	24
13	Hemlock,	50	5	10	16	22.5	22.5	..	..	..
13	2 White Oak,	32	4	8	12	32	..	..	32	..
13	4 Spruce or W. P	50	5	10	16	22.5	..	22.5	..	..
13	7 White Oak,	50	5	10	16	22.5	..	..	22.5	..
13	8 Hemlock,	50	5	10	14	25.7	25.7	..	..	..
13	9 Georgia Pine,	32	4	8	14	27.4	..	..	..	27.4
14	Spruce or W. P.	50	5	10	14	25.7	..	25.7	..	..
14	2 White Oak,	50	5	10	14	25.7	..	..	25.7	..
14	4 Hemlock,	50	5	10	12	30	30	..	..	..
14	4 Hemlock,	70	7	10	14	31.3	31.3	..	..	..
14	6 Georgia Pine,	32	4	8	12	32	..	..	..	32
14	8 Spruce or W. P	50	5	10	12	30	..	30	..	..
14	8 Spruce or W. P	70	7	10	16	30	..	30	..	..
14	11 Georgia Pine,	50	5	10	16	22.5	..	..	..	22.5

(*Continued*).



Area of section per sq. foot of floor.	Area of Section per square foot of floor for each wood.			
	Hemlock	Spruce or W. Pine.	White Oak.	Georgia Pine.
Inches.	Inches.	Inches.	Inches.	Inches.
30	..	..	30	..
30	..	..	30	..
34.3	34.3	..	..	..
34.3	..	34.3	..	..
25.7	..	..	..	25.7
34.3	..	..	34.3	..
27	27	..	..	..
40	40	..	..	..
27	..	27	..	..
40	..	40	..	..
30	..	..	..	30
30	..	..	..	30
27	..	..	27	..
30.9	30.9	..	..	..
40	..	..	40	..
30.9	..	30.9	..	..
32	..	..	32	..
34.3	..	..	..	34.3
36	36	..	..	..
36	36	..	..	..
36	..	36	..	..
36	..	36	..	..
27	..	..	..	27
36	..	..	36	..
36	..	..	36	..
41.1	41.1	..	..	..

Table XII *(Continued)*.



Length of Span		Material of Beam.	Area of cross section.	Thickness.	Depth.	Distance between centres of beams.	Area of Section per square foot of floor for each wood.			
							Hemlock	Spruce or W. Pine	White Oak.	Georgia Pine.
Ft.	Ins.		Ins.	Ins.	Ins.	Ins.	Inches.	Inches.	Inches.	Inches.
15		White Oak,	30	3	10	12	30	..	..	30
15		White Oak,	40	4	10	16	30	..	..	30
15		Hemlock,	40	4	10	14	34.3	34.3	..	..
15	4	Spruce or W. P	40	4	10	14	34.3	..	34.3	..
15	7	Georgia Pine,	30	3	10	14	27.7	..	..	27.7
15	8	White Oak,	40	4	10	11	34.3	..	34.3	..
15	8	Hemlock,	56	4	12	16	27	27	..	..
15	10	Hemlock,	40	4	10	12	40	40	..	..
16		Spruce or W. P	36	3	12	16	27	..	27	..
16	2	Spruce or W. P.	40	4	10	12	40	..	40	..
16	2	Georgia Pine,	24	2	12	12	30	..	..	30
16	3	Georgia Pine,	30	4	10	16	30	..	..	30
16	3	White Oak,	56	4	12	16	27	..	27	..
16	4	Hemlock,	28	4	12	14	30.9	30.9	..	..
16	5	White Oak,	40	4	10	12	40	..	40	..
16	9	Spruce or W. P	36	3	12	14	30.9	..	30.9	..
17		White Oak,	56	3	12	14	32	..	32	..
17	2	Georgia Pine,	30	4	10	14	34.3	..	..	34.3
17	..	Hemlock,	30	3	12	12	36	36	..	..
17	3	Hemlock,	18	4	12	16	36	36	..	..
17	7	Spruce or W. P	36	3	12	12	36	..	36	..
17	7	Spruce or W. P	18	4	12	16	36	..	36	..
17	11	Georgia Pine,	36	4	12	16	27	..	..	27
18		White Oak,	56	4	12	12	36	..	36	..
18		White Oak,	18	4	12	16	36	..	36	..
18		Hemlock,	18	3	12	14	41.1	41.1	..	..
18	1	Georgia Pine,	30	4	10	12	40	..	..	40
18	2	Hemlock,	42	3	14	16	31.5	31.5	..	..
18	5	Spruce or W. P	18	4	12	14	41.1	..	41.1	..
18	8	Georgia Pine,	36	4	12	14	30.9	..	..	30.9
18	8	Spruce or W. P	42	3	14	16	31.5	..	31.5	..
18	9	White Oak,	18	4	12	14	41.1	..	..	41.1
18	11	Hemlock,	18	4	12	12	48	48	..	..
19		White Oak,	42	3	14	16	31.5	..	31.5	..
19	1	Hemlock,	42	3	14	14	36	36	..	..
19	3	Spruce or W. P	18	4	12	12	48	..	48	..
19	6	Spruce or W. P	42	3	14	14	36	..	36	..
19	7	Georgia Pine,	36	3	12	12	36	..	..	36
19	7	Georgia Pine,	18	4	12	16	36	..	..	36
19	9	White Oak,	18	4	12	12	48	..	48	..
19	10	White Oak,	42	3	14	14	36	..	36	..
20	1	Hemlock,	42	3	14	12	42	42	..	..
20	1	Hemlock,	56	4	14	16	42	42	..	..
20	6	Spruce or W. P	42	3	14	12	42	..	42	..
20	6	Spruce or W. P	56	4	14	16	42	..	42	..
20	7	Georgia Pine,	18	4	12	14	41.1	..	..	41.1
20	11	Georgia Pine,	42	3	14	16	54.9	..	..	54.9
20	11	Hemlock,	18	3	16	16	36	36	..	..
21		White Oak,	42	3	14	12	42	..	42	..
21		White Oak,	56	4	14	16	42	..	42	..
21		Hemlock,	56	4	14	14	48	48	..	..
21	1	Spruce or W. P	18	3	16	16	36	..	36	..
21	3	Spruce or W. P	56	4	14	14	48	..	48	..
21	8	Georgia Pine,	18	4	12	12	48	..	..	48
21	9	White Oak,	18	4	16	16	36	..	36	..
21	10	Hemlock,	18	3	16	14	41.1	41.1	..	..
21	10	Georgia Pine,	42	3	14	14	36	..	..	36
21	11	White Oak,	56	4	14	14	48	..	48	..
22	2	Hemlock,	56	4	14	12	56	56	..	..
22	1	Spruce or W. P	18	3	16	14	47.1	..	41.1	..
22	8	White Oak,	18	3	16	14	41.1	..	41.1	..
22	8	Spruce or W. P	56	4	14	12	56	..	56	..
22	10	Georgia Pine,	42	3	14	12	42	..	..	42
22	10	Georgia Pine,	56	4	14	16	42	..	..	42
23		White Oak,	56	4	14	12	56	..	56	..
23		Hemlock,	18	3	16	12	48	48	..	..
23		Hemlock,	64	4	16	16	48	48	..	..
23	5	Spruce or W. P	18	3	16	12	48	..	48	..
23	5	Spruce or W. P	64	4	16	16	48	..	48	..
23	11	Georgia Pine,	18	4	16	16	36	..	..	36
24		White Oak,	18	3	16	12	48	..	48	..
24		White Oak,	64	4	16	16	48	..	48	..
24		Hemlock,	64	4	16	14	54.9	54.9	..	..
24		Georgia Pine,	56	4	14	14	48	..	..	48
24	7	Spruce or W. P	64	4	16	14	54.9	..	54.9	..
24	11	Georgia Pine,	18	3	16	14	41.1	..	..	41.1
25		White Oak,	64	4	16	14	54.9	..	..	54.9
25	3	Hemlock,	64	4	16	12	64	64	..	..
25	4	Georgia Pine,	56	4	14	12	56	..	..	56
25	10	Spruce or W.P.	64	4	16	12	64	..	64	..
26		Georgia Pine,	18	3	16	12	48	..	..	48
26		Georgia Pine,	64	4	16	16	48	..	..	48
26	3	White Oak,	64	4	16	12	64	..	..	64
27	5	Georgia Pine,	64	4	16	14	54.8	..	..	54.8
29		Georgia Pine,	64	4	16	12	64	..	..	64

XIII.

BRACED SIDEWAYS.

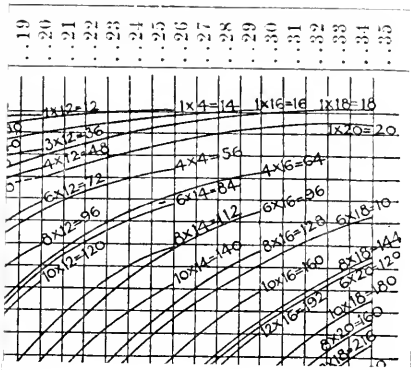
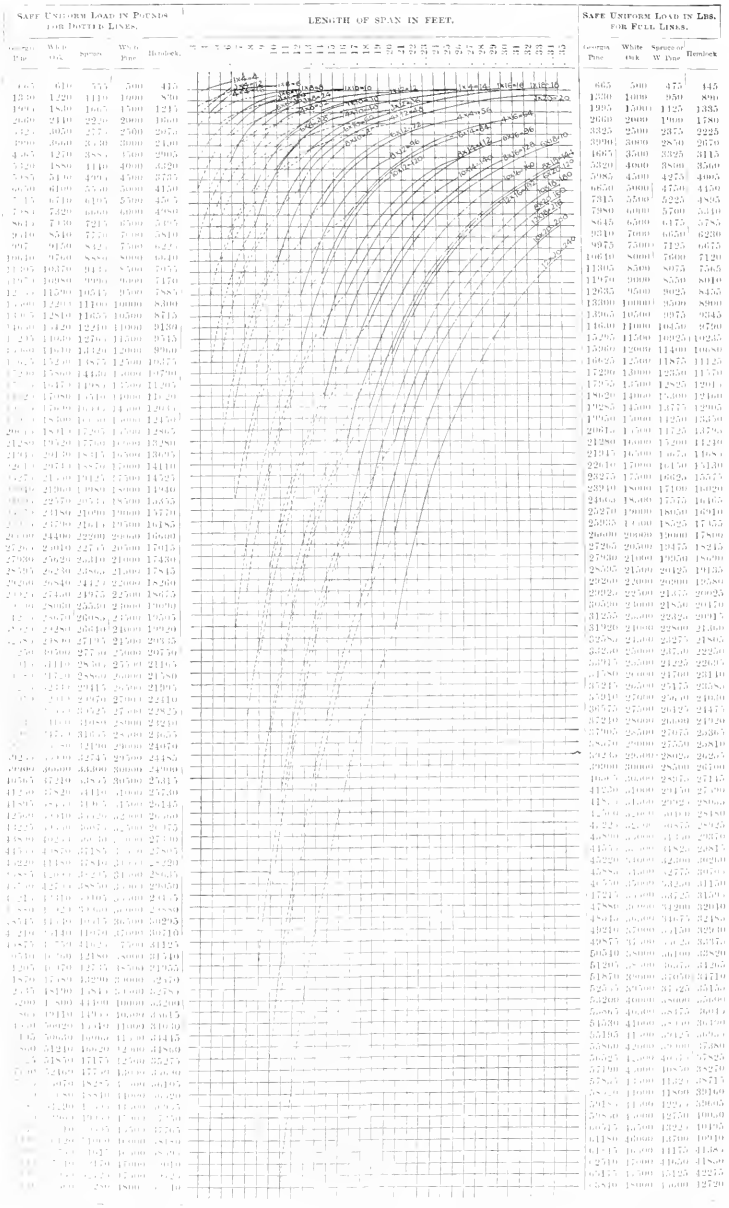
SPAN IN FEET.	Safe Uniform Load in Lbs. for Full Lines.			
	Georgia Pine.	White Oak.	Spruce or W. Pine.	Hemlock.
	665	500	475	445
	1330	1000	950	890
	1995	1500	1425	1335
	2660	2000	1900	1780
	3325	2500	2375	2225
	3990	3000	2850	2670
	4665	3500	3325	3115
	5320	4000	3800	3560
	5985	4500	4275	4005
	6650	5000	4750	4450
	7315	5500	5225	4895
	7980	6000	5700	5340

Table XIII.

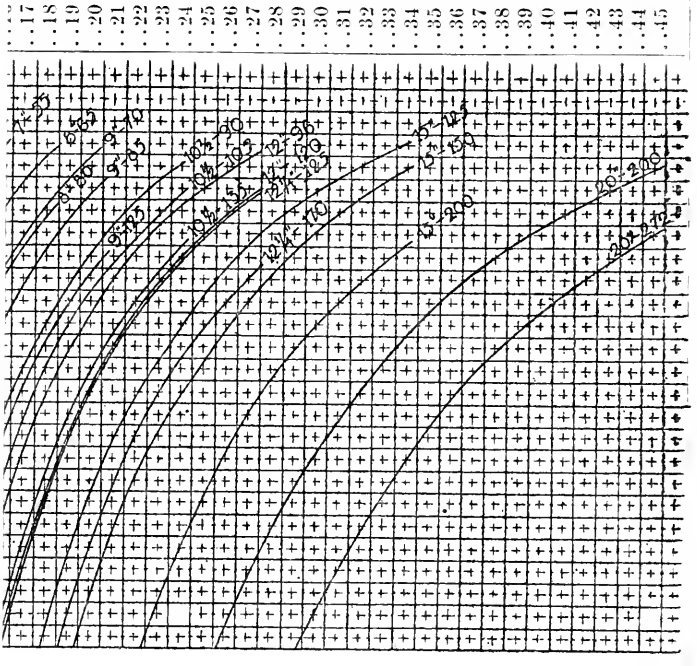
WOODEN GIRDERS, BRACED SIDEWAYS.



XIV.

FOR FLOORS.

LENGTH OF SPAN OF BEAM IN FEET.



or Iron Beams; but length of span in feet must not exceed twice the depth

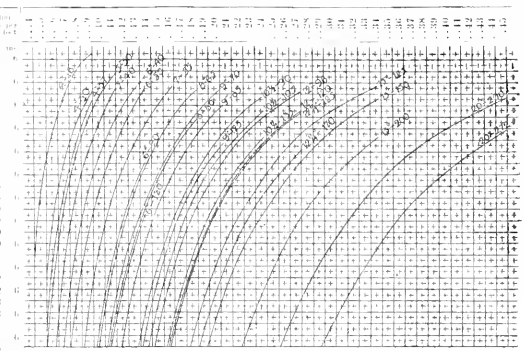
Table XIV.

IRON I BEAMS FOR FLOORS.

Distance between Centres of Beams.

300 lbs. per sq. foot	250 lbs. per sq. foot	200 lbs. per sq. foot	175 lbs. per sq. foot	150 lbs. per sq. foot	125 lbs. per sq. foot	100 lbs. per sq. foot
ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
10	1	1	1	1	1	2
1	1	2	1	1	2	3
1	2	1	1	2	2	4
1	4	1	2	2	3	5
1	6	1	2	2	3	6
1	8	2	2	2	3	7
1	10	2	2	2	3	8
2	2	2	3	3	4	9
2	2	2	3	3	4	10
2	4	2	3	3	4	11
2	6	2	3	4	4	12
2	8	2	4	4	5	13
2	10	3	4	4	5	14
3	2	3	4	5	6	15
3	4	3	4	5	6	16
3	6	3	5	6	7	17
3	8	3	5	6	7	18
3	10	3	5	6	7	19
4	2	4	6	7	8	20
4	4	4	6	7	8	21
4	6	4	6	7	9	22
4	8	4	6	7	9	23
4	10	4	6	7	9	24
5	2	5	7	8	10	25
5	4	5	7	8	10	26
5	6	5	7	8	10	27
5	8	5	7	8	10	28
5	10	5	7	8	11	29
6	2	6	7	8	11	30
6	4	6	7	8	11	31
6	6	6	7	9	11	32
6	8	6	7	9	11	33
6	10	6	7	9	11	34
7	2	7	8	9	12	35
7	4	7	8	9	12	36
7	6	7	8	9	12	37
7	8	7	8	9	12	38
7	10	7	8	9	12	39
8	2	8	9	10	13	40
8	4	8	9	10	13	41
8	6	8	9	10	13	42
8	8	8	9	10	13	43
8	10	8	9	10	13	44
9	2	9	10	11	14	45

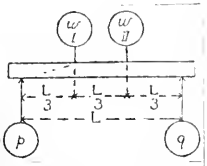
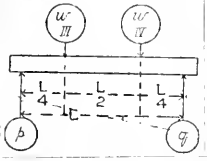
Length of span of beam in feet.



For Steel Beams, space one quarter distance their [illegible] larger than for Iron Beams; but length of span in feet must not exceed twice the depth of beam in inches, or deflection will be too great for plastering.

TABLES XX TO XXV.

H ARE UNIFORM ABOVE AND BELOW THE NEUTRAL AXIS.

MANNER OF LOADING.	TO OBTAIN SAFE LOAD IN POUNDS.	LENGTH OF SPAN NOT TO CRACK PLASTERING MUST NOT EXCEED.		GREATEST ACTUAL DEFLECTION WILL BE.	
		For Iron.	For Steel.	For Iron.	For Steel.
	$w_{\mathrm{I}} = w_{\mathrm{II}} = \frac{v}{2\frac{2}{3}} \cdot L$ or: — $w_{\mathrm{I}} + w_{\mathrm{II}} = \frac{v}{1\frac{1}{3}} \cdot L$	$L = 2\frac{3}{16} \cdot d$	$L = 1\frac{7}{8} \cdot d$	$\delta = \frac{L^2}{72\frac{3}{4} \cdot d}$	$\delta = \frac{L^2}{62\frac{1}{2} \cdot d}$
	$w_{\mathrm{III}} = w_{\mathrm{IIII}} = \frac{v}{2} \cdot L$ or: — $w_{\mathrm{III}} + w_{\mathrm{IIII}} = \frac{v}{L}$	$L = 2 \cdot d$	$L = 1\frac{3}{4} \cdot d$	$\delta = \frac{L^2}{67 \cdot d}$	$\delta = \frac{L^2}{57\frac{3}{5} \cdot d}$

ables XX to XXV; w = uniform load in lbs.; v = centre load in lbs.; $w_{\mathrm{I}} = w_{\mathrm{II}}$; also $w_{\mathrm{III}} = w_{\mathrm{IIII}}$ = concen-

refully, as steel varies greatly in strength. For *equal* deflections of steel and iron, add
ıe transverse values, the moduli of rupture used were: for iron, 12000 pounds per square

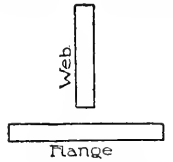
as follows: — In case of uniform load deduct *entire* weight of beam; in case of centre
m *each* load $\frac{3}{8}$ the weight of beam; in case of loads at quarters of span, deduct from

same dimensions. In ordering steel or iron, give either the required dimensions or the

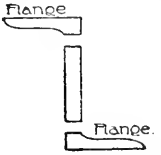
ble represent the following Rolling Mills: —

B — PHŒNIX IRON COMPANY, Philadelphia, Pa.
D — POTTSVILLE IRON AND STEEL COMPANY, Pottsville, Pa.
Pa. F — PASSAIC ROLLING MILL COMPANY, Passaic, N. J.

tes the Mill which rolls the exact shape given in the Table; the other letters give the



TEE.



TEE.

DECK BEAM.

TABLE XIX.

EXPLAINING USE OF TABLES XX TO XXV

HOW TO USE TRANSVERSE VALUE OF FOR SECTIONS WHICH ARE UNIFORM ABOVE AND BELOW THE NEUTRAL AXIS.

MANNER OF LOADING.	TO OBTAIN SAFE LOAD IN POUNDS.	LENGTH OF SPAN NOT TO EXCEED.		GREATEST ACTUAL DEFLECTION WILL BE.		MANNER OF LOADING.	TO OBTAIN SAFE LOAD IN POUNDS.	LENGTH OF SPAN NOT TO EXCEED.		GREATEST ACTUAL DEFLECTION WILL BE.	
		For Iron.	For Steel.	For Iron.	For Steel.			For Iron.	For Steel.	For Iron.	For Steel.
	$w = \frac{T}{L}$	$L = 2\frac{1}{2}\,d$	$L = 2\,d$	$\delta = \frac{L^2}{d}$	$\delta = \frac{L^2}{14\frac{1}{2}\,d}$		$w_1 = w_2 = \frac{T}{2\frac{1}{2}\,L}$ or $w_1 + w_2 = \frac{T}{1\frac{1}{4}\,L}$	$L = 3\frac{1}{10}\,d$	$L = 1\frac{1}{4}\,d$	$\delta = \frac{L^2}{72\frac{1}{4}\,d}$	$\delta = \frac{L^2}{624\frac{1}{2}\,d}$
	$w = \frac{T}{L}$	$L = 5\,d$	$L = 2\,d$	$\delta = \frac{L^2}{144\,d}$	$\delta = \frac{L^2}{800\,d}$		$w_1 = w_2 = \frac{T}{2\,L}$ or $w_1 + w_2 = \frac{T}{L}$	$L = 2\,d$	$L = 1\frac{1}{4}\,d$	$\delta = \frac{L^2}{612\,d}$	$\delta = \frac{L^2}{2178\,d}$

d = depth in inches, L = length in inches, w = transverse value given in Tables XX to XXV, w_1 = uniform load in lbs., w_2 = centre load in lbs., w_3 = distributed load, T = corresponding value of $\frac{S\,I}{d}$ in Tables XX to XXV, δ = deflection in inches, I = moment of inertia, S = safe strain per square inch, E = coefficient of elasticity.

NOTES.—In the transverse values as given for steel, are used, test each piece carefully, as steel varies greatly in strength. For *equal* deflections of steel and iron, add 6 2/3 to iron transverse values, instead of 2 as given in Tables. In calculating the transverse values, the moduli of rupture used were for iron, 12000 pounds per square inch, and for steel 40000 pounds per square inch.

From the "Safe Load" as obtained above, deduct the weight of beam in pounds, as follows:—In case of uniform load deduct *entire* weight of beam; in case of centre load, deduct one-half the weight of beam; in case of load at thirds of span, deduct from each load $\frac{7}{12}$ the weight of beam; in case of loads at quarters of span, deduct from each load $\frac{3}{4}$ the weight of beam.

Steel sections can be specified by their weight per foot in lbs., and iron sections of exactly the same dimensions. In ordering steel or iron, give either the required dimensions or the required weight of section per foot.

The capital letters in the first column headed "Mills Rolling Shape" in each Table represent the following Rolling Mills:—

A—NEW JERSEY STEEL AND IRON COMPANY, Trenton, N. J.
B—PHŒNIX IRON COMPANY, Phœnixville, Pa.
C—PENCOYD IRON WORKS, Pencoyd, Pa.
D—POTTSVILLE IRON AND STEEL COMPANY, Pottsville, Pa.
E—UNION IRON MILLS and HOMESTEAD STEEL WORKS, Pittsburgh, Pa.
F—PASSAIC ROLLING MILL COMPANY, Passaic, N. J.

In the columns headed "Mills Rolling Shape," the first letter on each line indicates the Mill which runs the exact shape given in the Table, the other letters give the Mills which roll an approximately similar shape.

In order that the various parts of sections, they were taken as shown below:—

: XX.

) STEEL I BEAMS.

OF THIS TABLE, SEE TABLE XIX.)

ıal to Web. M---I---N				Axis Parallel to Web. M--⊢⊣--N				
Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.	Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (ρ^2)	Transverse Value (v) in lbs. For Iron.	Transverse Value (v) in lbs. For Steel.
)	60.67	1320000	1650000	46.50	13.78	1.71	110000	137800
)	60.42	1160000	1450000	51.78	16.57	2.16	132600	165700
3.8	61.99	990000	1238000	26.62	8.87	1.33	71000	88700
4.6	59.68	917000	1146000	31.50	10.08	1.64	80600	100800
)	34.80	552000	690000	15.29	6.12	1.02	49000	61200
3.5	35.32	748000	935000	27.46	9.55	1.37	76000	95500
7.5	35.15	460000	575000	11.64	4.66	.94	37000	46600
3	32.52	864000	1080000	40.84	13.90	1.62	111200	139000
)	31.24	800000	1000000	29.90	10.29	1.25	82300	103000
2.5	34.70	740000	925000	33.79	12.12	1.69	97000	121000
1.5	31.47	655000	819000	20	7.50	1.02	60000	75000
).4	35.20	563000	704000	18.34	7.34	1.22	58700	73400
).5	35.76	556000	695000	16.91	6.60	1.17	52800	66000
7.3	34.40	458000	573000	13.13	5.34	1.05	42700	53400
3.5	23.32	508000	635000	25.41	9.24	1.52	74000	92400
3	22.60	504000	630000	20.90	7.96	1.23	63700	80000
7	23.35	376000	470000	11.54	4.82	.93	38500	48200
3.7	18.92	454000	567000	15.50	6.09	.86	48700	61000
3.7	22.46	510000	637000	24.08	8.76	1.42	70100	87600
7.1	22.60	377000	471000	12.98	5.46	1.04	43700	54600
3.9	23.98	375000	469000	16.76	6.10	1.43	49000	61000
3.3	21.80	290000	363000	8.74	4	.87	32000	40000
3.2	24.22	306000	382000	11.66	4.44	1.23	35500	44400
4.5	17.49	356000	445000	15.80	6.32	1.18	50500	63200
5.3	17.77	282000	353000	9.43	4.19	.90	33500	41900
1.4	17.39	251000	314000	8.01	3.53	.85	28200	35300
1.2	18.42	250000	312000	8.09	3.59	.91	28700	35900
7.5	13.91	300000	375000	11.30	4.74	.83	37900	47400
4.7	15.52	278000	347000	10.64	4.60	.96	36800	46000
2.2	15.35	258000	322000	11.08	4.79	1.05	38300	48000
).7	16.40	237000	297000	8.09	3.69	.90	29500	36900
2	12.63	336000	420000	23.16	8.62	1.54	69000	86000
5.3	11.70	282000	353000	14	5.67	1.02	45400	56700
3.5	12.23	268000	335000	11.23	4.99	.91	40000	49900
3	11.83	208000	260000	7.14	3.30	.72	26400	33000
3.4	13.10	211000	264000	8.44	3.86	.92	30900	38600
4.9	13.16	199000	249000	7.35	3.27	.86	26000	32700
).9	13.41	167000	209000	4.92	2.46	.70	19700	24600
2.6	8.64	181000	226000	6.96	3.25	.67	26000	32500
1	10.44	168000	210000	7.55	3.35	.93	26800	33500

TABLE XX.

LIST OF IRON AND STEEL I BEAMS.

(FOR INFORMATION AS TO THE USE OF THIS TABLE, SEE TABLE XIX.)

MILLS BUILDING SHAPE.	Depth of Beam. (d)	Weight per Yard.	Width of Flange. (b)	Thickness of Web.	Area of each Flange.	Area of Web.	Total Area. (a)	Axis Normal to Web. Moment of Inertia. (I)	Modulus of Section. (s)	Square of Radius of Gyration. (r²)	Transverse Value (v) in lbs. For Iron	For Steel.	Axis Parallel to Web. Moment of Inertia. (I)	Modulus of Section. (s)	Square of Radius of Gyration. (r²)	Transverse Value (v) in lbs. For Iron.	For Steel.
A B	20	272	6.75	0.69	7.87	11.46	27.20	1650.3	165	60.67	1320000	1650000	46.50	13.78	1.71	110600	127800
E	20	240	6	0.60	6.68	10.64	24	1450	145	60.42	1160000	1450000	31.78	10.57	2.16	132600	165700
A B	20	200	6	0.50	5.65	8.86	19.97	1238	123.8	61.99	990000	1238000	26.62	8.87	1.33	71000	88700
E	20	192	6.25	0.50	5.99	9.02	19.20	1146	114.6	59.68	917000	1146000	31.50	10.08	1.64	80800	100800
A F	15 3/4	150	5	0.50	4.31	6.36	15.04	523.5	69	34.80	552000	690000	15.29	6.12	1.02	49000	61200
A F	15 3/4	200	5.75	0.60	6.41	7.44	20.02	707.1	84.5	35.32	745000	935000	27.46	9.55	1.37	76000	95500
A	15 3/4	125	5	0.42	3.97	5.82	12.86	431.5	57.5	33.15	460000	575000	11.64	4.66	.54	37000	46600
D	15	250	5.875	0.75	7.22	10.56	25.	813.	108	32.52	864000	1080000	40.84	13.90	1.62	111200	139000
E	15	240	5.81	0.93	6.29	11.42	24	750	100	31 24	800000	1000000	39.90	10.29	1.25	82300	103000
B D E E	15	200	5.56	0.625	6.06	7.88	20	691	92.5	34.70	740000	925000	33.79	12.12	1.69	97000	121000
E	15	195	5.33	0.77	4.84	9.82	19.50	614	81.5	31.47	655000	819000	29	7.50	1.02	60000	75000
D E E	15	150	5	0.47	4.47	6.06	15	528	70.1	35.20	563000	704000	18.84	7.51	1.22	58700	73100
C	15	145	4.125	0.44	4.18	5.77	14.55	521.2	69.5	35.76	556000	695000	10.51	6.60	1.17	52800	66000
B E E	15	125	4.75	0.41	3.35	5.80	12.50	440	57 3	34.40	458000	573000	13.13	5.31	1.05	42700	53400
A	12 1/2	170	5.50	0.60	5.47	5.83	16.77	541.2	68.5	23.32	508000	635000	25.41	9.24	1.52	74000	92400
F	12 1/2	170	5.25	0.66	5.56	5.8	17	385	63	22.60	504000	630000	20.30	7.96	1.23	63700	80000
A F	12	125	4.79	0.47	4.58	4.77	12.33	288	47	24.55	376000	470000	11.54	4.82	.93	38500	48200
F	12	148	5.03	0.56	4.26	5.47	14	340	56.7	28.72	454000	567000	15.20	6.07	.86	48700	60800
E D	12	130	5.50	0.50	5.77	5.46	17	349.9	61.7	22.46	514000	617000	24.08	8.76	1.42	70100	87600
B D E ...	12	125	4.75	0.49	3.80	4.80	12.50	282.6	47.1	22.60	377000	471000	12.08	5.46	1.04	43700	54600
A C	12	120	5.50	0.39	3.87	3.99	11.73	281.3	46.9	23.98	375000	469000	16.76	6.10	1.43	49000	61000
D	12	100	4.44	0.44	2.71	4.58	10	218	36.3	21.80	290000	36.3000	8.74	4	.87	32000	40000
A B E ...	12	96	5.25	0.31	3.09	3.28	9.46	229.2	38.2	24.22	306000	382000	11.66	4.44	1.25	35500	44400
A B C D E E.	10 1/2	105	5	0.47	3.84	3.64	10.32	253.7	51 5	17.15	456000	445000	15.80	6.32	1.18	50500	63200
A B C D E ...	10 1/2	105	4.50	0.375	3.67	3 16	10.14	185.6	35.3	17.77	282000	353000	9.43	4.19	.90	33500	41900
E	10	95	4.54	0.41	2.96	3 68	9.50	165	31.4	17.59	251000	314000	8.01	3.53	.85	28200	35300
A B C D I	10 1/2	90	4.50	0.31	3.11	2 68	8.90	164	31 2	18.42	250000	312000	8.09	3.59	.91	28700	35900
E	10	105	4.77	0.77	3.07	6.16	13.50	187	37.5	13.91	300000	375000	11.30	4.74	.83	37900	47400
C	10	112	4.625	0.50	3.61	3.94	11.17	173.6	34.7	15.52	278000	347000	10.64	4.60	.96	36800	46000
D	10	104	4.625	0.50	3.24	3.42	10.50	174	32.2	16.55	258000	322000	11.08	4.79	1.05	38300	47900
C D E ...	10	90	4.375	0.44	3.14	2.76	9.04	118 3	29.7	16.40	237000	297000	8.02	3.59	.90	29500	36900
B E	9	150	5.375	0.69	5.59	5.82	15	189.1	42	12 83	336000	420000	23.16	8.62	1.54	69000	86000
L	9	115	4.94	0.75	4.22	5.06	13.50	150	35.3	11.70	282000	353000	14	5.67	1.02	45400	56700
A	9	125	4.50	0.57	4.11	3.51	12.33	150.8	33.5	12.23	268000	335000	11.23	4.99	.91	40000	49900
E	9	99	4.53	0.58	2.85	4.20	9.90	117	26	11.81	208000	260000	8.14	3.30	.72	26400	33000
C D	9	90	4.375	0.50	3.07	2.76	8.85	99.4	26.1	11.23	204000	255000	8.44	3.86	.92	30900	38600
A B D E ...	9	85	4.50	0.375	2.93	2.64	8.50	111.9	24.9	13.16	199000	249000	7.35	3.27	.86	26000	32500
A B C D E E.	9	70	4	0.36	2 10	2.20	7	93.9	20.9	13.41	167000	209000	4.92	2.46	.70	19700	24600
E	8	105	4.29	0.79	2.74	5 02	10 50	90.4	22 6	8 61	181000	226000	6.36	3.25	.67	26000	32500
A B C D E ...	8	80	4.50	0.375	2.82	2.39	8.03	83.9	21	10.43	168000	210000	7.55	3.35	.93	26800	33500
A B C D E E.	8	65	4	0.30	2 21	1.95	6.47	67.4	16.9	10.58	135000	169000	4.55	2.27	.71	18200	22700
A C	7	74	4.91	0.53	2.29	2 72	7.50	84.3	17 5	7 21	140000	175000	5.87	2.56	.66	20500	25000
B	7	60	4	0.375	2.47	1 96	6.90	55.7	15.9	8 08	127000	159000	5.42	2.71	.785	21680	27100
C D	7	65	3.81	0.41	2 08	2.42	6.58	49.8	14 2	7 56	113800	142000	4.15	2.18	.62	17400	21800
F	7	60	3.50	0.10	1.88	2.21	6	45	12 9	7 50	103200	129000	3.15	1.80	.53	14400	18000
A B D E ...	7	55	3.75	0.30	1 20	1.70	5 50	44 3	12.7	8 05	101600	127000	3.09	2.08	.71	16600	20800
C	7	52	3.61	0.234	1.92	1 30	5.14	43 1	12.3	8 55	9 5.00	123000	3.43	1.90	.67	15200	19000
C D	6	129	5.27	0.827	1.78	2.83	11 84	64.9	21.6	5 18	172000	216000	18.77	7.08	1.57	56600	70800
A F	6	90	5	0.56	3.31	2 38	8 70	49.8	16.6	5.72	132000	166000	10.78	4.32	1.24	34500	43200
L	6	54	3.46	0 16	1.61	2 18	5.40	28.1	9.5	5.20	76000	95000	2.31	1.45	.46	11600	14500
A B C D F ...	6	50	3.50	0.30	1.77	1 35	5 01	29	9.6	5.90	76800	96000	2.74	1.57	.56	12500	15700
A B C D E F.	6	40	3	0 25	1.42	1.17	4 01	23.5	7 8	5.96	62400	78000	1.61	1.07	.40	8500	10700
A B D E F ...	5	40	3	0.31	1.36	1.18	3 91	15.1	6.1	3.95	48800	61000	1.88	1.12	.43	9000	11200
E	5	36	3	0.30	1.20	1.20	3 60	14.9	5.96	4.34	47700	59600	1.74	1.16	.483	9260	11580
C	5	34	2.81	0.31	1.06	1.26	3 38	13 4	5.4	3.96	42800	53500	1.21	.85	.36	6800	8500
A B C D E E.	5	30	2.75	0.25	.99	1.01	2.99	12.1	4.8	4.04	38400	48000	1.04	.76	.35	6000	7600
F	4	37	3	0.31	1.10	.86	3 66	9 2	4 6	2.51	36800	46000	1.74	1.16	.48	9500	11600
A B D E F ...	4	30	2.75	0.25	1.08	.75	2.91	7.5	3.75	2.57	30000	37500	1.11	.81	.38	6500	8100
C	4	28	2.75	0.25	1.07	.75	2.90	7.7	3.84	2.65	30700	38400	1.17	.85	.40	6800	8500
D E	4	24	2.25	0.32	.71	.64	2.50	5.6	2.80	2.25	22400	28000	.58	.52	.22	4160	5200
A B C D F	4	18	2	0.19	.58	.61	1.77	4.5	2.25	2.54	18000	22500	.31	.31	.175	2500	3100
E	3	27	2.52	0.39	.93	.84	2.70	3.54	2.36	1.32	18900	23600	.84	.67	.31	5360	6700
B	3	23	2.50	0.25	.86	.53	2 25	3.29	2.16	1.46	17300	21600	.77	.62	.35	4960	6200
E	3	21	2.32	0.19	.85	.40	2.10	3.09	2.06	1.46	16500	20600	.55	.47	.26	3760	4700
C	3	17	2.25	0.156	.68	.25	1.71	2.66	1.77	1.56	14200	17700	.48	.43	.28	3440	4300
A	1 1/4	5	1.50	0.125	.19	.11	.72	.135	.21	.26	.....	.....	.069	.092	.013	.....	920

TABLE XXI.

AND STEEL CHANNELS.

(FOR USE OF THIS TABLE, SEE TABLE XIX.)

Axis Normal to Web.					Axis Parallel to Web.					
Moment of Inertia. (*i*)	Moment of Resistance. (*r*)	Square of Radius of Gyration. (ρ^2)	Transverse Value (*v*) in lbs.		Moment of Inertia. (*i*)	Moment of Resistance. (*r*)	Square of Radius of Gyration. (ρ^2)	Distance from Neutral Axis to Inside Face of Web.	Transverse Value (*v*) in lbs.	
			For Iron.	For Steel.					For Iron.	For Steel.
24.0	70.00	29.20	560000	700000	21.60	6.93	1.21	1.06	55600	69500
57.5	62.30	31.30	498400	623000	19.70	6.84	1.32	1.09	54720	68400
30.7	88.09	28.73	705000	881000	37.56	10.05	1.63	1.28	80500	100500
54.6	73.94	27.73	591500	739400	23.61	7.15	1.18	1.08	57200	71500
86.0	78.13	30.84	625000	781300	32.25	9.24	1.70	1.26	74000	92400
27.0	70.26	30.03	562080	702600	31.41	8.92	1.80	1.23	71360	89200
19.1	59.88	29.94	479000	598800	18.27	6.09	1.22	1.00	48720	60900
76.0	50.13	31.33	401000	501300	14.47	4.74	1.21	0.95	38000	47400
39.4	55.41	19.07	443300	554100	20.65	6.55	1.16	1.16	52500	65500
01.6	47.64	20.83	381000	476400	17.87	6.20	1.28	1.12	49600	62000
4.3	35.00	21.40	280000	350000	8.93	3.68	0.88	0.86	29440	36800
68.9	27.70	21.10	221600	277000	5.44	2.39	0.67	0.73	19120	23900
63.2	25.03	21.89	200000	250300	5.04	2.24	0.72	0.755	18000	22400
35.7	39.29	15.72	314200	392900	8.44	3.13	0.56	0.80	25040	31300
31.5	30.25	20.16	242000	302500	7.11	3.29	0.79	0.84	26320	32900
23.7	20.62	20.79	164960	206200	3.22	1.62	0.55	0.62	12960	16200
29.4	24.64	12.32	197000	246400	4.96	1.98	0.47	0.69	16000	19800
6.0	22.10	15.50	176800	221000	5.20	2.55	0.71	0.80	20400	25500
68.4	16.84	14.73	134700	168400	3.84	1.81	0.64	0 63	14500	18100
67.0	16.60	15.90	132800	166000	3.20	1.70	0.58	0.70	13600	17000
0.0	28.00	10.82	224000	280000	7.79	2.93	0.61	0.84	23440	29300
8.6	25.72	11.59	205760	257200	5.26	2.35	0.47	0.76	18800	23500
0.5	20.10	10.92	161000	201000	3.02	1.31	0.33	0.626	10500	13100
7.4	19.47	12.98	155760	194700	3.51	1.78	0.47	0 66	14240	17800
3.2	14.63	12.84	117040	146300	1.97	1.15	0.35	0.53	9200	11500
4.0	12.80	13.33	102400	128000	2.20	1.14	0.46	0.565	9100	11400
8.7	24.16	9.97	193300	241600	7.30	2.75	0.67	0.91	22000	27500
9.1	19.80	9.92	158400	198000	4.25	2.02	0.47	0.73	16160	20200
2.1	18.24	11.73	146000	182400	5.35	2.35	0.76	0.85	18800	23500
8.8	13.07	11.76	104600	130700	2.53	1.35	0.51	0.63	10800	13500
2.0	9.33	11.42	74640	93300	1.52	0.99	0.41	0.55	7920	9900
4.5	16.14	7.67	129100	161400	4.00	1.98	0.48	0.73	15840	19800
0.1	15.03	8.12	120200	150300	3.53	1.69	0.48	0.755	13560	16900
8.4	12.10	7.81	97000	121000	1.94	0.98	0.31	0.584	7840	9800
4.5	11.12	9.89	89000	111200	2.54	1.46	0.56	0.76	11700	14600
8.2	7.06	9.55	56480	70600	1.06	0.71	0.36	0.50	5680	7100
7.9	10.83	6.30	86640	108300	2.85	1.53	0.48	0.68	12240	15300
5.6	7.31	5.57	58500	73100	1.11	0.62	0.24	0.51	4960	6200
7.1	7.74	7.53	62000	77400	1.96	1.10	0.54	0.715	8800	11000
7.6	5.03	7.05	40240	50300	0.75	0.49	0.30	0.470	3920	4900
8.0	9.33	4.24	74600	93300	2.95	1.43	0.45	0.788	11440	14300
3.4	7.80	4.33	62400	78000	1.84	0.95	0.34	0.662	7600	9500
1.7	7.23	4.82	58000	72360	2.12	1.20	0.47	0.725	9600	12000
7.2	5.73	5.21	46000	57300	1.30	0.80	0.39	0.630	6400	8000
0.4	3.47	4.74	27760	34700	0.62	0.46	0.28	0.400	3680	4600
3.4	5.34	3.17	42720	53400	1.50	0.94	0.36	0.640	7520	9400
0.3	4.12	3.03	33000	41200	0.63	0.44	0.19	0.493	3520	4400
9.5	3.80	2.73								

| | | |

XXII.

EVEN-LEGGED ANGLES.

THIS TABLE, SEE TABLE XIX.)

Axis Parallel to One Side. M—N						Axis at 45°. M—N	
Inertia. (*i*)	Moment of Resistance. (*r*)	Square of Radius of Gyration. (p^2)	Distance of Centre of Gravity from Base.	Transverse Value (*v*) in lbs.		Moment of Inertia. (*i*)	Square of Radius of Gyration. (p^2)
				For Iron.	For Steel.		
46	7.54	3.24	1.860	60350	75400	15	1.37
91	7.01	3.28	1.820	56100	70100	13.10	1.34
91	4.61	3.46	1.685	36900	46100	7.75	1.35
22	3.90	3.42	1.580	31200	39000	6.77	1.35
64	4.97	2.19	1.610	39760	49700	8.67	.96
70	3.94	2.37	1.550	31550	39400	6.07	.98
35	2.64	2.52	1.460	21100	26400	3.77	1.02
20	3.28	1.80	1.396	26250	32800	4.88	.772
20	2.24	1.92	1.286	18000	22400	2.65	.707
66	2.47	1.40	1.271	19800	24700	3.45	.634
18	2.32	1.39	1.220	18550	23200	3.01	.624
36	1.52	1.52	1.138	12200	15200	1.86	.650
38	2.40	1.21	1.220	19200	24000	2.40	.470
58	1.74	1.08	1.122	13920	17400	2.04	.470
36	1.15	1.15	1.013	9200	11500	1.20	.484
30	.90	1.12	.930	7200	9000	.95	462
77	1.13	.76	.996	9040	11300	1.05	338
23	.96	.79	.930	7680	9600	.95	.336
24	.58	.86	.842	4640	5800	.52	.361
83	.79	.66	.887	6320	7900	.61	.300
15	.59	.71	.802	4720	5900	.50	.309
05	.48	.72	.780	3840	4800	.39	.303
22	.61	.52	.770	4880	6100	.52	.221
28	.66	.57	.806	5280	6600	.51	.227
70	.39	.59	.717	3120	3900	.30	.252
52	.34	.59	.700	2720	3400	.25	.240
32	.46	.45	.740	3680	4600	.35	.194
78	.45	.44	.720	3600	4500	.35	.197
50	.31	.47	.654	2480	3100	.22	.208
40	.26	.50	.690	2080	2600	.17	.212
78	.51	.38	.730	4080	5100	.28	.150
45	.30	.33	.634	2400	3000	.21	.154
81	.22	.33	.580	1765	2200	.13	.158
27	.19	.38	.570	1520	1900	.11	.160
48	.35	.31	.640	2800	3500	.18	.120
27	.20	.27	.550	1600	2000	.12	.120
18	.15	.29	.507	1200	1500	.08	.129
.9	.17	.19	.510	1360	1700	.09	.096
.6	.14	.19	.487	1120	1400	.07	.083
.4	.12	.19	.450	960	1200	.06	.081
.1	.10	.21	.444	800	1000	.05	.094
09	.085	.21	.440	680	850	.04	.081
.23	.126	.152	.460	1010	1260	.05	.070
077	.079	.138	.404	632	790	.04	.071
044	.050	.147	.358	400	500	.02	.067
037	.047	.084	.340	376	470	.02	.045
030	.040	.084	.310	320	400	.01	.040
022	.031	.096	.296	250	310	.01	.043
019	.029	.066	.286	232	290	..	..
014	.023	.070	.264	184	230	..	..
012	.021	.048	.254	168	210	..	..
009	.017	.053	.233	136	170	..	..

Table XXII.

LIST OF IRON AND STEEL EVEN-LEGGED ANGLES.

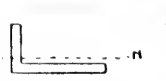
(For Information as to Use of this Table, see Table XIX.)

Mills Rolling Shape.	Size of Angle.	Weight per Yard.	Actual Length of Legs.	Thickness.	Area of each Leg.	Total Area. (a)	Axis Parallel to One Side. M—N						Axis at 45°. M—N	
							Moment of Inertia (I)	Moment of Resistance (r)	Square of Radius of Gyration (p^2)	Distance of Centre of Gravity from Base.	Transverse Value (c) in lbs. For Iron.	Transverse Value (c) in lbs. For Steel.	Moment of Inertia. (I)	Square of Radius of Gyration (p^2)
C,E.	6 ×6	110	6.562	1	5.50	11.	35.46	7.54	3.24	1.860	60350	75400	15	1.37
A,B,C,D,E.	6 ×6	97.3	6.375	.875	4.865	9.73	31.91	7.01	3.28	1.820	56100	70100	13.10	1 34
A,B,C,D,E,F.	6 ×6	57.5	6	.500	2.875	5.75	19.91	4.61	3.46	1.685	36900	46100	7.75	1.35
B,C	6 ×6	50.3	6	.438	2.515	5.03	17.22	3.90	3.42	1.580	31200	39000	6.77	1.35
C	5 ×5	90	5.562	1	4.50	9	19.64	4.97	2.19	1.610	39760	49700	8.67	.96
B,C,F	5 ×5	62	5.282	.688	3.10	6.20	14.70	3.94	2.37	1.550	31550	39400	6.07	.98
B,I	5 ×5	37	5.	.406	1.85	3.70	9.35	2.64	2.52	1.460	21100	26400	3.77	1.02
A	4½ ×4½	61.9	4.812	.750	3.095	6 19	11.20	3.28	1.80	1.396	26250	32800	4.88	.772
A	4½ ×4½	37.5	1.500	.438	1.875	3.75	7 20	2.24	1.92	1.286	18000	22400	2.65	.707
A,C,E,F	4 ×4	54.4	4.375	.750	2.72	5.44	7 66	2.47	1.40	1.271	19800	24700	3.45	.634
B,A,C,E,F	4 ×4	51.6	1.313	.688	2.58	5.16	7.18	2.32	1.39	1.220	18550	23200	3.01	.624
A,B,C,D,E,F.	4 ×4	28.6	4	.375	1.43	2.86	4.36	1 52	1.52	1.138	12200	15200	1.86	.650
E	3½ ×3½	51	3.875	.750	2.55	5.10	6.38	2.40	1.21	1.220	19200	24000	2.40	.470
A,B,C,E,F	3½ ×3½	43.4	3 813	.688	2.17	4 34	4.68	1.74	1.08	1.122	13920	17400	2.04	.470
A,B,C,D,E,F.	3½ ×3½	24 8	3.500	.375	1.21	2 48	2.86	1.15	1.15	1.013	9200	11500	1.20	.484
B	3½ ×3½	20.5	3.500	.313	1.025	2 05	2.30	.90	1.12	.930	7200	9000	.95	462
A,C,E,F	3 ×3	36.5	3.438	.688	1.825	3.65	2.77	1.13	.76	.996	9040	11300	1.05	338
B,A,C,D,E,F.	3 ×3	28.1	3.250	.500	1.405	2.81	2 23	.96	.79	.930	7680	9600	.95	.336
A,B,C	3 ×3	11.1	3	.250	.72	1.44	1.24	.58	.86	.842	4640	5800	.52	.361
A,B,C,D,E	2½ ×2½	27.7	3	.563	1.385	2.77	1.83	.79	.66	.887	6320	7900	.61	.300
A,B,C,E	2½ ×2½	16.2	2.750	.313	.81	1.62	1 15	.59	.71	.802	4720	5900	.50	.309
C,B	2½ ×2½	13.1	2.750	.250	.655	1.31	.95	.48	.72	.780	3840	4800	.39	.303
B,E,F	2½ ×2½	23.6	2.781	.500	1.18	2.36	1 22	.61	.52	.770	4880	6100	.52	.221
A,B,C,D,E,F.	2½ ×2½	22.5	2.750	.500	1.125	2.25	1.28	.66	.57	.806	5280	6600	.51	.227
A,B,C,D,E,F.	2½ ×2½	11.9	2.500	.250	.595	1.19	.70	.39	.59	.717	3120	3900	.30	.252
B	2½ ×2½	10.5	2.500	.219	.525	1.05	.62	.34	.59	.700	2720	3400	.25	.240
B,E,F	2½ ×2½	18.3	2.500	.438	.915	1.83	.82	.46	.45	.740	3680	4600	.35	.194
A,B,C,D,E,F.	2½ ×2½	17.8	2.438	.438	.89	1.78	.78	.45	.44	.720	3600	4500	.35	.197
A,B,C,D,E,F.	2½ ×2½	10.6	2.250	.250	.53	1.06	.50	.31	.47	.654	2480	3100	.22	.208
B	2½ ×2½	8	2.250	.188	.40	.80	.40	.26	.50	.690	2080	2600	.17	.212
F	2½ ×2½	19.2	2.250	.500	.96	1.92	.78	.51	.38	.730	4080	5100	.28	.150
A,B C,D,E,F.	2 ×2	13.6	2.156	.375	.68	1.36	.45	.30	.33	.634	2400	3000	.21	.154
A,B C,D,E,F.	2 ×2	9.4	2	.219	.47	.91	.31	.22	.33	.580	1765	2200	.13	.158
C,B	2 ×2	7.1	2	.188	.355	.71	.27	.19	.38	.570	1520	1900	.11	.160
E,F	1¾ ×1¾	15	2	.438	.75	1.50	.48	.35	.31	.640	2800	3500	.18	.120
A,B,C,D,E,F.	1¾ ×1¾	10	1.875	.313	.50	1	.27	.20	.27	.550	1600	2000	.12	.120
A,B,C,D,E,F.	1½ ×1½	6.21	1.750	.188	31	.62	.18	.15	.29	.507	1200	1500	.08	.129
C,E,F	1½ ×1½	9.80	1.688	.375	.49	.98	.19	.17	.19	.510	1360	1700	.09	.096
A,C,E,F	1¼ ×1¼	8.40	1.625	.313	.42	.84	.16	.14	.19	.487	1120	1400	.07	.083
B,A,C,E,F	1¼ ×1¼	7.10	1.594	.250	.355	.71	.14	.12	.19	.450	960	1200	.06	.081
A,B,C,D,E,F.	1¼ ×1¼	5.27	1.500	.188	.295	.53	.11	.10	.21	.444	800	1000	.05	.094
B,D	1¼ ×1¼	4.40	1.500	.156	.22	.44	.09	.085	.21	.440	680	850	.04	.081
F	1¼ ×1¼	7.50	1.438	.313	.375	.75	.123	.126	.152	.460	1010	1260	.05	.070
A,C,E,F	1¼ ×1¼	5.06	1.375	.250	.28	.56	.077	.079	.138	.404	632	790	.04	.071
A,B C,D,E,F	1¼ ×1¼	3	1.250	.125	.15	.30	.044	.050	.147	.358	400	500	.02	.067
A,C,F	1 ×1	4.40	1.125	.250	.22	.44	.047	.047	.084	.340	376	470	.02	.045
B,A,C,D,E,F.	1 ×1	3.60	1.063	.188	.18	.36	.030	.040	.084	.310	320	400	.01	.049
A,B,C,D,E,F.	1 ×1	2.50	1	.125	.115	.23	.022	.031	.096	.296	250	310	.01	.043
A,F	¾ ×¾	2.04	.938	.188	.345	.29	.019	.029	.066	.286	232	290	..	..
A F	¾ ×¾	2.03	.875	.125	.109	20	.014	.023	.070	.264	184	230	..	..
A,F	¾ ×¾	2 46	.813	.188	.125	.25	.012	.021	.048	.254	168	210	..	..
A,F	¾ ×¾	1.72	.750	.125	.085	.17	.009	.017	.053	.233	136	170	..	..

TABLE

AND STEEL GLES.

TION AS TO USE OF



Parallel to Short Leg	to Long Leg.	Moment of Resistance. (r')	Square of Radius of Gyration. (p^2)	Distance of Centre of Gravity from Back.	Transverse Value (r) in lbs. For Iron.	Transverse Value (r) in lbs. For Steel.	Moment of Inertia. (i)	Square of Radius of Gyration. (q^2)
9.72	4.80	2. .77	.96	20880	26100	6.70	0.71
6.82	4.88	2. .85	.82	15680	19600	4.45	0.72
8.25	4.08	2. 1.17	1.13	25680	32100	8.35	0.86
6.87	4.24	2. 1.19	1.03	21600	27000	6.75	0.90
4.07	4.33	2. 1.32	.91	14000	17500	3.60	0.88
7	3.42	2. 1.19	1.17	25360	31700	7.46	0.83
5.75	3.53	2. 1.25	1.08	21450	26800	5.72	0.83
3.83	3.70	1. 1.34	.96	14720	18400	3.55	0.85
3.40	3.72	2 1.37	.97	13200	16500	2.89	0.79
6.79	3.46	2. .85	1.01	18880	23600	5.75	0.66
4.92	3.57	2. .92	.90	14480	18100	3.78	0.67
3.24	3.72	2. 1	.82	10100	12600	2.11	0.62
4.09	2.99	1. .94	.91	14000	17500	3.35	0.64
2.75	3.13	1. 1.10	.82	9760	12200	2.14	0.66
4.68	2.28	1. 1.28	1.25	24000	30000	6.10	0.74
3.49	2.40	1. 1.37	1.11	18480	23100	3.93	0.74
2.33	2.53	1. 1.44	1.04	12400	15500	2.20	0.69
3.82	2.39	1. .93	.99	14880	18600	3.72	0.64
2.30	2.55	1. 1.05	.86	9680	12100	1.96	0.64
2.03	2.59	1. 1.06	.79	8560	10700	1 94	0.71
3.72	2.40	1. .64	.84	11040	13800	2.58	0.48
3.32	2.43	1. .66	.81	10000	12500	2.11	0.45
1.88	2.56	1. .72	.74	6160	7700	.97	0.41
3.81	2.10	1. .77	.94	14800	18500	2.54	0.48
2.66	1.96	1. .67	.83	9600	12000	2.45	0.48
1.83	2.05	1. .74	.74	7040	8800	1.25	0.47
3.08	1.66	1. 1.21	1.16	20000	25000	2.80	0.57
1.99	1.49	1. 1.06	.99	12480	15600	2.29	0.58
1.43	1.56	1. 1.10	.89	9120	11400	1.61	0.61
3	1.69	1. .86	.99	14400	18000	2.05	0.40
1.97	1 54	1. .74	.86	9120	11400	1.56	0.42
1.23	1.61	1. .78	.76	5856	7320	.77	0.37
2.40	1.28	1. .88	1.04	14400	18000	1.86	0.39
1.48	1.14	1. .76	.88	8640	10800	1.35	0.40
.74	1.23	1. .83	.79	4720	5900	.61	0.39
1.35	1.21	1. .52	.72	5760	7200	.80	0.28
.79	1.25	1. .53	.62	3440	4300	.43	0.29
.69	1.26	1. .14	.32	1150	1440	.16	0.134
1.30	1.14	1. .36	.61	4480	5600	.80	0.31
.63	1.08	1. .32	.48	2080	2600	.41	0.32
1.02	.84	1. .52	.77	5680	7100	.91	0.33
.56	.89	. .57	.66	3220	4020	.36	0.28
.87	.86	1. .28	.58	2920	3650	.47	0.21
.73	.86	1. .31	.53	2800	3500	.39	0.22
.48	.93	. .24	.48	1840	2300	.24	0.23
.58	.55	. .31	.62	3100	3860	.37	0.18
.41	.66	. .35	.54	2030	2530	.20	0.18
.34	.40	. .16	.44	1350	1680	.15	0.12
.23	.50	. .18	.38	920	1150	.08	0.12
.36	.42	. .17	.44	1520	1900	.11	0.095
.23	.45	. .15	.37	960	1200	.08	0.106
.21	.35	. .17	.32	712	890	.08	0.084
.16	.40	. .15	.31	600	750	.05	0.096

TABLE XXIII.

LIST OF IRON AND STEEL UNEVEN-LEGGED ANGLES.

(FOR INFORMATION AS TO USE OF THIS TABLE, SEE TABLE XIX.)

Mills Rolling Shape.	Size of Angle.	Weight per Yard.	Thickness of Legs.	Actual Length of Long Leg.	Actual Length of Short Leg.	Allowed Long Leg.	Allowed Short Leg.	Total Area. (a)	Axis Parallel to Short Leg.					Axis Parallel to Long Leg.					Axis Parallel to A-B.	
									Moment of Inertia.	Modulus of Section.	Value of Radius of Gyration. (r)	Distance of Centre of Gravity from Edge.	Transverse Value of ab in lbs. For Iron.	For Steel.	Moment of Inertia.	Modulus of Section.	Value of Radius of Gyration. (r)	Distance of Centre of Gravity from Edge.	Transverse Value of ab in lbs. For Iron.	For Steel.	Moment of Inertia.	Value of Radius of Gyration. (r)
	7 × 3½	16	1	7.38	3.88	6.50	3	9.50	15.37	4.02	1.80	2.71	77760	972000	5.73	2.41	.77	.96	20880	261000	6.70	0.74
C	7 × 3½	14	7/8	7.38	3.88	6.50	3	8.25	13.42	3.62	1.80	2.73	68040	850500	5.03	2.13	.78	.98	18270	228375	5.87	0.74
C	7 × 3½	12	3/4	7.38	3.88	6.50	3	7.00	11.47	3.09	1.80	2.75	58320	729000	4.31	1.82	.79	.99	15660	195750	5.03	0.74
C	7 × 3½	10	5/8	7.38	3.88	6.50	3	5.88	9.52	2.55	1.80	2.77	48600	607500	3.56	1.50	.78	.98	13050	163125	4.16	0.74
C	7 × 3½	8	1/2	7.38	3.88	6.50	3	4.75	7.58	2.01	1.80	2.78	38880	486000	2.81	1.19	.77	.97	10440	130500	3.30	0.74
C	7 × 3½	7	7/16	7.38	3.88	6.50	3	4.19	6.63	1.75	1.80	2.79	34020	425250	2.45	1.03	.76	.96	9135	114187	2.87	0.74
C	7 × 3½	6	3/8	7.38	3.88	6.50	3	3.63	5.74	1.48	1.80	2.80	29160	364500	2.08	.87	.76	.96	7830	97875	2.43	0.74
C	7 × 3½	5	5/16	7.38	3.88	6.50	3	3.06	4.84	1.21	1.80	2.81	24300	303750	1.71	.71	.75	.95	6525	81562	1.99	0.74
C	7 × 3½	4	1/4	7.38	3.88	6.50	3	2.44	3.85	.94	1.80	2.82	19440	243000	1.33	.55	.74	.94	5220	65250	1.55	0.74
A.B.C.	6 × 4	14	7/8	6.06	4.14	5.34	3.72	8.25	11.41	3.62	1.57	2.33	59400	742500	5.03	2.13	.78	.98	18270	228375	5.87	0.90
A.B.C.	6 × 4	12	3/4	6.06	4.14	5.34	3.72	7.00	9.72	3.09	1.57	2.35	51840	648000	4.31	1.82	.79	.99	15660	195750	5.03	0.90
A.B.C.	6 × 4	10	5/8	6.06	4.14	5.34	3.72	5.88	8.07	2.55	1.57	2.37	43200	540000	3.56	1.50	.78	.98	13050	163125	4.16	0.90
A.B.C.	6 × 4	8	1/2	6.06	4.14	5.34	3.72	4.75	6.42	2.01	1.57	2.38	34560	432000	2.81	1.19	.77	.97	10440	130500	3.30	0.90
A.B.C.	6 × 4	7	7/16	6.06	4.14	5.34	3.72	4.19	5.62	1.75	1.57	2.39	30240	378000	2.45	1.03	.76	.96	9135	114187	2.87	0.90
A.B.C.	6 × 4	6	3/8	6.06	4.14	5.34	3.72	3.63	4.86	1.48	1.57	2.40	25920	324000	2.08	.87	.76	.96	7830	97875	2.43	0.90
A.B.C.	6 × 4	5	5/16	6.06	4.14	5.34	3.72	3.06	4.10	1.21	1.57	2.41	21600	270000	1.71	.71	.75	.95	6525	81562	1.99	0.90
A.B.C.	6 × 4	4	1/4	6.06	4.14	5.34	3.72	2.44	3.26	.94	1.57	2.42	17280	216000	1.33	.55	.74	.94	5220	65250	1.55	0.90
A.B.C.D.E.F.	6 × 4	14	7/8	6.06	4.14	5.34	3.72	8.25	11.41	3.62	1.57	2.33	59400	742500	5.03	2.13	.78	.98	18270	228375	5.87	0.90
A.B.C.D.E.F.	6 × 4	12	3/4	6.06	4.14	5.34	3.72	7.00	9.72	3.09	1.57	2.35	51840	648000	4.31	1.82	.79	.99	15660	195750	5.03	0.90
A.B.C.D.E.F.	6 × 4	10	5/8	6.06	4.14	5.34	3.72	5.88	8.07	2.55	1.57	2.37	43200	540000	3.56	1.50	.78	.98	13050	163125	4.16	0.90
A.B.C.D.E.F.	6 × 4	8	1/2	6.06	4.14	5.34	3.72	4.75	6.42	2.01	1.57	2.38	34560	432000	2.81	1.19	.77	.97	10440	130500	3.30	0.90
A.B.C.D.E.F.	6 × 4	7	7/16	6.06	4.14	5.34	3.72	4.19	5.62	1.75	1.57	2.39	30240	378000	2.45	1.03	.76	.96	9135	114187	2.87	0.90
A.B.C.D.E.F.	6 × 4	6	3/8	6.06	4.14	5.34	3.72	3.63	4.86	1.48	1.57	2.40	25920	324000	2.08	.87	.76	.96	7830	97875	2.43	0.90
A.B.C.D.E.F.	6 × 4	5	5/16	6.06	4.14	5.34	3.72	3.06	4.10	1.21	1.57	2.41	21600	270000	1.71	.71	.75	.95	6525	81562	1.99	0.90
A.B.C.D.E.F.	6 × 4	4	1/4	6.06	4.14	5.34	3.72	2.44	3.26	.94	1.57	2.42	17280	216000	1.33	.55	.74	.94	5220	65250	1.55	0.90
A.B.C.D.E.F.	6 × 4	14	7/8	6.06	4.14	5.34	3.72	8.25	11.41	3.62	1.57	2.33	59400	742500	5.03	2.13	.78	.98	18270	228375	5.87	0.90
A.B.C.D.E.F.	6 × 4	12	3/4	6.06	4.14	5.34	3.72	7.00	9.72	3.09	1.57	2.35	51840	648000	4.31	1.82	.79	.99	15660	195750	5.03	0.90
A.B.C.D.E.F.	6 × 4	10	5/8	6.06	4.14	5.34	3.72	5.88	8.07	2.55	1.57	2.37	43200	540000	3.56	1.50	.78	.98	13050	163125	4.16	0.90
A.B.C.D.E.F.	6 × 4	8	1/2	6.06	4.14	5.34	3.72	4.75	6.42	2.01	1.57	2.38	34560	432000	2.81	1.19	.77	.97	10440	130500	3.30	0.90
A.B.C.D.E.F.	6 × 4	7	7/16	6.06	4.14	5.34	3.72	4.19	5.62	1.75	1.57	2.39	30240	378000	2.45	1.03	.76	.96	9135	114187	2.87	0.90
A.B.C.D.E.F.	6 × 4	6	3/8	6.06	4.14	5.34	3.72	3.63	4.86	1.48	1.57	2.40	25920	324000	2.08	.87	.76	.96	7830	97875	2.43	0.90
A.B.C.D.E.F.	6 × 4	5	5/16	6.06	4.14	5.34	3.72	3.06	4.10	1.21	1.57	2.41	21600	270000	1.71	.71	.75	.95	6525	81562	1.99	0.90
A.B.C.D.E.F.	6 × 4	4	1/4	6.06	4.14	5.34	3.72	2.44	3.26	.94	1.57	2.42	17280	216000	1.33	.55	.74	.94	5220	65250	1.55	0.90

XXIV.

AND STEEL TEES.

E OF THIS TABLE, SEE TABLE XIX.)

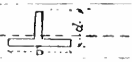
to Web. 					Axis Parallel to Web.				
Square of Radius of Gyration. (g)	Distance of Centre of Gravity from Base.	Transverse Value (r) in lbs.			Moment of Inertia. (i)	Moment of Resistance. (r)	Square of Radius of Gyration. (gg')	Transverse Value (r) in lbs.	
		For Iron.	For Steel.					For Iron.	For Steel.
1.32	.99	16560	20700		9.04	3.01	1.99	24080	30100
1.12	1.08	16640	20800		5.25	2.10	1.19	16800	21000
1.10	1.05	17360	21700		5.31	2.12	1.08	16960	21200
.64	.73	8800	11000		5.70	2.30	1.46	18400	23000
.62	.77	8880	11100		5.24	2.10	1.49	16800	21000
.43	.61	6400	8000		5.23	2.09	1.49	16720	20900
.44	.58	5680	7100		4.60	1.80	1.46	14400	18000
.48	.66	6480	8100		3.94	1.58	1.37	12640	15800
1.14	1.13	17440	21800		3.90	1.70	.86	13600	17000
.77	.76	6880	8600		2.39	1.06	.96	8480	10600
2.46	1.57	24400	30500		2.70	1.40	.64	11206	14000
1.93	1.37	19840	24800		2.70	1.40	.67	11200	14000
1.48	1.18	15760	19700		2.62	1.31	.70	10480	13100
1.10	1.09	15520	19400		3.23	1.61	.77	12880	16100
.76	.80	7680	9600		2.30	1.10	.81	8800	11000
.49	.62	4800	6000		2.00	1	.86	8000	10000
.28	.46	2800	3500		1.80	.91	.92	7280	9100
1.51	1.24	14960	18700		1.80	1	.52	8000	10000
1.12	1.06	11920	14900		1.82	1.04	.56	8320	10400
1.14	1.03	10560	13200		1.53	.87	.53	6960	8700
.77	.85	8000	10000		1.60	.93	.59	7440	9300
1.54	1.35	16800	21000		1.30	.87	.36	6960	8700
1.12	1.15	13360	16700		1.40	.92	.38	7360	9200
1.10	1.10	10000	12500		1.06	.71	.37	5680	7100
.80	.93	8560	10700		1.15	.77	.42	6160	7700
.83	.89	6670	8340		.97	.65	.46	5200	6500
.82	.84	6080	7600		.75	.50	.38	4000	5000
.58	.82	6800	8500		.94	.63	.40	5040	6300
.52	.70	4800	6000		.89	.60	.44	4800	6000
.53	.69	4160	5200		.74	.49	.42	3920	4900
.31	.54	2960	3700		.85	.567	.49	1540	5670
.32	.52	2590	3240		.68	.153	.47	3630	4530
.12	.40	1600	2000		.57	.455	.38	3640	4550
.17	.37	1360	1700		.56	.372	.50	2976	3720
.40	.75	5280	6600		.63	.460	.30	3680	4600
.30	.66	4080	5100		.62	.450	.34	3600	4500
.86	.96	6480	8100		.50	.400	.26	3200	4000
.71	.84	5280	6600		.50	.400	.28	3200	4000
.61	.75	6000	7500		.58	.460	.30	3680	4600
.57	.76	4500	5630		.49	.390	.28	3120	3900
.58	.74	3860	4830		.40	.320	.27	2560	3200
.10	.30	800	1000		.33	.260	.37	2080	2600
.47	.66	2820	3520		.26	.231	.22	1850	2310

TABLE XXIV.

LIST OF IRON AND STEEL TEES.

(FOR INFORMATION AS TO THE USE OF THIS TABLE, SEE TABLE XIX.)

Mills Rolling Shape.	Width of Flange (f)	Depth over all (d)	Weight per Yard	Thickness of Flange	Area of Flange	Area of Web.	Total Area (a)	Axis Normal to Web. Moment of Inertia (I)	Moment of Resistance (R)	Square of Radius of Gyration (P)	Distance of Centre of Gravity from Inside (Y)	Transverse Value (r) in lbs. For Iron	Transverse Value (r) in lbs. For Steel	Axis Parallel to Web. Moment of Inertia (I)	Moment of Resistance (R)	Square of Radius of Gyration (P)	Transverse Value (r) in lbs. For Iron	Transverse Value (r) in lbs. For Steel
C	5	4	17	0.500	3	1.7	4.75	6.25	2.97	1.32	99	16560	20700	9.04	3.01	1.99	24080	30100
C	5	4	14	0.500	2.50	1.91	4.41	6.24	2.08	1.42	1.08	16640	20800	5.25	2.10	1.19	16800	21000
C	5	4	18	0.500	2.50	2.5	4.85	5.37	2.17	1.10	1.05	17360	21700	5.31	2.12	1.08	16960	21200
D F	5	3	17	0.500	2.80	1.1	3.90	2.50	1.10	1.4	73	8800	11000	5.70	2.30	1.46	18400	23000
B	5	2½	15	0.500	2.53	1.97	3.50	2.21	1.11	1.62	77	8880	11100	5.24	2.10	1.19	16800	21000
A C E F	5	2½	15	0.500	2.50	1	3.50	1.60	1.80	1.4	61	6400	8000	5.23	2.09	1.19	16720	20900
E C F	5	2½	20	0.500	2.67	1.01	3.68	1.40	1.71	1.4	58	5680	7100	4.60	1.80	1.46	14400	18000
B	5	2½	2	0.375	1.88	1.02	2.90	1.39	1.81	1.18	66	6480	8100	3.94	1.58	1.37	12640	15800
E C	4½	3½	15	0.500	2.25	2.25	4.50	7.20	2.18	1.11	1.14	17440	21800	3.90	1.70	.86	13600	17000
B	4½	3½	2	0.315	1.43	1.03	2.50	1.91	.86	.77	76	6880	8600	2.59	1.06	.96	8480	10600
E	4	4	12	0.625	2	2.20	4.20	10.50	3.05	2.46	1.57	24400	30500	2.70	1.40	.64	11200	14000
E C	4	4½	10	0.625	2	2.05	4.05	7.80	2.18	1.93	1.37	19800	24800	2.70	1.40	.67	11200	14000
A C E I	4	4	17	0.500	2	1.75	3.75	5.56	1.97	1.18	1.18	15760	19700	2.62	1.31	.70	10480	13100
E	4	4	17	0.438	2.25	1.5	4.18	4.65	1.94	1.10	1.09	15520	19400	2.23	1.61	.77	12880	16100
E C D	4	4	27	0.470	1.75	1.0	2.75	2.40	.96	.76	80	7680	9600	2.30	1.10	.81	8800	11000
E	4	2½	22	0.438	1.50	1.5	2.25	1.10	.90	.44	1.2	4800	6000	2.00	1	.86	8000	10000
E B C F	4	2	17	0.375	1.37	1.5	1.75	.54	.5	.28	46	2800	3500	1.80	.91	.92	7280	9100
E	3½	4	15	0.500	1.70	1.68	3.38	5.15	1.87	1.51	1.31	14960	18700	1.80	1	.52	8000	10000
A C E F	3½	3	12	0.500	1.75	1.00	2.25	3.64	1.19	1.12	1.06	11920	14900	1.82	1.01	.56	8320	10400
A B D E F	3½	3	28	0.438	1	1.31	2.87	.9	1.32	1.11	1.05	10560	13200	1.53	.87	.53	6960	8700
E D	3½	3	27	0.500	1.64	1.11	2.75	2.11	1	.77	85	8000	10000	1.60	.93	.59	7440	9300
E I	3	4	30	0.625	1.69	1.99	3.68	3.35	2.10	1.34	1.35	16800	21000	1.30	.87	.36	6960	8700
E	3	3½	30	0.500	1.82	1.71	3.5	3.15	1.67	1.12	1.15	13360	16700	1.40	.92	.38	7360	9200
E	3	3½	28	0.500	1.41	1.12	2.85	3.12	1.25	1.10	1.10	10000	12500	1.06	.71	.37	5680	7100
A C E I	3	3	27	0.500	1.50	1.25	2.75	2.1	1.07	.90	95	8560	10700	1.15	.77	.42	6160	7700
A B C D E F	3	3	21	0.375	1.13	.95	2.11	1.76	.73	.55	89	6670	8340	.97	.65	.46	5200	6500
C I	3	3	19	0.313	1.03	.90	1.93	1.53	.70	.52	84	6080	7600	.95	.50	.38	4000	5000
E	3	2½	25	0.438	1.20	1.18	2.38	1.38	.8	.58	82	6800	8500	.94	.63	.40	5040	6300
C E	3	2½	20	0.438	1.21	.85	2.06	1.08	.60	.52	70	4800	6000	.89	.60	.44	4800	6000
C F	3	2½	17	0.375	1.02	.71	1.76	.94	.52	.53	69	4160	5200	.74	.49	.42	3920	4900
A I	3	2½	17	0.375	1.12	.61	1.73	.44	.37	.31	71	2960	3700	.85	.67	.49	4540	5670
A	3	2	11	0.313	.94	.52	1.46	.17	.32	.32	62	2560	3240	.68	.153	.47	3630	4540
E	3	1½	15	0.375	1.15	.38	1.51	.22	.20	.12	40	1600	2000	.57	.155	.38	3640	4550
E	3	1½	11	0.250	.75	.37	1.12	.19	.17	.17	37	1360	1700	.56	.172	.40	2976	3720
C E	2½	2	21	0.344	.90	1.20	2.10	.83	.66	.70	75	5280	6600	.63	.460	.30	3680	4600
C E	2½	1½	18	0.313	.90	.98	1.88	.56	.51	.30	66	4080	5100	.62	.450	.54	3600	4500
E	2½	2	11	0.440	.97	.98	1.95	1.66	.81	.86	96	6480	8100	.50	.400	.26	3200	4000
E C	2½	2	18	0.375	.94	.86	1.80	1.26	.66	.71	81	5280	6600	.50	.400	.28	3200	4000
C I	2½	2½	19	0.467	1.09	.86	1.95	1.12	.75	.61	75	6000	7500	.48	.460	.30	3680	4600
A B C E F	2½	2½	17	0.375	.93	.80	1.73	.98	.56	.57	76	4500	5630	.49	.400	.28	3120	3900
A F	2½	2	17	0.313	.78	.63	1.47	.85	.48	.58	74	3860	4830	.40	.320	.27	2560	3200
E C	2½	1½	9	0.313	.65	.25	.90	.09	.10	.10	30	800	1000	.35	.360	.37	2080	2600
A C E	2½	1½	11	0.281	.65	.36	1.19	.36	.35	.17	66	2820	3520	.26	.231	.22	1840	2310
E	2½	1½	8	0.250	.62	.25	.81	.09	.097	.01	32	776	970	.24	.214	.30	1712	2140
A	2½	1½	7	0.250	.56	.22	.78	.06	.072	.077	29	580	720	.18	.160	.23	1280	1600
E	2½	1	6	0.250	.50	.03	.65	.01	.026	.011	18	2080	2600	.24	.213	.37	1704	2150
E	2	1½	9	0.188	.59	.26	.65	.81	1.060	1.50	37	8480	10600	.15	.140	.025	1120	1400
A C I	2	2	11	0.313	.65	.32	1.15	.11	.092	.092	41	2420	3020	.21	.210	.182	1680	2100
A B D E I	2	2	9	0.250	.50	.11	.91	.35	.247	.172	53	1980	2470	.16	.160	.170	1280	1600
A C E	2	1½	9	0.281	.56	.31	.90	.17	.170	.190	50	1360	1700	.18	.180	.200	1440	1800
A	2	1	6	0.250	.50	.19	.65	.04	.051	.055	6	1440	540	.14	.140	.203	1120	1400
E	2	1	5	0.250	.50	.09	.59	.01	.026	.017	17	2080	2600	.17	.170	.292	1360	1700
E	1	1½	8	0.250	.43	.38	.82	.235	.192	.232	32	1536	1920	.14	.125	.157	1000	1250
C F	1	1	7	0.250	.39	.32	.71	.210	.168	.232	30	1344	1680	.10	.114	.137	912	1140
A C D E I	1½	1½	6	0.250	.38	.31	.69	.150	.125	.188	46	1000	1250	.07	.093	.101	745	930
A	1½	1	5	0.250	.37	.19	.56	.040	.055	.071	28	440	550	.07	.093	.125	745	930
A	1½	1	5	0.250	.31	.25	.56	.070	.089	.136	46	710	890	.042	.067	.075	535	670
A C E F	1½	1½	4	0.250	.27	.22	.49	.065	.075	.135	38	600	750	.034	.054	.070	430	540
A C F	1	1	3	0.188	.3	1	.33	.030	.044	.091	31	350	440	.016	.072	.048	255	320
E	1	1	2	0.150	.1	.5	.25	.024	.034	.085	295	270	340	.012	.024	.013	190	240
E	1	1	2	0.125	.12	.11	.23	.022	.034	.096	295	248	310	.011	.022	.005	176	220

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